ratios makes an interpretation in terms of tunneling difficult. On the other hand, the preference for neutron transfer compared to alpha transfer in the Li^6+Li^6 reactions argues against compound nucleus formation since the outgoing particles are a He^5 and a deuteron, respectively, and penetrability considerations favor deuteron emission.

On the basis of the above disagreements with the data of the simple tunneling theory, and compound-nucleus formation, it is suggested that the reactions proceed by a surface interaction between the long-range nuclear tails. This would require that the population ratios remain constant with change in bombarding energy. At the same time, no contradiction with the data ensues from the larger cross section Li^{7*} compared to B^{10*}. The yield curves are well fitted (see Figs. 3 and 6) by assuming the yield is a function of the penetrability of the Coulomb barrier of the Li^6 nuclei by the other nucleus, with the nuclear radius given by $r=1.75A^{1/3}$ F. Penetrabilities were taken from published graphs of Coulomb functions.[12] The present results do not establish this interpretation, but they are consistent with it.

ACKNOWLEDGMENT

The author appreciates the advice and encouragement of Professor R. R. Carlson.

[12] W. T. Sharp, H. E. Gove, and E. B. Paul, "Graphs of Coulomb Functions," Chalk River Project, 1955 (unpublished).

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Effects of Angular Momentum and Gamma-Ray Emission on Excitation Functions*

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Excitation functions for many compound nucleus reactions are strongly dependent on the competition between gamma-ray and particle emission in the final particle-emission step. This competition depends, in turn, mainly on the energies and spins of a relatively few levels in the product nucleus, namely, on the lowest energy level at every angular momentum J (these energies are herein designated by E_J). A method for making approximate calculations is described in which the influence of the competitive gamma-ray emission on excitation functions well above threshold is estimated, using assumed plausible distributions of the E_J's. It is found that larger values of the level density parameter a are required to achieve agreement of calculations with experimental data when the competitive gamma-ray emission is included than when it is neglected (i.e., practically equivalent to setting $E_J=0$ for all angular momenta). An approximate analysis of experimental excitation functions for the reaction-pair $Ag^{109}(\alpha,n)In^{112}$ and $Ag^{109}(\alpha,2n)In^{111}$ suggests that $a>12$ MeV^{-1}, assuming the level density expression $\omega(E)\propto E^{-2}\exp[2\surd(aE)]$, or $a>7$ MeV^{-1} assuming $\omega(E)\propto\exp[2\surd(aE)]$. Alternatively, assuming $a\approx 16$ MeV^{-1} (in the first formula), consistent with values calculated by Lang from level spacings observed near neutron binding energies, it appears that the average value of the E_J's for angular momenta of $(11/2)\hbar$ to $(17/2)\hbar$ in In^{111} is roughly 2 to 2.5 MeV. It is suggested that instead of trying to extract a from excitation functions, it is perhaps more appropriate to try to extract information on the E_J distribution, using for this purpose values of a from other types of experiment.

INTRODUCTION

Excitation functions for compound nucleus processes are expected to provide data from which a knowledge of the variation of nuclear state density with excitation energy may be obtained. In analyses of such data, it is necessary to include the effects of angular momentum, and of gamma-ray emission in competition with particle emission[1]; this, in turn, requires a knowledge of the energies, spins and parities of all levels populated in the product nuclei. For most medium to heavy nuclei this information is known only for the first few levels, so that the calculations can seldom be carried beyond one or two MeV above threshold.

An extension to many MeV above threshold is described in this report, in which the influence of the unknown levels is estimated. These approximate calculations are applied to experimental data, to find whether the required information about the product levels is also obtainable from excitation function data, or whether the lack of this information blocks extraction of the desired knowledge about the energy variation of the nuclear state density.

DESCRIPTION OF THE PROBLEM AND AN APPROXIMATE SOLUTION

Emission of charged particles is neglected, for it would complicate the discussion while adding nothing essential. Parity is omitted for the same reason. Symbols used in this paper for nuclei important in the reactions

* Research performed under the auspices of the U. S. Atomic Energy Commission.
[1] J. R. Grover, Phys. Rev. 123, 267 (1961).

to be discussed are given by the following scheme:

$$T \xrightarrow{+X} C \xrightarrow{-n} U_1 \xrightarrow{-n} U_2 \xrightarrow{-n} \cdots \xrightarrow{-n} F \xrightarrow{-n} P. \quad (1)$$

Projectile X amalgamates with the target nucleus T to form the compound nucleus C. C emits a neutron to form U_1 which emits a neutron in turn to form U_2, and so on to F, the last nucleus that can emit a neutron. If F emits a neutron, product nucleus P is formed.

The populated levels in nucleus F are distributed both in energies E_f and in spins J_f. This is illustrated[2] by the schematic "contour plot" of Fig. 1. At lower energies than the neutron binding energy B, the levels de-excite by gamma-ray emission forming F as a product. For $E_f > B$ they either emit a neutron to form P, or emit gamma rays only, remaining F. At any point (E_f, J_f), the branching ratio depends on the energies and spins of the energetically available levels in P.

