examined: the radiation gauge ($\nabla \cdot \mathbf{b}^a=0$) and the gauge $b_3{}^a=0$. The radiation gauge leads to canonical commutation relations similar to those of electrodynamics. It has, however, the drawback that the constraint equations cannot be solved rigorously. The above consistency requirements were shown, however, to be satisfied in lowest-order perturbation theory. The gauge where $b_3{}^a=0$ is more complex than the radiation gauge due to the loss of three-space rotational invariance. This gauge does have the advantage of affording a rigorous solution of the constraint equations and hence a complete verification of the consistency conditions.[19] It might also be mentioned that in this gauge, the constraint variable E^3 (and b_0) depend only linearly on the canonical variables and linearly on the isotopic current operator [see Eqs. (5.8) and (5.9)]. One might hope, then, that E^3 and b_0 are operators with well-defined matrix elements.

For a nonlinear theory such as the Yang-Mills field, the imposition of a gauge condition is a nontrivial operation. This is due to the fact mentioned earlier (in Sec. II), that the theory is invariant, in general, under only c-number gauge transformations. Two different q-number related gauges, will, more likely than not, represent two physically different theories[20] (with different predictions for cross sections, etc.). As may easily be seen, the radiation gauge and $b_3{}^a=0$ gauge are indeed q-number related and so the theories of Secs. IV and V may have different physical content. Should this be the case, and should both gauges be Lorentz invariant, one would presumably have to resort to experiment to decide which gauge was correct. In electrodynamics, a valid gauge is the radiation gauge.[21] While the theoretical origin of this result is not clear, the radiation gauge *does* possess a preferred position in electrodynamics. It is the gauge in which the total vector potential A_μ equals the gauge-invariant part of A_μ (and hence is the gauge where the gauge-variant part of A_μ has been set to zero). Due to the more complicated nature of the Yang-Mills gauge transformation [Eq. (2.7)], neither of the gauges considered in this paper have this property. It would be of interest to discover the nature of the analogous preferred gauge for the Yang-Mills field.

[19] One of us (S.I.F.) has investigated in this gauge the case of the Yang-Mills field coupled to the nucleon field [Bull. Am. Phys. Soc. **7**, 80 (1962)]. The consistency conditions can be completely verified here also.

[20] However, the gauge transformations associated with Lorentz transformations have q-number parameters and a valid theory must be invariant under these.

[21] It is not known whether or not, in a given Lorentz frame, there exist other experimentally correct gauges in electrodynamics that are q-number related to the radiation gauge.

PHYSICAL REVIEW VOLUME 127, NUMBER 5 SEPTEMBER 1, 1962

Suggested Quantum Numbers for Bosons of Mass $\approx 4m_\pi$†

E. M. HENLEY AND B. A. JACOBSOHN
Physics Department, University of Washington, Seattle, Washington

(Received April 30, 1962)

The decay modes of the ζ ($T=1$) and η ($T=0$) bosons of mass $\approx 4m_\pi$ are discussed, together with other experiments bearing on quantum number assignments for these mesons. Using an effective interaction which includes the influence of angular momentum barriers, we have made numerical estimates of branching ratios. Arguments based on these estimates lead to the most likely spin and parity assignments of $J^{PG}=0^{+\pm}$ for ζ and 0^{--} for η. Study of the reaction $\pi+\mathrm{He}\rightarrow\zeta+\mathrm{He}$ is proposed as a test of the 0^+ assignment.

RECENTLY, renewed evidence has been presented for an isospin 1 resonance at about 565 MeV $\approx 4m_\pi$.[1] We present arguments below in support of a suggestion that the quantum numbers for this resonance (ζ) are either $J^{PG}=0^{++}$ or 0^{+-}. We furthermore discuss the relationship of the neutral component ζ^0 to the η which has $T=0$ and decays into three pions, and reexamine the quantum numbers for the latter.

Since the η and the ζ^0 have about the same mass, all presently existing experimental discussions of the decay modes refer to some combination of the two particles. For brevity, in this paper we use the name X for those phenomena which refer to whatever mixture of η and ζ^0 has been measured.

