Low-Energy Limit of the Photodisintegration of the Deuteron

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The exact transition amplitude for the photodisintegration of the deuteron at the zero-energy limit of the incident γ ray is presented as a function of the electric charges and the magnetic moments of the proton, neutron, and deuteron, the effective range for triplet n-p scattering, and the binding energy of the deuteron. The method used in this article is based on the theory of composite particles in quantum theory which has been developed by Nishijima, Zimmermann, and Haag. The low-energy limit presented here is the generalization of the Kroll-Ruderman theorem of pion photoproduction to the problem, including a composite particle.

I. INTRODUCTION

DISPERSION relations have been applied to the photodisintegration of the deuteron by assuming the analyticity of the transition amplitude as a function of energy for a fixed difference of squares of the momentum transfers of the two final nucleons.[1] The dispersion relations of electric and magnetic dipole amplitudes thus obtained have a certain low-energy limit. Therefore, it is desirable to prove the low-energy limit theorem from another standpoint in order to lend support to these dispersion relations. In proving the low-energy limit theorem, we use the theory of the composite particle developed by Nishijima, Zimmerman, and Haag.[2] As a consequence of this theory, we can derive the low-energy limit of the photodisintegration of the deuteron for both of the electric and magnetic dipole amplitudes, so that we can compare the theory of the composite particle with experiment.

The low-energy limit theorem for the Compton scattering by a spin-1/2 particle has been proved by several authors.[3] A remarkable point of this theorem is that the structure of the target particle contributes to the matrix element in the low-energy limit only through its magnetic moment. Therefore, we might have a hope that the transition amplitudes of the photodisintegration of the deuteron at the low-energy limit also can be expressed as a function of measurable quantities such as the electric charge and the magnetic moments.

As a preliminary, in Sec. II, we define the deuteron operator $B_\mu(x)$, which is a pseudovector, and we derive the integral representation of its one-body Green's function. The electromagnetic vertex function of the deuteron and np-d vertex function are also given.

The proof of the low-energy limit theorem is given in Sec. III, along the lines of the proof of the Kroll-Ruderman theorem.[4] The exact transition matrix element at the long-wavelength limit of the incident γ ray is given as a function of the electric charges and magnetic moments of the proton, neutron, deuteron, the effective range of the triplet n-p scattering, the ratio of the wave function of the deuteron in the ^3S state to the ^3D state at infinite distance, and the binding energy of the deuteron.

In the final section we remark briefly about the application of the low-energy limit theorem.

II. DEUTERON FIELD OPERATOR AND ITS GREEN'S FUNCTION AND VERTEX FUNCTIONS[5]

As a preparation to deriving the low-energy limit theorem, we define the deuteron field operator based on the theory of the composite particle and present the integral representation of the Green's function and the general forms of the γ-d and np-d vertex functions.

In order to describe the deuteron in a covariant formalism, it is convenient to consider the deuteron as a pseudovector particle whose four-vector spin function (or polarization vector) is orthogonal to the four-momentum vector of the deuteron

$$d_\nu U_\nu^{(i)}(\mathbf{d})=0, \tag{2.1}$$

where d_ν and $U_\nu^{(i)}(\mathbf{d})$ are the four-momentum vector and the spin function with spin direction i. The reason for describing the spin function as a pseudovector satisfying the above equation is that the deuteron is in a triplet even state. We normalize the spin function by

$$U_\nu^{(i)*}U_\nu^{(j)}=\delta_{ij}, \tag{2.2}$$

so that

$$\sum_{i=1}^{3} U_\mu^{(i)}(\mathbf{d})U_\nu^{(i)*}(\mathbf{d})=\delta_{\mu\nu}+\frac{d_\mu d_\nu}{M^2}, \tag{2.3}$$

where M is the mass of the deuteron.

Let $\psi_\alpha(x)$ be the nucleon field operator, in which α denotes the ordinary four-component spinor label and the two component isospinor label. In order to generate

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[1] B. Sakita and C. J. Goebel, preceding paper [Phys. Rev. **126**, 1787 (1962)].

[2] K. Nishijima, Phys. Rev. **111**, 995 (1958); W. Zimmermann, Nuovo cimento **10**, 597 (1958); R. Haag, Phys. Rev. **112**, 669 (1958).

[3] W. Thirring, Phil. Mag. **41**, 1193 (1950); F. E. Low, Phys. Rev. **96**, 1428 (1954), M. Gell-Mann and M. L. Goldberger, *ibid.* **96**, 1433 (1954); A. Klein, *ibid.* **99**, 998 (1955).

[4] N. M. Kroll and M. A. Ruderman, Phys. Rev. **93**, 233 (1954); A. Klein, Phys. Rev. **99**, 998 (1955).