Plotted as solid circles in Fig. 1 are the lowest energy level at each spin, J_p in nucleus P, for the first few spins[3] (the excitation energy E_p and spin J_p coordinates of nucleus P are labeled at top and right, respectively, the two energy scales being so adjusted that $E_p = 0$ when $E_f = B$). For most nuclei a few such levels are known or can be estimated, but many of the corresponding lowest levels, especially those at higher J_p's, remain unknown. It is postulated that the unknown points should display a "trend", most of them being found in a restricted region such as the shaded zone[4] in Fig. 1. The energies corresponding to the lowest product level at each spin are designated in this paper by the symbol E_j, where the spin J_p is designated by the subscript.

Branching into products F and P is conveniently discussed in terms of the E_j, using Fig. 1. Notice that Fig. 1 has been so constructed that the emission energy $\epsilon = E_f - (E_p + B)$ is represented by the horizontal distance between points (E_f, J_f) and (E_p, J_p), while the minimum angular momentum carried away by the neutron is represented by the vertical distance (the neutron's intrinsic spin, 1/2, is neglected). Three cases are considered.

Case 1. $B < E_f < E_j + B$; indicated by a square. Compared with its energy the neutron must usually carry off a large angular momentum to reach any available level. The centrifugal barrier then reduces the rate of neutron emission[5] below that of gamma-ray emission and the final product is usually F.

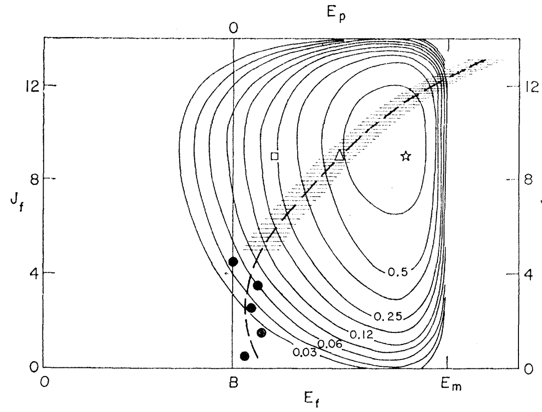

FIG. 1. The "contours" refer to the population distribution with respect to E_f and J_f in nucleus F. They are drawn roughly to scale for $a = 5$ MeV^{-1}, and for 25-MeV helium ions incident on silver. The solid points represent the "known" values[3] of E_j, while the shaded zone represents the region in which one expects to find most of the unknown E_j's. The dashed line represents $\bar{E}_j$, and above $J_p = 5$ is arbitrarily drawn for $\mathcal{I} = 0.27\,\mathcal{I}_r$ in In^{111}. See text.

Case 2. $E_f < E_j + B$; indicated by a star. De-excitation favors neutron emission if, as here, s- and p-wave neutrons can be emitted with a few tenths of a MeV or more to populate several to many product levels. This can be seen by the insertion into Eq. (18) in reference 1 of reasonable values for the transmission coefficients T_l (e.g., 3×10^{-2} to 3×10^{-1} for $\epsilon = 0.1$ MeV and $l = 0$ or 1), and for the ratio of average radiation width to average level spacing Γ_γ/D (e.g., 10^{-4} to 10^{-2} if evaluated at $E_f = B$ and $J_f = 0$). For J_f const as E_f grows larger, the neutron emission energy, and therefore the $\sum_l T_l$ for each product level increases, while additional levels also become available for s- and p-wave neutrons. Meanwhile Γ_γ/D increases less rapidly, about as fast as the level density itself increases. Also, as E_f increases, an additional branching to P arises because gamma-ray emission will increasingly populate levels in F which again have sufficient excitation energy to emit neutrons. Thus, neutron emission soon overtakes gamma-ray emission.[6]

Case 3. $E_f \approx E_j + B$; indicated by a triangle. In this region, neutron emission may or may not predominate, depending in a detailed way on whether the available product levels permit a favorable combination of spin-change and emitted energy.

From the above argument one sees that one needs a reasonably good knowledge of the levels E_j for the calculation of approximate excitation functions, while the higher levels need not be known as well.

[2] The use of contour lines to illustrate the two-dimensional distribution of population in the levels of nucleus F is adapted from T. D. Thomas (unpublished).

[3] The illustrated points are based on a definite example, the first few known and estimated levels in In^{111}.

[4] The width and location of the indicated zone correspond to suggestions made later in this paper.

[5] A rough rule-of-thumb for E_f not too much higher than B and for J_f not too large is that if $l \gtrsim 1 + 2\sqrt{\epsilon}$, where $l\hbar$ is the minimum angular momentum which can be carried away by a neutron of energy ϵ (in Mev, where $\epsilon > 0.25$ MeV) to populate a single level, then gamma-ray emission will be faster than the neutron emission to that level. For increasing E_f, the above rule increasingly underestimates the effectiveness of the competitive gamma-ray emission.