The arguments which we have used to arrive at the above assignment for the ζ are based on the following pieces of experimental evidence: (1) The decay of the ζ^+ is primarily into $\pi^+\pi^0$ with a width less than 15 MeV.[1,2] (2) The branching ratio for the X produced in $K^-+p\rightarrow\Lambda+X$ is $\lesssim 1/20$ for $\pi^+\pi^-$ as well as for $\pi^+\pi^-\gamma$, as indicated by the absence of these modes in the data of Bastien *et al.*[3] (3) This same X has[3] $\Gamma(\text{neutrals})/\Gamma(\pi^+\pi^-\pi^0)\approx 3/1$. (4) The experiment of

† Supported in part by the U. S. Atomic Energy Commission.

[1] B. Sechi Zorn, Phys. Rev. Letters **8**, 282, 386(E) (1962), in which references for earlier evidence are found.

[2] B. Sechi Zorn, Bull. Am. Phys. Soc. **7**, 349 (1962).

[3] P. L. Bastien, J. P. Berge, O. I. Dahl, M. Ferro-Luzzi, D. H. Miller, J. J. Murray, A. H. Rosenfeld, and M. B. Watson, Phys. Rev. Letters **8**, 114, 302(E) (1962).

TABLE I. Principal decay modes for the isospin one ζ meson of mass $4m_\pi$. The +, −, 0 refer to π^+, π^-, π^0, and the subscript to the barrier L value; the superscript s identifies strong interaction decays.

J^{PG}	Decay modes	Relative decay rates $m=2$	$m=3$
ζ^+, $0^{+\pm}$	$(+\ 0)_0$	1	1
	$(+\ 0)_1\,\gamma$	0.07	0.008
	$(+)\,\gamma\,\gamma$	0.003	0.00035
ζ^+, 1^{--}	$(+)\,\gamma$	1	1
	$(+\ 0)_1$	0.006	0.006
	$(+\ 0)_0\,\gamma$	0.0045	0.002
	$(+\ +\ -)_4{}^s$	0.001	0.00004
	$(+\ 0\ 0)_4{}^s$	0.001	0.00004
ζ^0, 0^{++}	$(+\ -)_2\,\gamma$	x	x
	$(0\ 0)_2\,\gamma$	$1-x$	$1-x$
ζ^0, 0^{+-}	$(+\ -)_0$	x	x
	$(0\ 0)_0$	$1-x$	$1-x$
	$\gamma\,\gamma$	0.3	0.1
	$(+\ -)_1\,\gamma$	0.04	0.004
ζ^0, 1^{--}	$(+\ -\ 0)_4{}^s$	1	1
	$(+\ -)_1\,\gamma$	0.5	2.1
	$(0)\,\gamma\,\gamma$	0.015	0.10
	$(0\ 0\ 0)_8{}^s$	0.0002	0.00001

Rosenfeld *et al.*[4] shows that if there is appreciable production of X in the decay of $\rho^+ \to X+\pi^+$, then this X decays into neutrals with a very small branching ratio.

For $J \leqslant 1$, the requirement that ζ^+ should be able to decay into $\pi^+\,\pi^0$ limits us to the quantum numbers $0^{+\pm}$ and $1^{-\pm}$. The 1^{-+} assignment would have the ζ^0 decay strongly into $\pi^+\,\pi^-$. This is ruled out by point (2) since we shall make the explicit assumption that the production rates of ζ^0 and η are similar in all cases where selection rules do not favor one or the other.[5]

To proceed further we must make quantitative estimates of branching ratios for the various decay possibilities. Our method is similar to that used by several authors.[4,6] We use a crude formula based on a point interaction in which each boson field ϕ appears in the dimensionless combination ϕR (where R is a distance) and in which the "barrier factor" L (i.e., the power of the momentum which appears in the matrix element) is approximately accounted for by the insertion shown as the last factor in Eq. (1) below. The parameter $m\equiv(Rm_\pi)^{-1}$ is the only one left free in our formula, since the dimensionless coupling constant g is assigned the same value for all strong processes and is modified so as to include the correct power of e for real and virtual electromagnetic processes. In units in which $\hbar=c=m_\pi=1$, the decay rate of a particle of mass M into a state with n_1 pions and n_2 gamma rays is $(n_1+n_2=n)$

$$\Gamma=\frac{\pi n_1!n_2!}{2^nM}g^2S\rho\left[\frac{\bar{p}^{2L}}{(\bar{p}^2+m^2)^L}\right]. \quad (1)$$

In this equation $\bar{p}$ is the average momentum of each particle,

$$\rho=(2\pi)^{3(1-n)}\int\frac{d\mathbf{p}_1\cdots d\mathbf{p}_n\,\delta(\sum\mathbf{p}_i)\delta(\sum\omega_i-M)}{\omega_1\cdots\omega_n}, \quad (2)$$

and S is a statistical factor which depends on the number of gamma rays and the number of different isospin states in the final configuration.