[5] We use units of $\hbar=c=1$, $e^2/4\pi=1/137$, and take the scalar product of four-vectors as $a\cdot b=\mathbf{a}\cdot\mathbf{b}-a_0b_0$.

the deuteron field out of the nucleon field operators, we introduce the pseudovector field operator $B_\mu(x,\xi)$ defined by[6]

$$B_\mu(x,\xi)=\mathcal{O}_{\mu;\alpha\beta}(\partial_x)T(\psi_\alpha(x+\tfrac{1}{2}\xi)\psi_\beta(x-\tfrac{1}{2}\xi)), \quad (2.4)$$

where $\mathcal{O}_{\mu;\alpha\beta}(\partial_x)$ is given by

$$\mathcal{O}_\mu(\partial)=\mathcal{C}^{-1}(a\gamma_\mu+b\sigma_{\mu\nu}\partial_\nu+c\partial_\mu), \quad (2.5)$$

with constant a, b, c, and[7]

$$\mathcal{C}=i\tau_2C. \quad (2.6)$$

The constants a, b, and c in Eq. (2.6) are to be adjusted so that

$$\begin{aligned}\langle 0|B_\mu(x,\xi)|\mathbf{d},i\rangle&\neq 0,\\ \partial_\mu\langle 0|B_\mu(x,\xi)|\mathbf{d},i\rangle&=0,\end{aligned} \quad (2.7)$$

where $|\mathbf{d},i\rangle$ is a deuteron state with momentum $\mathbf{d}$ and spin direction i while $|0\rangle$ is the vacuum state. We shall assume that this adjustment is possible and the first equation of (2.7) is finite. We can easily demonstrate the usual translation relation

$$e^{-ip\cdot a}B_\mu(x,\xi)e^{i\cdot pa}=B_\mu(x+a,\,\xi). \quad (2.8)$$

In the limit $\xi\to 0$, B_μ behaves like a local pseudovector field operator under a Lorentz transformation.

Due to the Lorentz covariance, the first equation of (2.8) can be written as

$$\langle 0|B_\mu(x,\xi)|\mathbf{d},i\rangle=\frac{e^{id\cdot x}}{(2\pi)^{3/2}(2d_0)^{1/2}}f_\mu^{(i)}(\xi,d). \quad (2.9)$$

Using the relation (2.1) and the second equation of (2.7), we can see that

$$f_\mu^{(i)}(\xi,d)=U_\mu^{(i)}(\mathbf{d})f(\xi^2,\,\xi\cdot d), \quad (2.10)$$

where f is a function only of ξ^2 and $\xi\cdot d$ since $d^2=-M^2$. Thus

$$\lim_{\xi\to 0}f(\xi^2,\,\xi\cdot d)=f_0=\text{const.} \quad (2.11)$$

Let us define the deuteron field operator $B_\mu(x)$ by

$$\lim_{\xi\to 0}\frac{B_\mu(x,\xi)}{f_0}=B_\mu(x), \quad (2.12)$$

so that $B_\mu(x)$ satisfies

$$\langle 0|B_\mu(x)|\mathbf{d},i\rangle=(2\pi)^{-\frac{3}{2}}(2d_0)^{-\frac{1}{2}}e^{id\cdot x}U_\mu^{(i)}(\mathbf{d}). \quad (2.9)$$

Applying the weak reflection invariance[8] to Eq. (2.9), we obtain the condition that the constants a/f_0, b/f_0 and c/f_0 in Eq. (2.6) must be real.[9] This leads to the following equation for the charge conjugation of $B_\mu(x)$:

$$R_CB_\mu(x)R_C^{-1}=B_\mu^*(x). \quad (2.13)$$

For the strong reflection,[8] we obtain

$$R_SB_\mu(x)R_S^{-1}=-B_\mu(-x). \quad (2.14)$$

By the prescription of Nishijima and Zimmermann,[2] we can derive the following reduction formulae for the deuteron:

$$\langle\mathbf{d},i|T(\psi_A\psi_B\cdots)|0\rangle=\frac{iU_\mu^{(i)*}(\mathbf{d})}{(2\pi)^{3/2}(2d_0)^{1/2}}\int e^{-id\cdot x}dx\,K_x \times\langle 0|T(\psi_A\psi_B\cdots B_\mu(x))|0\rangle, \quad (2.15a)$$

$$\langle 0|T(\psi_A\psi_B\cdots)|\mathbf{d},i\rangle=\frac{iU_\mu^{(i)}(\mathbf{d})}{(2\pi)^{3/2}(2d_0)^{1/2}}\int e^{id\cdot x}dx\,K_x \times\langle 0|T(\psi_A\psi_B\cdots B_\mu^*(x))|0\rangle, \quad (2.15b)$$

where

$$K_x=M^2-\Box_x.$$

A. One-Body Green's Function of the Deuteron

The Green's function of the deuteron is defined by

$$G_{\rho\lambda}(x-y)=\langle T(B_\rho(x)B_\lambda^*(y))\rangle_0. \quad (2.16)$$

The method of the derivation of the integral representations of the Green's function given by Lehmann[10] can be applied to our problem if we use the relation (2.14) in order to relate the negative-frequency part of the Green's function to the positive-frequency part. Thus, we obtain

$$G_{\rho\lambda}(x)=\left(\delta_{\rho\lambda}-\frac{\partial_\rho\partial_\lambda}{M^2}\right)\Delta_F(x;\,M^2)+\int_{(2m)^2}^{\infty}d\kappa^2 \times\left(\sigma_1(\kappa^2)\delta_{\rho\lambda}-\sigma_2(\kappa^2)\frac{\partial_\rho\partial_\lambda}{M^2}\right)\Delta_F(x;\,\kappa^2), \quad (2.17)$$

where σ_1 and σ_2 are weight functions which depends in a complicated way on the possible intermediate states and m is a nucleon mass.