[6] This can also be seen qualitatively from an inspection of (n,γ) excitation functions (see reference 27) in which, at $\epsilon = 0.1$ MeV, the competing neutron emission from the compound nucleus proceeds mainly via a mixture of s and p waves. At $\epsilon = 0.1$ MeV the ratio $\sigma(n,\gamma)/[\frac{1}{2}\sigma(\text{total})]$ almost always has values in the neighborhood of 0.005 to 0.2, even for targets heavier than $A \approx 50$. In almost all cases this ratio decreases rapidly as ϵ increases.

As most of the E_j are unknown, they must be estimated. It seems reasonable to approximate the E_j with a "smooth" function $\bar{E}_j$, shown in Fig. 1 as a dashed line; the estimation of $\bar{E}_j$ and a discussion of the scatter of E_j about $\bar{E}_j$ is deferred to a separate subsection.

One anticipates that using the known levels in the calculation described in reference 1 (called hereinafter I), an excitation function which "fits" the data could be calculated as far as possible above threshold, and then at higher energies the excitation function could be approximated using the $\bar{E}_j$'s and estimated higher levels in a continuation of the same calculation. This calculation would, however, be impractically elaborate. In addition, the parameter adjustment necessary to secure a good fit near threshold seems irrelevant to the purpose of this study, because, as has already been implied, well above threshold the excitation functions are relatively insensitive to one of the parameters important near threshold, i.e., to the average rate of gamma-ray emission. Since it is adequate for this exploratory investigation that the calculated excitation functions be reasonable approximations only well above threshold, the $\bar{E}_j$'s were used in place of even the known E_j's, so that the calculations could be carried out according to the simplified scheme described in the following paragraph.

Rather than calculating branching ratios for every point (E_f,J_f), it seems sufficiently accurate, if one is already forced to use the approximation $\bar{E}_j$, to assign entirely to product F all the population for which $E_f<B+\bar{E}_j$, and entirely to product P all the population for which $E_f>B+\bar{E}_j$. Thus, the cross-section ratio $\sigma_p/(\sigma_p+\sigma_f)$ may be written

$$\sigma_p/(\sigma_p+\sigma_f)\approx\int_0^\infty\int_{B+\bar{E}_j}^{E_m}W(E,J)dEdJ\Big/\int_0^\infty\int_0^{E_m}W(E,J)dEdJ,\quad(2)$$

where $W(E,J)$ is the distribution of population in F, with respect to E and J, where $E_m=E_b-Q_f$ is the maximum energy F can have, E_b being the center-of-mass bombarding energy and Q_f the threshold for forming F, and where σ_f and σ_p are the cross sections for forming F and P, respectively, as final products. It is to be understood that $W(E,J)$ also depends on E_b.

The calculation of $W(E,J)$ and $\bar{E}_j$ is discussed in the two following subsections.

Calculation of $W(E,J)$

The tedious calculation of $W(E,J)$ outlined in I is unnecessarily elaborate for use with Eq. (2), so it seems appropriate to use one of the approximate methods also described in I, instead. Thus, for neutron emission from heavy nuclei, $W(E,J)$ may be approximated by the product of two functions, $V(J)$ and $Y(E)$, which depend only on J and E, respectively, provided the effective nuclear moment of inertia is not much less than the rigid-sphere value[7]; this is assumed to be the case for excitation energies higher than the neutron binding energy.[8,9] Thus,

$$W(E,J)\approx Y(E)V(J).\quad(3)$$

It is compatible with the present accuracy to obtain $Y(E)$ from Jackson's formula,[10] which yields

$$Y(E)\propto(E_m-E)^{2\nu-1}\exp(E/\tau),\quad(4)$$

where $\nu\geqslant1$ is the number of neutrons emitted from C to form F, and τ is an adjustable parameter sometimes called the "nuclear temperature". The value of τ is a measure of the rate of increase of the nuclear level density with increasing excitation energy. Its relationship to the parameters appearing in the commonly used level density expressions is not straightforward, and is discussed for the special case $\nu=1$ in a following section.

For the medium-energy bombardments being considered here, a neutron evaporation does not appreciably change the already rather broad distribution of population in J, because most of the neutrons are emitted with small angular momenta.[11,12] Thus, for ν not too large one might directly use for $V(J)$ the distribution calculated for the compound nucleus[13] (neglecting the intrinsic spin of the neutron):

$$V(J)\propto\sigma_c(J,E_b)=\pi\lambda\!\!\!^{-2}\frac{2J+1}{(2J_t+1)(2s+1)}\sum_{S=|J_t-s|}^{J_t+s}\sum_{l=|J-S|}^{J+S}T_l(E_b),\quad(5)$$

where $\sigma_c(J,E_b)$ is the cross section for forming C with spin J using projectiles of incident energy E_b. The spins of the target and projectile are J_t and s, respectively, and $T_l(E_b)$ is the transmission coefficient[13] for projectiles incident with orbital angular momentum $\hbar l$.