We are conscious of the imperfections of such a formula.[7] However, it does correctly take account of phase space, factors of 137, and barrier factors. In the absence of a known reason for an inhibition or an enhancement of a given decay, it is hard to close one's eyes to a disagreement for reasonable values of m between the predictions of Eq. (1) and experiment by a factor of more than 10.

In Table I, we list the branching ratios for ζ^+ and ζ_0 decays with $m=2$ and 3 for the quantum numbers mentioned above. Decay into 4 pions, which is allowed for the 0^{++}, has not been included because the phase space for it is extremely small if the mass of the ζ is $\approx 4m_\pi$.[8] The important modes are listed in the second column. Each pion configuration carries a subscript which indicates the barrier factor L; although they are not listed, such factors also occur for gamma rays. These barrier factors are connected with the symmetry requirements of the meson matrix elements and can have important effects in reducing Γ—see, for example, the strong $(L=4)$ three-pion decay for the 1^{--} ζ^+.

We estimate that the apparent absence of the decay $\zeta^+\to\pi^+\,\gamma$ in the data[1,2] implies a branching ratio $\lesssim 0.2$ for this mode. The 1^{--} fails by a factor of 10^3 to meet this requirement and we rule it out, although its neutral component would have satisfactory properties.

The parameters indicated by x for ζ^0 $(0^{+\pm})$ can lie between 0 and 1 because the pions are produced in an arbitrary mixture of isospin $T=0$ and $T=2$ states. A statistical mixture of the two would correspond to $x=0.5$. If x were $\lesssim 0.1$ (or in the 0^{+-} case, if the $\gamma\,\gamma$ mode were somewhat enhanced), then the ζ^0 would decay mostly into neutrals.

Since observations have presumably been made on an approximately equal fraction of ζ^0 and η, the $0^{+\pm}$ assignment for the ζ requires that the relative decay rate of the η (a) into neutrals be $\lesssim 0.5$, (b) into $\pi^+\,\pi^-\,\pi^0$ be $\gtrsim 0.5$ and (c) into $\pi^+\,\pi^-$ and $\pi^+\,\pi^-\,\gamma$ be very small. Table II presents the decay characteristics for various properties of the η. No case meets all the requirements enumerated above, but the 0^{--} at least comes within reach of doing so. Indeed, because of the large barrier factors in some of the 0^{--} modes there is considerable sensitivity to the range parameter. Thus, for $m=1$, $\Gamma(\pi^+\,\pi^-\,\gamma)/(\pi^+\,\pi^-\,\pi^0)_6$ is only 0.12 and $\Gamma(\pi^0\,\pi^0\gamma)/$

[4] A. H. Rosenfeld, D. D. Carmony, and R. T. Van de Walle, Phys. Rev. Letters **8**, 293 (1962).

[5] Preliminary evidence indicates an amplitude for ζ production comparable to that for π production; see references 1 and 2.

[6] B. T. Feld, Phys. Rev. Letters **8**, 181, 386(E) (1962)

[7] Rosenfeld *et al.* (reference 4) cite one case in which estimates made with an equation similar to (1) yield a branching ratio differing by a factor of 25 from that calculated with a specific dispersion theory model.

[8] The strong decay for 0^{+-} is into five pions.

TABLE II. Principal two- and three-particle decay modes for the isospin zero η-meson of mass $4m_\pi$. For explanation of symbols see Table I.

J^{PG}	Decay modes	Relative decay rates $m=2$	$m=3$
0^{-+}	$\gamma\gamma$	1	1
	$(+\,-)_1\,\gamma$	0.1	0.035
	$(0\,0\,0)_0$	0.005	0.006
	$(+\,-\,0)_0$	0.0035	0.004
0^{--}	$(+\,-)_2\,\gamma$	1	1
	$(0\,0)_2\,\gamma$	0.5	0.5
	$(+\,-\,0)_6{}^s$	0.7	0.2
	$(+\,-\,0)_2$	0.1	0.4
1^{++}	$(+\,-)_1\,\gamma$	1	1
	$(+\,-\,0)_1$	0.02	0.05
	$(0)\,\gamma\,\gamma$	0.04	0.04
1^{+-}	$(0)\,\gamma$	1	1
	$(+\,-\,0)_3{}^s$	0.006	0.0006
	$(+\,-)_0\,\gamma$	0.003	0.001
	$(0\,0)_0\,\gamma$	0.0015	0.0006
1^{-+}	$(+\,-)_1\,\gamma$	1	1
	$(0)\,\gamma\gamma$	0.04	0.04
	$(+\,-\,0)_4$	0.0001	0.00006
1^{--}	$(0)\,\gamma$	1	1
	$(+\,-\,0)_2{}^s$	0.04	0.0065
	$(+\,-)_1$	0.006	0.006
	$(+\,-)_0\,\gamma$	0.003	0.001
	$(0\,0)_0\,\gamma$	0.0015	0.0006