B. γ-d Vertex Functions

The electromagnetic vertex function of the deuteron is defined by

$$\langle T(B_\nu(x)B_\lambda^*(y)A_\mu(z))\rangle_0 =-e\int d\xi\int d\eta\int d\zeta\;G_{\nu\nu'}(x-\xi)\Lambda_{\nu'\lambda';\mu'}(\xi-\zeta;\,\zeta-\eta) \times G_{\lambda'\lambda}(\eta-y)D_{\mu'\mu}(\zeta-z), \quad (2.18)$$

[6] We use here the general form suggested by Nishijima and Zimmermann. However, we shall be interested only in space-like ξ so that time ordering indicated by T is not necessary.

[7] C is the charge-conjugation Dirac matrix, which has the properties $C^*=-C$, $C^\dagger=C$, $C^{-1}\gamma_\mu C=-\gamma_\mu^T$.

[8] W. Pauli, *Niels Bohr and the Development of Physics* (McGraw-Hill Book Company, Inc., New York, 1955), p. 30.

[9] We assume that the limiting process of ξ in (2.11) and (2.12) is taken on a space-like surface. Then, what actually is shown by weak reflection is that the phases of these three constants are equal and opposite of the phase of U. If we choose a real spin function U, therefore, the statement in the text follows.

[10] H. Lehmann, Nuovo cimento **11**, 342 (1954).

where A_μ is the electromagnetic field operator and $D_{\mu'\mu}(\zeta-z)$ is defined by

$$D_{\mu'\mu}(\zeta-z)=\langle T(A_{\mu'}(\zeta)A_\mu(z))\rangle_0.$$

Takahashi[11] has shown that in order to derive the generalized Ward's identity for quantum electrodynamics it is sufficient to assume

$$[\psi(x),j_0(x')]\delta(x_0-x_0')=e\delta(x-x')\tfrac{1}{2}(1+\tau_3)\psi(x), \quad (2.19)$$

which is actually satisfied in the usual case of minimal electromagnetic interaction. Also in a certain class of the strong interactions,

$$\begin{gathered}[f(x),j_0(x')]\delta(x_0-x_0')=e\delta(x-x')\tfrac{1}{2}(1+\tau_3)f(x),\\ (\gamma\partial+m)\psi(x)=f(x),\end{gathered} \quad (2.20)$$

will be satisfied. We will assume that in our case Eq. (2.19) and Eq. (2.20) for the nucleon field are satisfied. By means of this equation and the definition of the deuteron field $B_\mu(x)$, it is not difficult to prove the corresponding equation for $B_\mu(x)$:

$$[B_\mu(x),j_0(x)]\delta(x_0-x_0')=e\delta(x-x')B_\mu(x). \quad (2.21)$$

by following the same procedure as Takahashi, we obtain the generalized Ward's identity for the electromagnetic vertex function of the deuteron[12]:

$$G_{\nu\lambda}(d')-G_{\nu\lambda}(d)=-i(d'-d)_\mu G_{\nu\nu'}(d')\Lambda_{\nu'\lambda';\mu}(d';d)G_{\lambda'\lambda}(d). \quad (2.22)$$

Using Eq. (2.13) and the invariance of the theory under charge conjugation, we obtain

$$\langle T(B_\nu(x)B_\lambda^*(y)A_\mu(z))\rangle_0=-\langle T(B_\lambda(y)B_\nu^*(x)A_\mu(z))\rangle_0.$$

The above relation and the definition of the vertex function $\Lambda_{\nu\lambda;\mu}$, Eq. (2.18), give the condition:

$$\Lambda_{\nu\lambda;\mu}(d';d)=-\Lambda_{\lambda\nu;\mu}(-d;-d'). \quad (2.23)$$

The most general form of $\Lambda_{\nu\lambda;\mu}$ is

$$\begin{aligned}\Lambda_{\nu\lambda;\mu}(d',d)=i[&\delta_{\nu\lambda}(d+d')_\mu\lambda_1+\delta_{\nu\lambda}(d'-d)_\mu\lambda_2+\{\delta_{\nu\mu}(d'-d)_\lambda-\delta_{\lambda\mu}(d'-d)_\nu\}\lambda_3+\{\delta_{\nu\mu}(d'-d)_\lambda+\delta_{\lambda\mu}(d'-d)_\nu\}\lambda_4\\&+\{\delta_{\nu\mu}(d'+d)_\lambda-\delta_{\lambda\mu}(d'+d)_\nu\}\lambda_5+\{\delta_{\nu\mu}(d'+d)_\lambda+\delta_{\lambda\mu}(d'+d)_\nu\}\lambda_6+(d_\nu d_\lambda+d_\nu'd_\lambda')(d'+d)_\mu\lambda_7\\&+(d_\nu d_\lambda-d_\nu'd_\lambda')(d'+d)_\mu\lambda_8+(d_\nu d_\lambda+d_\nu'd_\lambda')(d'-d)_\mu\lambda_9+(d_\nu d_\lambda-d_\nu'd_\lambda')(d'-d)_\mu\lambda_{10}\\&+(d_\nu d_\lambda'+d_\nu'd_\lambda)(d'+d)_\mu\lambda_{11}+(d_\nu d_\lambda'-d_\nu'd_\lambda)(d'+d)_\mu\lambda_{12}+(d_\nu d_\lambda'+d_\nu'd_\lambda)(d'-d)_\mu\lambda_{13}\\&+(d_\nu d_\lambda'-d_\nu'd_\lambda)(d'-d)_\mu\lambda_{14}],\end{aligned} \quad (2.24)$$

where λ_i $(i=1\cdots14)$ are functions of d^2, d'^2 and $(d'-d)^2$. Eq. (2.23) implies that $i=1$, 3, 6, 7, 10, 11 and 14 are even functions with respect to the interchange of d^2 and d'^2 while the other λ's are odd functions. By using the generalized Ward's identity (2.22), we obtain a certain relation among the various λ's. Especially in the limit $d^2\to-M^2$, $d'^2\to-M^2$ and $(d'-d)^2\to0$, we obtain

$$\begin{gathered}\lambda_1(-M^2,-M^2,0)=1,\\ \lambda_6(-M^2,-M^2,0)=\tfrac{1}{2}.\end{gathered} \quad (2.25a)$$