If a sharp-cutoff approximation is used for $T_l(E_b)$, i.e., taking $T_l(E_b)=1$ for $l\leqslant l_m$ and $T_l(E_b)=0$ for $l>l_m$, then, neglecting J_t and s, Eq. (5) becomes

$$V(J)\propto\sigma_c(J,E_b)=\pi\lambda\!\!\!^{-2}(2J+1)\quad\text{for }J\leqslant l_m(E_b)=J_m(E_b),\quad(6)$$
$$V(J)\propto\sigma_c(J,E_b)=0\quad\text{for}\quad J>J_m(E_b).$$

[7] Alternatively, this approximation is also applicable if the product of the nuclear temperature and the moment of inertia is energy independent, provided that most of the neutrons are emitted with low angular momenta; the moment of inertia can then be smaller than the rigid-sphere value.

[8] D. W. Lang, Nuclear Phys. **26**, 434 (1961).

[9] C. D. Bowman, E. G. Bilpuch, and H. W. Newson, Ann. Phys. (New York) **17**, 319 (1962); however there is also some evidence that near 7-MeV excitation the moment of inertia could be substantially lower than the rigid-body value; see J. H. Carver and G. A. Jones, Nuclear Phys. **19**, 184 (1960).

[10] J. D. Jackson, Can. J. Phys. **34**, 767 (1956).

[11] T. D. Thomas (unpublished).

[12] C. T. Bishop, Argonne National Laboratory Report ANL-6405, 1961 (unpublished).

[13] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), Chap. VIII.

From Eq. (6),

$$\sigma_c(E_b)=\sum_J \sigma_c(J,E_b)=\pi\lambda\!\!\!^{-2}[J_m(E_b)+1]^2, \tag{7}$$

from which

$$J_m(E_b)=[\sigma_c(E_b)/(\pi\lambda\!\!\!^{-2})]^{1/2}-1, \tag{8}$$

where $\sigma_c(E_b)$ is the cross section for forming a compound nucleus. The approximation represented by Eqs. (6) to (8) is often appropriate for sufficiently high energies of sufficiently massive incident particles, but must be used with discretion.[12,14]

Calculation of $\bar{E}_j$

Let $\omega_1(E,J)dE$ be the probability that the lowest level of spin J in P occurs between energies E and $E+dE$. Then,

$$\bar{E}_j=\int_0^\infty E\omega_1(E,J)dE. \tag{9}$$

Likewise, interpreting the spin-dependent level density $\omega(E,J)$ to be a function such that $\omega(E,J)dE$ is the probability of finding a level of spin J between energies E and $E+dE$, one finds that[15]

$$\omega_1(E,J)=\omega(E,J)\exp\left[-\int_0^E \omega(x,J)dx\right]. \tag{10}$$

Only limited guidance to the form of $\omega(E,J)$ near the lowest level of spin J is provided by available experimental data. For a few nuclei (mainly medium mass) most of the levels having energies less than four or five MeV have been observed. Ericson[16] has examined these data from a statistical viewpoint, choosing to define the level density $\omega(E)$ [i.e., where $\omega(E)=\sum_J \omega(E,J)$] above the tenth level. He finds the data consistent with a simple exponential increase of level density with excitation energy,

$$\omega(E)\propto\exp(E/T), \tag{11}$$

corresponding to an energy-independent "nuclear temperature" T. The scatter of the observed level spacings about the values predicted by extrapolations of $\omega(E)$ into the region below the tenth level seems to be no more than is compatible with the interpretation that $\omega(E)dE$ is the probability that a level occurs between E and $E+dE$. Therefore, in the calculations reported here, the simple exponential form of $\omega(E)$ given by Eq. (11) is used below the tenth level, although it is applied for a somewhat heavier than medium mass nucleus. There seem to be no comparable experimental data to help indicate how $\omega(E)$ should be broken into its constituent $\omega(E,J)$'s. For purposes of orientation, the theoretical expression[17] for the J dependence of the *level* density $\omega(E,J)$,

$$\omega(E,J)\cong\omega(E,0)(2J+1)\exp[-J(J+1)/2\sigma^2], \tag{12}$$

is used, although the energies below which, and/or the spins above which it becomes invalid[18] seem not to have been investigated experimentally. Here $\sigma^2=(\mathcal{I}/\hbar^2)T$, where $\mathcal{I}$ is a moment of inertia. For the reported calculations, T and $\mathcal{I}$ are assumed independent of E and J.