$\Gamma(\pi^+\pi^-\pi^0)_6$ is 0.06. We require a ratio of this size for a good fit and assume it to hold in the remainder of the discussion, without committing ourselves to a choice between an anomalously long range or a failure of Eq. (1) by one order of magnitude.

It is interesting to note that for $m=2$ or 3 the strong $L=6$ and the second-order electromagnetic $L=2$ three-pion decays compete on an equal footing. As a consequence, the Dalitz plot contains one arbitrary parameter. In the nonrelativistic limit, the density of points on a Dalitz plot for 0^{--} is proportional to

$$x^2|3x^2-(3y-1)^2+\alpha/3\sqrt{2}|^2, \qquad (3)$$

where x and y are the customary Dalitz variables,[3] and $\alpha=1$ if the electromagnetic mode occurs with strength equal to that for the strong one.

The 0^{--} assignment is in agreement with requirement (4), above; although the strong process $\rho\to\eta+\pi$ is allowed, our proposed η will not go into many neutrals. Decay of the ρ into ζ^0, which does predominantly decay into neutrals, is forbidden by parity and angular momentum conservation.

If 1^{--} or 1^{+-} were assigned to the η, the strong decay of $\rho\to\eta+\pi$ would again be allowed. To force these into agreement with experiment the predictions of Table II for $\Gamma(\pi^0\gamma)$ would have to be decreased by $\gtrsim 200$. Other candidates have far too large a $\pi^+\pi^-\gamma$ relative decay rate.

Although the entries are not included in the tables, we have also examined the assignments $2^{+\pm}$ for the ζ; in both cases the charged component decays mainly into $\pi^+\gamma$ in disagreement with the estimated upper limit on the experimental branching ratio.

To summarize: We have shown that, if simple statistical estimates which account for barrier factors can be trusted, then the J^{PG} assignments $0^{+\pm}$ are the only ones for the $T=1$ ζ boson which are not at variance with experiments already published. Similar estimates for the $T=0$ η indicate a favored assignment 0^{--}, although the predicted branching ratios take some forcing to gain agreement with the facts. The nonunique Dalitz plot is predicted to vanish along the line $x=0$, in apparent disagreement with experiment.[3] The parity difference between ζ^0 and η precludes any modification of our conclusions because of electromagnetic mixing.

One test for the proposed J and P of the ζ is furnished by the reaction $\pi^\pm+\mathrm{He}\to\zeta^\pm+\mathrm{He}$, since this is strictly forbidden for 0^+. Selection rules in the production of ζ are affected by its G value only when the initial state corresponds to a definite G; while the latter is true for ρ and ω, angular momentum and parity are sufficient to forbid the decay of either of these into $0^{+\pm}$. To choose between $G=+1$ and -1, there remain only the decays for ζ^0: The former would in our scheme go rarely into $\pi^+\pi^-\gamma$ and frequently into $\pi^0\pi^0\gamma$, the latter rarely into $\pi^+\pi^-$ and frequently into $\pi^0\pi^0$ or $\gamma\gamma$.

Note added in proof. Feinberg and Pais[9] have also examined the quantum number assignments for the ζ and η; some of their arguments are similar to ours. Shaw and Wong[10] suggest 0^{--} for the η, using a calculation based on a particular set of diagrams. See also Peierls and Treiman.[11]

---

[9] G. Feinberg and A. Pais, Phys. Rev. Letters **8**, 341 (1962).
[10] G. L. Shaw and D. Y. Wong, Phys. Rev. Letters **8**, 336 (1962).
[11] R. F. Peierls and S. B. Treiman, Phys. Rev. Letters **8**, 339 (1962).