For the lowest order of the electromagnetic interaction, we obtain

$$\begin{aligned}\langle\mathbf{d}',j|j_\mu(x)|\mathbf{d},i\rangle&=\langle\mathbf{d}',j|-\Box A_\mu(x)|\mathbf{d},i\rangle\\&=-i\frac{U_\nu^*(\mathbf{d}')U_\lambda(\mathbf{d})}{(2\pi)^3(2d_02d_0')^{\frac{1}{2}}}e^{-i(d'-d)\cdot x}\Lambda_{\nu\lambda;\mu}(d';d).\end{aligned}$$

This relation gives us the value of λ on the mass shell,[1] i.e.,

$$\lambda_3(-M^2,-M^2,0)=(M/m)\mu_D, \quad (2.25b)$$

$$\lambda_{11}(-M^2,-M^2,0)=-Q/2\sqrt{10}, \quad (2.25c)$$

where μ_D is the magnetic moment of the deuteron in units of the nuclear Bohr magneton and Q is the quadrupole moment.

C. *np-d* Vertex Function[13]

The *np-d* vertex function $\Omega_{\nu;\alpha\beta}(\xi;\eta)$ is defined by

$$\begin{aligned}\langle T(\psi_\alpha(x)\psi_\beta(y)B_\nu^*(z))\rangle_0&=-\int d\xi\int d\eta\int d\zeta\,(S_F'(x-\xi))_{\alpha\alpha'}(S_F'(y-\eta))_{\beta\beta'}\\&\quad\times\Omega_{\nu;\alpha'\beta'}(\xi-\zeta;\eta-\zeta)D_{\nu'\nu}(\zeta-z),\end{aligned} \quad (2.26)$$

where $S_F'(x)$ is a nucleon one-body Green's function which is defined by

$$(S_F'(x-y))_{\alpha\beta}=\langle T(\psi_\alpha(x)\bar\psi_\beta(y))\rangle_0.$$

Interchanging x and y in Eq. (2.26), we obtain the following relation by the definition of the T product:

$$\Omega_\nu(\xi;\eta)=-\Omega_\nu^T(\eta;\xi), \quad (2.27)$$

which is a statement of the generalized Pauli principle in this formulation. The Fourier transform of $\Omega_\nu(\xi;\eta)$ has, in general, the following form:

$$\begin{aligned}\Omega_\nu(p',p)=[&\Sigma_+(p')\omega_\nu^{(0)}(p',p)\Sigma_+(p)+\Sigma_-(p')\omega_\nu^{(1)}(p',p)\\&\times\Sigma_+(p)+\Sigma_+(p')\omega_\nu^{(2)}(p',p)\Sigma_-(p)\\&+\Sigma_-(p')\omega_\nu^{(3)}(p',p)\Sigma_-(p)]\mathcal{C},\\ \Sigma_\pm(p)=[&\mp i\gamma\cdot p+(-p^2)^{1/2}]/2(-p^2)^{1/2},\end{aligned} \quad (2.28)$$

[11] Y. Takahashi, Nuovo cimento 6, 371 (1957); K. Nishijima, Phys. Rev. **119**, 485 (1960).

[12] The Ward identity for the composite particle is discussed by Nishijima [K. Nishijima, Phys. Rev. **122**, 298 (1961)].

[13] This vertex function was investigated by Blankenbecler and Cook [R. Blankenbecler and L. F. Cook, Phys. Rev. **119**, 1745 (1960)].

where $\omega_\nu^{(i)}$ ($i=0$, 1, 2, and 3) is given by

$$\omega_\nu^{(i)}(p',p)=i\gamma_\nu a^{(i)}(p'^2,p^2,(p+p')^2)+\frac{(p'-p)_\nu}{2m}b^{(i)}(p'^2,p^2,(p+p')^2)+\frac{(p'+p)_\nu}{2m}\tilde{b}^{(i)}(p'^2,p^2,(p+p')^2). \quad (2.29)$$

The relation (2.27), the Pauli principle, implies that $a^{(0)}$ and $b^{(0)}$ are even functions under the interchange of p and p' while $\tilde{b}^{(0)}$ is an odd function.