In terms of Eqs. (11) and (12), the *state* density[19] $\rho(E)$ is also exponential; i.e.,

$$\rho(E)=\sum_J (2J+1)\omega(E,J)=C\exp(E/T). \tag{13}$$

The spin- and energy-independent constants C and T can usually be reliably estimated or obtained from experimental data. From Eqs. (12) and (13) the level density becomes

$$\omega(E,J)=(K_j/T)\exp(E/T), \tag{14}$$

where K_j is defined by

$$K_j=\frac{CT(2J+1)}{[\sqrt{(8\pi)}]\sigma^3\exp[1/(8\sigma^2)]}\exp\left[-\frac{J(J+1)}{2\sigma^2}\right]. \tag{15}$$

For $\mathcal{I}>(1/10)\mathcal{I}_r$, where $\mathcal{I}_r$ is the rigid-sphere moment of inertia, and for almost all cases of interest, $K_j<1$ and usually $K_j\ll1$.

Finally, insertion of Eq. (14) into Eq. (9) via Eq. (10) gives, after integration,

$$\bar{E}_j=(-T\exp K_j)\,\mathrm{Ei}(-K_j), \tag{16}$$

where $\mathrm{Ei}(x)$ is the exponential integral, values of which may be found tabulated in standard reference books.[20]

Equation (16) is conveniently discussed in terms of the following formula, a simplification of Eq. (16) for $K_j\ll1$;

$$\bar{E}_j\approx\frac{\hbar^2}{2\mathcal{I}}J(J+1)+T\ln\left[\frac{\sqrt{(\pi T)}}{C\exp\mathfrak{C}}\left(\frac{2\mathcal{I}}{\hbar^2}\right)^{3/2}\frac{1}{2J+1}\right], \tag{17}$$

where $\mathfrak{C}\approx0.577$ is Euler's constant. The first right-hand term of Eq. (17) has the form of a "rotational energy", while the second term, which is positive at low J (for reasonable values of $\mathcal{I}$), could be regarded as the upward

[14] R. Vandenbosch, H. Warhanek, and J. R. Huizenga, Phys. Rev. **124**, 846 (1961).

[15] If $P(E,J)$ is the probability that a level of spin J does not lie between energies 0 and E, then $dP=-P\omega dE$, $P=\exp[-\int_0^E\omega dE]$, and $\omega_1=\partial(1-P)/\partial E$, which leads to Eq. (10).

[16] T. Ericson, Nuclear Phys. **11**, 481 (1959). Ericson works with the function $N(E)$, the total number of levels with energies less than E. He points out that if $N(E)$ depends on E like a pure exponential [consistent with the data he analyzes, for $N(E)>10$] then $\omega(E)$ does also.

[17] H. A. Bethe, Revs. Modern Phys. **9**, 84 (1937); and many others (see reference 19, and appropriate references in I).

[18] T. Ericson, in *Proceedings of the International Conference on Nuclear Structure, Kingston, Canada* (University of Toronto Press, Toronto, 1960), p. 697; Suppl. Phil. Mag. **9**, 425 (1960).

[19] J. M. B. Lang and K. J. LeCouteur, Proc. Phys. Soc. (London) **A67**, 586 (1954).

[20] E. Jahnke and F. Emde, *Tables of Functions* (Dover Publications, New York, 1945), pp. 1–9. U. S. National Bureau of Standards, National Applied Mathematics Laboratories, Computation Laboratory, *Tables of Sine, Cosine and Exponential Integrals*, Vol. 1 (1940), Tech. Dir., A. N. Lowan.

adjustment necessary for compatibility with the known level density. (For deformed even-even nuclei whose first few even spin-even parity excited states have excitation energies proportional to $J(J+1)$, such an upward adjustment might be small or unnecessary.) As J increases, the second term decreases and finally becomes negative. Beyond this point Eq. (17) would require the nucleus to have more energy in rotation than it has excitation energy. However, before J achieves values this high, Eq. (12) is theoretically expected to become inaccurate[18] and overestimate $\omega(E,J)$; from Eq. (9) one sees that $\bar{E}_j$ calculated by Eqs. (16) or (17) would then be too small because too much probability would be given to the low energies. Quasi-classically, if high E_j and J reflect a situation in which all of the nuclear excitation has gone into rotating the nucleus, it seems reasonable to expect that for sufficiently large E_j

$$\bar{E}_j \approx (\hbar^2/2\mathcal{I})J(J+1) = E_r. \tag{18}$$

Sperber[21] has estimated the lowest energy eigenvalue corresponding to any given high angular momentum for four different models (liquid drop, "rotating bag full of fermions", Fermi gas in a harmonic oscillator potential, shell model for closed shell nuclei), and in each case finds that the energy of the lowest level depends approximately quadratically on angular momentum. He defines an effective nuclear moment of inertia in terms of the constant of proportionality, through an expression similar to Eq. (18).

The $\bar{E}_j$ used in the investigation reported here were,

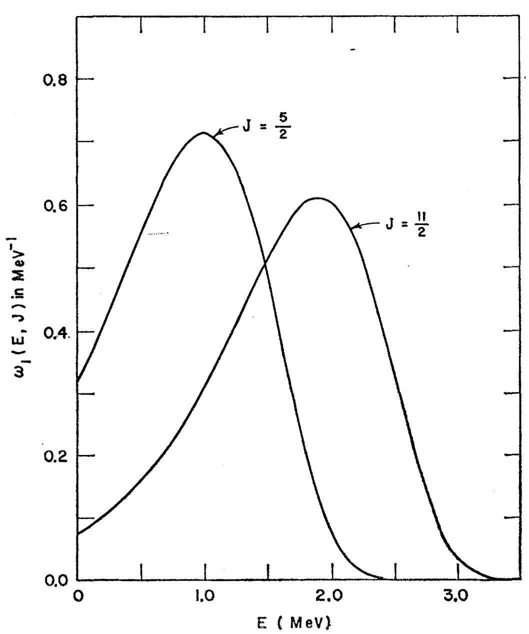

FIG. 2. Examples of the function $\omega_1(E,J)$.

[21] D. Sperber, Ph.D. thesis, Princeton University, 1961 (unpublished). Also see Bull. Am. Phys. Soc. **6**, 77 (1961).

therefore, calculated as follows: (a) Use Eq. (16) if it gives $\bar{E}_j > E_r$; (b) use Eq. (18) otherwise.

An idea of the amounts by which the E_j are expected to scatter about the calculated $\bar{E}_j$ may be obtained from Fig. 2, in which are plotted examples of $\omega_1(E,J)$ for In^{111}, calculated using Eq. (14) for $\omega(E,J)$, setting $\mathcal{I} = 0.27\mathcal{I}_r$, $C = 8.9$ MeV^{-1} and $T = 0.65$ MeV. One sees that the full width at half-maximum is about $2.3T$. At sufficiently high J that one must use Eq. (18) instead of Eq. (16), $\omega(E,J)$ decreases more steeply with decreasing energy than Eq. (14) would predict, near $E \approx \bar{E}_j$; it can be seen from Eq. (10) that the width at half-maximum should then be less than $2.3T$. The shaded zone in Fig. 1 is drawn to scale to correspond to a constant width of $2.3T$ and therefore overestimates the expected scatter somewhat at the larger J's.

In view of the uncertainty in the estimate of $\omega(E,J)$, the calculation of $\bar{E}_j$ embodied in Eqs. (16) and (18) must be regarded as only a first orientation, or simply as a convenient way to parametrize the unknown distribution of E_j's for the purpose of making exploratory studies. How the parameter $\mathcal{I}$ may be related to the nuclear moments of inertia measured by other experimental methods is problematical. In particular, the value of $\mathcal{I}$ used for the calculation of $\bar{E}_j$ when $\bar{E}_j \approx B$ probably need not be the same as that of the moment of inertia which was assumed to have a value near $\mathcal{I}_r$ in the justification of Eq. (3). In the absence of experimental knowledge of the connection (if any) between the moment of inertia appropriate for expressing the dependence of level density on spin, and the "moment of inertia" appropriate for expressing the "cutoff" of nuclear levels which is approximated by $\bar{E}_j$, the two moments of inertia are treated in this calculation as if they were independent of each other.

COMPARISON WITH EXPERIMENT

The data of Bleuler *et al.*[22] for the reactions $Ag^{109}(\alpha,n)$-In^{112} and $Ag^{109}(\alpha,2n)In^{111}$ were used for a test calculation.

Owing to the large atomic number and relatively high proton binding energy of In^{112}, charged particle emission is suppressed, and may be neglected. Justification for this is suggested by the small cross sections observed for the reactions[23] $Ag^{107}(\alpha,pn)Cd^{109}$ and $Ag^{107}(\alpha,\alpha n)Ag^{106}$ (8.3-day isomer only). One sees that $\sigma(\alpha,pn)/\sigma(\alpha,2n) \lesssim 0.05$ and $\sigma(\alpha,\alpha n) \ll \sigma(\alpha,pn)$ within 8 MeV above the $(\alpha,2n)$ threshold. Also, one expects charged particle emission to be even less important for In^{112*} than for In^{110*}.

Contributions from direct interactions are neglected. Although this might be a serious omission for the (α,n) reaction well above the maximum in its excitation

[22] E. Bleuler, A. K. Stebbins, and D. J. Tendam, Phys. Rev. **90**, 460 (1953).

[23] S. Fukushima, S. Hayashi, S. Kume, H. Okamura, K. Otozai, K. Sakamoto, and Y. Yoshizawa, Osaka University, Osaka, Japan, privately circulated report, 1962 (unpublished).

function,[24] the data used here cover an energy range up to only about 4 MeV above threshold, near the maximum of the (α,n) curve where the direct-interaction contribution is expected to be unimportant.

That these data were taken near enough to threshold for an internal estimate of the threshold energy[25] to be made is particularly important, because range-energy curves are not yet sufficiently accurate to allow critical comparisons of calculations made using handbook threshold values with experimental data in which the accelerator-beam energy is determined via range measurements, as is the case in much reported work.