In order to see the magnitudes of the constants $a^{(0)}$ and $b^{(0)}$ on the mass shell, that is, when $-p^2=-p'^2=m^2$ and $-(p+p')^2=M^2$, let us consider the matrix element

$$\bar{u}(\mathbf{p}')\int dx\, e^{-ip'\cdot x}\langle\mathbf{p}|f(x)|\mathbf{d}\rangle,$$

where

$$f=(\gamma\partial+m)\psi.$$

By using the reduction formulas and the definition of the np-d vertex function, we obtain

$$\bar{u}(\mathbf{p}')\int dx\, e^{-ip'\cdot x}\langle\mathbf{p}|f(x)|\mathbf{d}\rangle=(2\pi)^4\delta(p+p'-d)\left[\frac{m}{(2\pi)^6p_0 2d_0}\right]^{1/2}\times\bar{u}(\mathbf{p})\left[i\gamma\cdot Ua^{(0)}+\frac{(p-p')\cdot U}{2m}b^{(0)}\right]\mathcal{C}\bar{u}(\mathbf{p}'). \quad (2.30)$$

These constants $a^{(0)}$ and $b^{(0)}$ on the mass shell in the form of (2.30) have already been obtained from the deuteron pole for np-scattering.[1] Here we quote only the results:

$$a^{(0)}(-m^2,-m^2,-M^2)=A\cong-\Gamma(1+\alpha),$$
$$b^{(0)}(-m^2,-m^2,-M^2)=B\cong 3(m^2/\gamma^2)\Gamma\alpha,$$
$$\Gamma=\left[\frac{8\pi(\gamma/m)}{(1+2\alpha^2)(1-\rho\gamma)}\right]^{1/2}, \quad (2.31)$$

where $\gamma=(mB)^{1/2}$ (B is the binding energy of the deuteron), ρ is the effective range of np scattering, and $\sqrt{2}\alpha$ is the ratio of the amplitude of the ^3S state wave function of the deuteron to that of the ^3D state at infinity.[14]

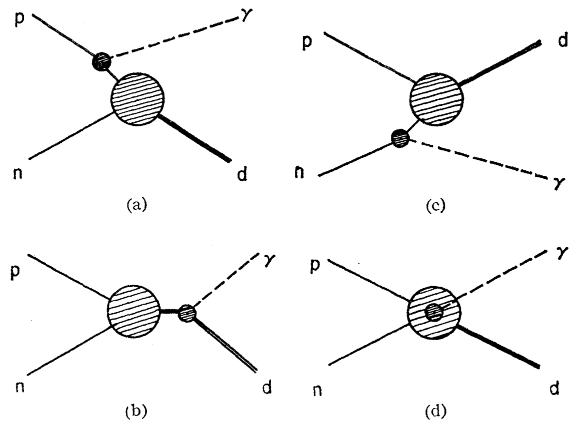

FIG. 1. Feynman diagram for the photodisintegration of the deuteron.

III. LOW-ENERGY LIMIT THEOREM OF THE PHOTODISINTEGRATION OF THE DEUTERON

The transition matrix element for photodisintegration of the deuteron can be expressed in terms of reduction formulas as

$$\langle\mathbf{p},\mathbf{p}'|T|\mathbf{k},\mathbf{d}\rangle=-i\left[\frac{m^2}{(2\pi)^6(2d_0)(2k_0)p_0p_0'}\right]^{1/2}e_\mu U_\nu(\mathbf{d})\bar{u}_\alpha(\mathbf{p})\bar{u}_\beta(\mathbf{p}')\int e^{-ip\cdot x-ip'\cdot y+ik\cdot r+id\cdot z}dxdydzdr \times\vartheta_{\alpha\alpha'}(x)\vartheta_{\beta\beta'}(y)K_z\langle T(\psi_{\alpha'}(x)\psi_{\beta'}(y)B_\nu{}^*(z)j_\mu(r))\rangle_0 \quad (3.1)$$

$$\vartheta=(\gamma\cdot\partial+m),$$

where e_μ and U_ν are the polarization vectors of the incident photon and of the deuteron, respectively, and p, p', k, and d are four-momentum vectors of the final nucleons, of the incident photon, and of the deuteron, respectively.

Let us introduce and external electromagnetic potential, A^e. Then the field operators ψ and B_μ deviate from the case of no external electromagnetic potential, so that these field operators are functionals of A^e. Let us denote these functional operators and Green's functions by bold letters: $\boldsymbol{\psi}$, $\mathbf{B}$, $\mathbf{S}_F'$, etc. The electromagnetic vertex function of the deuteron, defined by Eq. (2.18), can be written in the lowest order approximation of the electromagnetic interaction, using the functional derivative of the Green's functions, as

$$\left[\frac{\delta}{\delta A_\mu{}^e(r)}\mathbf{G}_{\nu\lambda}(x-y)\right]_{A^e\to0}=e\int d\xi\int d\eta\, G_{\nu\nu'}(x-\xi)\Lambda_{\nu'\lambda';\mu}(\xi-r;r-\eta)G_{\lambda'\lambda}(\eta-y). \quad (3.2)$$

[14] This constant was calculated by Wong and by Blankenbecler and Cook. D. Y. Wong, Phys. Rev. Letters 2, 406 (1959).

Similarly, we obtain the electromagnetic vertex function of the nucleon, Γ_μ:

$$\left[\frac{\delta}{\delta A_\mu^e(r)}\mathbf{S}_F'(x-y)\right]_{A^e\to 0}=e\int d\xi\int d\eta\, S_F'(x-\xi)\Gamma_\mu(\xi-r;r-\eta)S_F'(\eta-y). \tag{3.3}$$