Since the cited experimental work was published, an isomer of In^{111} has been reported,[26] which has a half-life of about 10 min, a spin parity expected to be $1/2-$, and which lies 0.53 MeV above the ground state. Estimates based on the known levels in Cd^{111} indicate that positron- and electron capture-decay accounts for less than (and probably much less than) 15% of the disintegrations of In^{111m}. No attempt has been made to correct the data for this effect.

Values of C and T for In^{111} were calculated from the following estimates: (i) from the systematics of low-lying levels in neighboring odd-even nuclei[26] it was estimated that $\rho(0.64)\approx 24$ states per MeV; (ii) from the level spacings observed between neutron resonances[27] for several neighboring even-odd nuclei it was estimated that $\omega(6.8,0)\approx 5000$ levels per MeV [i.e., $\omega(6.8,1/2)\approx 2\omega(6.8,\,1/2+)\approx 10\,000$ levels/MeV], assuming that this level density is also applicable for a nearby odd-even nucleus. The rigid-sphere moment of inertia $\mathcal{I}_r$ was calculated using the formula $\mathcal{I}_r=(2/5)MR^2$, where M is the mass of the nucleus and $R=(1.2\times 10^{-13})A^{1/3}$ cm, taken from Hofstadter's review paper.[28] It is not possible to find a unique pair of values for C and T from these estimates, because one must know the moment of inertia to calculate $\rho(E)$ from $\omega(E,0)$, i.e.,

$$\rho(E)\approx(8\pi)^{1/2}(\mathcal{I}/\hbar^2)^{3/2}T^{3/2}\omega(E,0). \quad (19)$$

Using Eqs. (12) and (13), and the estimates given above, the relation between T and $\mathcal{I}$ works out to be

$$6.16/T-(3/2)\ln T=12.30+(3/2)\ln(\mathcal{I}/\mathcal{I}_r). \quad (20)$$

[24] I. Dostrovsky, Z. Fraenkel, and G. Friedlander, Phys. Rev. **116**, 683 (1959); R. L. Hahn and J. M. Miller, Phys. Rev. **124**, 1879 (1961).

[25] Threshold determinations performed by extrapolations of excitation function data according to formulations devised neglecting competitive gamma-ray emission should be re-examined. In this case, the cross sections appear to have been measured close enough to threshold that the derived threshold value is not very sensitive to the form of extrapolation used.

[26] K. Way, *Nuclear Data Sheets*, National Academy of Sciences, National Research Council (U. S. Government Printing Office, Washington, D. C., 1961); K. Way, N. B. Gove, C. L. McGinnis, and R. Nakasima, in *Landolt-Börnstein, Energy Levels of Nuclei: A=5 to A=257*, edited by A. M. Hellwege and K. H. Hellwege (Springer-Verlag, Berlin, 1961), Vol. I, p. 252.

[27] *Neutron Cross Sections*, compiled by D. J. Hughes and R. B. Schwartz, Brookhaven National Laboratory Report, BNL-325 (U. S. Government Printing Office, Washington, D. C., 1958).

[28] R. Hofstadter, Ann. Rev. Nuclear Sci. **7**, 231 (1957).

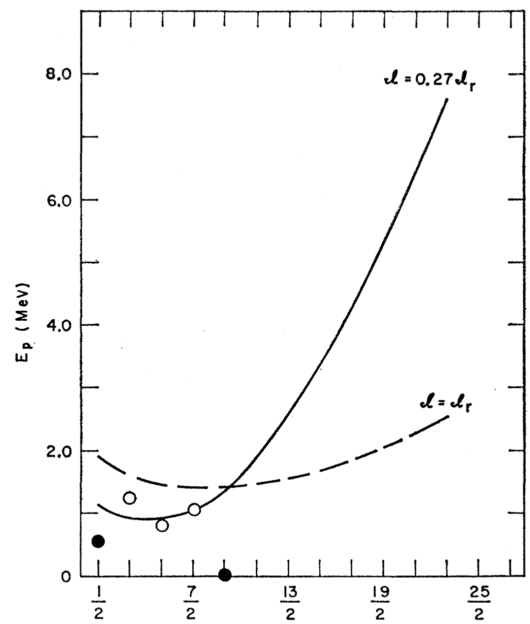

FIG. 3. Examples of $\bar{E}_j$ calculated for In^{111}. The distribution of $\bar{E}_j$ is indicated by the solid line for $\mathcal{I}=0.27\mathcal{I}_r$ and by the dashed line for $\mathcal{I}=\mathcal{I}_r$.

The dependence of $\bar{E}_j$ on $\mathcal{I}$ through T (and C) implied by Eq. (20) is, however, very weak compared to the dependence of $\bar{E}_j$ on $\mathcal{I}$ through σ^3 in Eq. (15), or on $\mathcal{I}$ directly in Eq. (18), and does not affect the final results appreciably.