It is well known that the Fourier transform of Γ_μ must have the following form:

$$\Gamma_\mu(p',p)=[\Sigma_+(p')\gamma_\mu^{(0)}(p',p)\Sigma_+(p)+\Sigma_-(p')\gamma_\mu^{(1)}(p',p)\Sigma_+(p)+\Sigma_+(p')\gamma_\mu^{(2)}(p',p)\Sigma_-(p)+\Sigma_-(p')\gamma_\mu^{(3)}(p',p)\Sigma_-(p)], \tag{3.4}$$

where

$$\gamma_\mu^{(i)}(p',p)=\gamma_\mu\rho^{(i)}(p'^2,p^2,(p'-p)^2)-[\sigma_{\mu\nu}(p'-p)_\nu/2m]\mu^{(i)}(p'^2,p^2,(p'-p)^2)+[(p'-p)_\mu/2m]\tau^{(i)}(p'^2,p^2,(p'-p)^2), \tag{3.5}$$

with

$$\begin{aligned}\rho^{(0)}(-m^2,-m^2,0)&=\tfrac{1}{2}(1+\tau_3),\\ \mu^{(0)}(-m^2,-m^2,0)&=\eta^++\eta^-\tau_3,\\ \eta^\pm&=\tfrac{1}{2}[\eta_p\pm\eta_n],\end{aligned} \tag{3.6}$$

η_p and η_n being the anomalous magnetic moments of the proton and neutron, respectively.[15]

Using this method, the vacuum expectation value of Eq. (3.1) can be written as

$$\begin{aligned}\langle T(\psi(x)\psi(y)B_\nu^*(z)j_\mu(r))\rangle_0&=\left[i\frac{\delta}{\delta A_\mu^e(r)}\langle T(\boldsymbol{\psi}(x)\boldsymbol{\psi}(y)\mathbf{B}_\nu^*(z))\rangle_0\right]_{A^e\to 0}\\ &=-i\int d\xi\int d\eta\int d\zeta\left[\frac{\delta}{\delta A_\mu^e(r)}\{\mathbf{S}_F'(x-\xi)\mathbf{S}_F'(y-\eta)\boldsymbol{\Omega}_{\nu'}(\xi-\zeta;\eta-\zeta)\mathbf{G}_{\nu'\nu}(\zeta-z)\right]_{A^e\to 0}.\end{aligned} \tag{3.7}$$

Performing the functional derivative and using Eqs. (3.2) and (3.3), we obtain

$$\begin{aligned}\mathfrak{M}_{\mu\nu}=&\Gamma_\mu(p,\,p-k)S_F'(p-k)\Omega_\nu(p-k,\,p')+\Omega_\nu(p,\,p'-k)S_F'^T(p'-k)\Gamma_\mu^T(p',\,p'-k)\\ &+\Omega_{\nu'}(p,p')G_{\nu'\nu''}(d+k)\Lambda_{\nu''\nu;\mu}(d+k,\,d)+\mathcal{K}_{\mu\nu}(p,p',d,k),\end{aligned} \tag{3.8}$$

where $\mathfrak{M}$ is defined by

$$\langle\mathbf{p},\mathbf{p}'|T|\mathbf{k},\mathbf{d}\rangle=(2\pi)^4\delta(p+p'-d-k)i\left[\frac{m^2}{(2\pi)^6(2d_0)(2k_0)p_0p_0'}\right]^{1/2}e_\mu U_\nu(\mathbf{d})\bar{u}_\alpha(\mathbf{p})\bar{u}_\beta(\mathbf{p}')[\mathfrak{M}_{\mu\nu}]_{\alpha\beta}, \tag{3.9}$$

and $\mathcal{K}_{\mu\nu}$ is given by

$$(2\pi)^4\delta(p+p'-d-k)\mathcal{K}_{\mu\nu}(p,p',d,k)=-i\int d\xi\int d\eta\int d\zeta\int dr\, e^{-i\xi\cdot p-i\eta\cdot p'+ik\cdot r+id\cdot\zeta}\left[\frac{\delta}{\delta A_\mu^e(r)}\boldsymbol{\Omega}_\nu(\xi-\zeta;\eta-\zeta)\right]_{A^e\to 0}. \tag{3.10}$$

Each term in Eq. (3.8) can be represented by diagrams shown in Fig. 1, which are self-explanatory.

In order to see the behavior of $\mathfrak{M}_{\mu\nu}$ in the low-energy limit, we must know the behavior of $\mathcal{K}_{\mu\nu}$ in this limit. To see this, let us consider $\langle T(\psi_\alpha(x)\psi_\beta(y)B_\nu^*(z)j_\mu(r))\rangle_0$, take a four-dimensional divergence with respect to r and apply the Eqs. (2.19) and (2.21), then we obtain[16]

$$\begin{aligned}&\frac{\partial}{\partial r_\mu}\langle T(\psi_\alpha(x)\psi_\beta(y)B_\nu^*(z)j_\mu(r))\rangle_0\\ &\quad=-e\langle T(\psi_{\alpha'}(x)\psi_{\beta'}(y)B_\nu^*(z))\rangle_0\{\tfrac{1}{2}(1+\tau_3)_{\alpha\alpha'}\delta_{\beta\beta'}\delta(x-r)+\delta_{\alpha\alpha'}\tfrac{1}{2}(1+\tau_3)_{\beta\beta'}\delta(y-r)-\delta_{\alpha\alpha'}\delta_{\beta\beta'}\delta(z-r)\}.\end{aligned}$$

Taking the Fourier transform of the above equation and applying the generalized Ward's identity for the nucleon

[15] From the charge-conjugation invariance we obtain $\Gamma_\mu(p',p)=-\Gamma_\mu^T(-p,-p')$, which implies that $\rho^{(0)}$ and $\mu^{(0)}$ are symmetric with respect to p^2 and p'^2 while $\tau^{(0)}$ is antisymmetric. The same holds for $\rho^{(3)}$, $\mu^{(3)}$, and $\tau^{(3)}$ but $\rho^{(1)}\rightleftarrows\rho^{(2)}$, $\mu^{(1)}\rightleftarrows\mu^{(2)}$, and $\tau^{(1)}\rightleftarrows\tau^{(2)}$ by changing p and p'.

[16] This is the W-T equation obtained by Nishijima. See references 11 and 12.

and the deuteron, we obtain

$$
\begin{aligned}
-k_\mu \int e^{-ip\cdot x-ip'\cdot y+ik\cdot r+id\cdot z}dxdydzdr\left[\frac{\delta}{\delta A_\mu{}^e(r)}\int d\xi\int d\eta\int d\zeta\, \mathbf{S}_F'(x-\xi)\mathbf{S}_F'(y-\eta)\mathbf{\Omega}_{\nu'}(\xi-\eta;\eta-\zeta)\mathbf{G}_{\nu'\nu}(\zeta-z)\right]_{A^e\to 0} \\
=(2\pi)^4 i\delta(p+p'-d-k)e[-ik_\mu\tfrac{1}{2}(1+\tau_3)S_F'(p)\Gamma_\mu(p,\,p-k)S_F'(p-k)\Omega_{\nu'}(p-k,\,p')S_F'^T(p')G_{\nu'\nu}(d) \\
-ik_\mu S_F'(p)\Omega_{\nu'}(p,\,p'-k)S_F'^T(p'-k)\Gamma_\mu{}^T(p',\,p'-k)S_F'^T(p')G_{\nu'\nu}(d)-ik_\mu S_F'(p)\Omega_{\nu'}(p,p')S_F'^T(p') \\
\times G_{\nu'\nu''}(d+k)\Lambda_{\nu''\nu''';\mu}(d+k,\,d)G_{\nu'''\nu}(d)+S_F'(p)\{\tfrac{1}{2}(1+\tau_3)\Omega_{\nu'}(p-k,\,p')+\Omega_{\nu'}(p,\,p'-k)\tfrac{1}{2}(1+\tau_3) \\
-\Omega_{\nu'}(p,p')\}S_F'^T(p')G_{\nu'\nu}(d)].
\end{aligned}
$$

Comparing the above expression with (3.7), (3.8), and (3.10), we obtain

$$-k_\mu\mathcal{K}_{\mu\nu}=\tfrac{1}{2}(1+\tau_3)\Omega_\nu(p-k,\,p')+\Omega_\nu(p,\,p'-k)\tfrac{1}{2}(1+\tau_3)-\Omega_\nu(p,p').$$

Therefore,

$$\lim_{k\to 0}\mathcal{K}_{\mu\nu}(p,p',k,d)=\left[\frac{\partial}{\partial p_\mu}+\frac{\partial}{\partial p_\mu{}'}+\left(\frac{\partial}{\partial p_\mu}-\frac{\partial}{\partial p_\mu{}'}\right)\tau_3\right]\Omega_\nu(p,p'). \tag{3.11}$$

Now let us divide the transition matrix element (3.8) into two parts:

$$\mathfrak{M}_{\mu\nu}=\mathfrak{M}_{\mu\nu}{}^B+\mathfrak{M}_{\mu\nu}{}'.$$

One is the Born term, $\mathfrak{M}_{\mu\nu}{}^B$, which is the term containing the isolated pole of the one-body Green's function in the first three terms in Eq. (3.8) and is given by

$$
\begin{aligned}
\mathfrak{M}_{\mu\nu}{}^B=\left(i\gamma_\mu\rho-\frac{i\sigma_{\mu\rho}k_\rho}{2m}\mu\right)\frac{-i\gamma\cdot(p-k)+m}{(p-k)^2+m^2}\left(i\gamma_\nu a+\frac{(p-k-p')_\nu}{2m}b\right)\mathfrak{C}+\left[i\gamma_\nu a+\frac{(p+k-p')_\nu}{2m}b\right] \\
\times\mathfrak{C}\left[\left\{i\gamma_\mu\rho-\frac{i\sigma_{\mu\rho}k_\rho}{2m}\mu\right\}\frac{-i\gamma\cdot(p'-k)+m}{(p'-k)^2+m^2}\right]^T+\left[i\gamma_\nu a+\frac{(p-p')_\nu}{2m}b\right]\frac{\delta_{\nu'\nu''}+(d+k)_{\nu'}(d+k)_{\nu''}/M^2}{(d+k)^2+M^2} \\
\times\{(2d+k)_\mu\delta_{\nu''\nu}\lambda_1+[k_{\nu''}\delta_{\mu\nu}-k_\nu\delta_{\mu\nu''}]\lambda_3\}+\frac{\delta_{\mu\nu}}{2m}b,
\end{aligned}
\tag{3.12}
$$

where the last term is added in order to make it be gauge invariant. The other term, $\mathfrak{M}_{\mu\nu}{}'$, contains all the other parts of the matrix element.

Then we expand $\mathfrak{M}_{\mu\nu}$ in powers of ω where ω is the energy of the γ ray. The Born term starts from the term $\sim 1/\omega$, while $\mathfrak{M}_{\mu\nu}{}'$ starts from the term of zeroth power in ω. Due to (3.11), the term of zeroth order in ω in $\mathfrak{M}_{\mu\nu}{}'$ does not depend on k at all so that it is independent of the direction of the incident γ ray $\hat{k}$, while the term of zeroth power in the Born term does depend on $\hat{k}$. Therefore, we can say that the $1/\omega$ term of the Born approximation is exact and also that the part of the Born term which approaches a constant as $\omega\to 0$ and which is odd under the operation $\hat{k}\to-\hat{k}$ is exact in the limit $\omega\to 0$. The former corresponds to the $E1$ transition while the latter corresponds to the $M1$. The last term added to Eq. (3.12) is an even term, so that if we keep only the odd part for the zeroth power of ω in Eq. (3.11) it does not contribute at all.

Along this line, if we make in (3.12) a nonrelativistic expansion and neglect γ^2/m^2 and p^2/m^2, then we obtain

$$
\begin{aligned}
\mathfrak{F}=\frac{e\Gamma}{16\pi m^2}i\sigma_2 i\tau_2\Big\{-(1+\alpha)[\mu_p+\mu_n-\tfrac{1}{2}(M/n)\mu_D](\mathbf{e}\cdot\boldsymbol{\sigma})(\mathbf{U}\cdot\hat{k})+\tau_3\Big[\left(\frac{2p}{\omega}\right)[(1+\alpha)(\mathbf{e}\cdot\hat{p})(\mathbf{U}\cdot\boldsymbol{\sigma})-3\alpha(\mathbf{e}\cdot\hat{p})(\mathbf{U}\cdot\hat{p})(\boldsymbol{\sigma}\cdot\hat{p})] \\
+(\mu_p-\mu_n)\{(1-2\alpha)i[\mathbf{e}\cdot\mathbf{U}\times\hat{k}]-3\alpha i(e\cdot p)[\mathbf{U}\cdot\hat{\mathbf{p}}\times\hat{\mathbf{k}}]\}\Big]\Big\},
\end{aligned}
\tag{3.13}
$$

where $\hat{p}$ is the direction of the relative momentum of two final nucleons in the center-of-mass system while $\hat{k}$ is the direction of the incident γ ray. $\mathfrak{F}$ is the transition amplitude, such that the differential cross section of the photodisintegration of the deuteron in the center-of-mass system is

$$\frac{d\sigma}{d\Omega}=\overline{\sum}\left(\frac{p}{\omega}\right)|\langle\mathfrak{F}\rangle|^2, \tag{3.14}$$

where the matrix element indicated is taken between the two final Pauli spinors. $\overline{\sum}$ is an abbreviation for the sum of the ordinary spin and isospin for the final state and the average of the polarization for the initial γ ray and deuteron.

The amplitude $\mathfrak{F}$ in Eq. (3.13) is the exact amplitude in the long-wavelength limit of the incident γ ray except the small corrections of order of B/m.

IV. APPLICATION

The low-energy limit derived in the previous section is the result of the theory of composite particles and of gauge invariance. It is interesting, therefore, to apply this result to the *n-p* capture (*M*1 transition) in order to compare the theory with experiment.

As we have shown,[1] the effect of rescattering in the *n-p* state can be obtained as a function of the *n-p* phase shift by using unitarity. The low-energy limit obtained in the previous section and the enhancement factor due to the rescattering effect will give a good approximation for the *M*1 amplitude and allows comparison with experiment. This subject has been discussed in detail in reference 1.

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G Parity and the Interactions of Heavy Mesons*

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Some implications of the *G* parity of heavy mesons are discussed. It is pointed out that the *G* parity of a meson may determine whether it can contribute to a pole term in the nucleon-nucleon scattering amplitude. Since *G* parity may not be conserved in the decays of some heavy mesons, *G* must be determined indirectly. One method is to measure the charge parity of the decay products of a neutral meson, a quantity which determines *G* is the isospin of the meson is known. Selection rules for the decay of neutral and charged mesons are given. Results are applied to the ζ and η mesons.

RECENTLY a number of heavy mesons of strangeness zero have been discovered, the ω, ρ, η, and ζ,[1] and there may be more to come. Four quantum numbers are required to specify such mesons (in addition to the strangeness which is zero): the spin J, parity P, isospin T, and G parity. We wish to point out some consequences of the G parity for the interactions of these mesons with nucleons, and to emphasize that it may not always be trivial to measure G. We illustrate the problem by considering the ζ and η mesons and mention some (admittedly difficult) experiments which can distinguish between the alternative possibilities.

The G parity of n pions is $G=(-1)^n$. Thus, if a meson decays into pions with conservation of G, its G parity is determined simply by counting the number of final-state pions. However, as has been discussed by Feld[2] and others, G may not be conserved in the decay of these mesons because of coupling to the electromagnetic field. Despite this fact, if the interaction which causes the decay is invariant under charge conjugation C, then the properties of the decay products under C can be used to obtain the G parity of the meson. In the following, we consider only decays with lifetimes which are very short compared to typical weak interaction lifetimes and assume that P and C are conserved.

Before discussing the measurement of G, we shall point out how the meson G parity affects its interactions with nucleons. If the interaction is linear in the meson field, the meson should have the same quantum numbers as a bound state of a nucleon-antinucleon pair. The parity and G parity of such a pair are[3]

$$P=-(-1)^L,\quad G=(-1)^{S+T+L}, \tag{1}$$

where S, T, and L are the spin, isospin, and orbital angular momentum of the pair. For neutral mesons (including the neutral members of multiplets), G and C are related by

$$G=C(-1)^T. \tag{2}$$

From (1) and (2), we obtain for neutral mesons which interact linearly with nucleons the following relations

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