Examples of $\bar{E}_j$ calculated for In^{111} are shown in Fig. 3. The solid curve is for $\mathcal{I}=0.27\mathcal{I}_r$, $C=8.9$ MeV^{-1} and $T=0.65$ MeV, while the dashed curve is for $\mathcal{I}=\mathcal{I}_r$, $C=7.5$ MeV^{-1} and $T=0.55$ MeV. The two known levels, at $J=1/2$ and $J=9/2$, are plotted as solid circles, and three estimated levels are plotted as open circles; these five levels are assumed to be the lowest five levels. Considering the expected scatter, (see Fig. 2) both curves are consistent with the few known and estimated E_j's. However, that all the points are low for $\mathcal{I}=\mathcal{I}_r$ is slightly prejudicial for using $\mathcal{I}<\mathcal{I}_r$ in Eqs. (16) and (18). Perhaps a slightly stronger argument for $\mathcal{I}<\mathcal{I}_r$ arises because $\bar{E}_j$ for $\mathcal{I}=\mathcal{I}_r$ is rather flat to quite high J's, so that, considering the expected scatter, one might expect to find levels with high spins (of the order of 15/2 to 25/2) rather frequently below 1.5 MeV in the neighboring nuclei, if the $\bar{E}_j$ function appropriate for them is similar to that appropriate for In^{111}; such levels do not seem to be common. In the absence of clearly contradictory experimental evidence, however, the possibility that $\mathcal{I}\approx\mathcal{I}_r$ was not rejected in performing the reported calculations.

In most of the calculations, the sharp-cutoff approximation of Eqs. (6)–(8) was used for $V(J)$, and Eq. (4) with $\nu=1$ was used for $Y(E)$. For the compound nucleus-formation cross section, it was assumed that $\sigma_c(E_b)\approx\sigma(\alpha,n)+\sigma(\alpha,2n)$ up to $E_b=18.5$ MeV (the

highest energy attained by Bleuler *et al.*[22]), while from $E_b=15.4$ up to $E_b=23.3$ MeV the data of Porges[29] (which match the data of Bleuler *et al.* almost exactly where they overlap) for the reactions of Ag^{107} with helium ions were used, assuming that $\sigma_c(E_b)\approx\sigma(\alpha,n)+\sigma(\alpha,2n)+\sigma(\alpha,pn)$; a smooth curve drawn through these data was used for $\sigma_c(E_b)$. The associated values of $J_m(E_b)$ increased from 6.1 at $E_b=16.0$ MeV (c.m.) to 12.7 at $E_b=23.3$ MeV.

Typical calculated excitation functions are shown as solid curves in Fig. 4. Because of the invalidity of Eq. (2) near threshold, the calculated curves are not extended below $\mathcal{E}=2$ MeV, where $\mathcal{E}$ is the center-of-mass bombarding energy in excess of the $(\alpha,2n)$ threshold. For comparison, the experimental data are plotted as open circles.

No combination of τ and $\mathcal{I}$ gives a unique fit; the locus of points $(\tau,\mathcal{I})$ which give a "fit" at $\mathcal{E}=3.7$ MeV is shown as a solid line in Fig. 5(a). The shaded zone reflects the quoted experimental uncertainty. To test the sharp-cutoff approximation used for $V(J)$, a single point was calculated using Eq. (5) and the transmission coefficients of Huizenga and Igo[30]; the result is plotted as an open circle in Fig. 5(a) and 5(b).

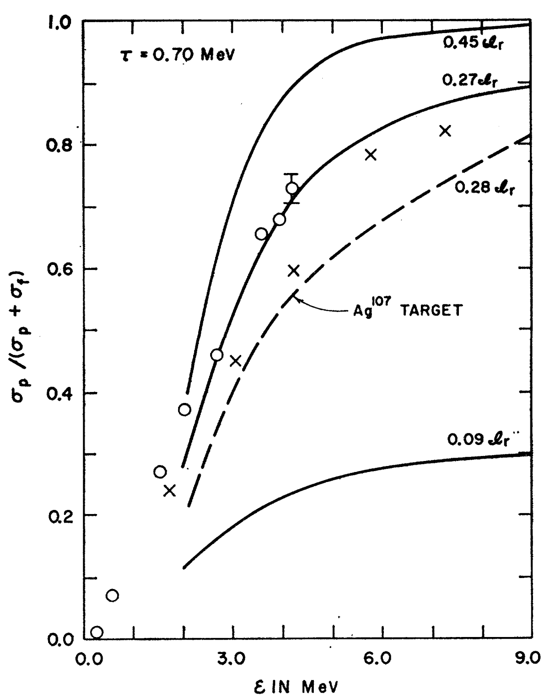

FIG. 4. Examples of some calculated excitation functions for the reactions $Ag^{109}(\alpha,2n)In^{111}$ (solid lines) and $Ag^{107}(\alpha,2n)In^{109}$ (dashed line). The open circles are the data of Bleuler *et al.* (reference 22) for a Ag^{109} target, and the ×'s are the data of Fukushima *et al.* (reference 23) for Ag^{107}.

Experimental results for the reaction-pair $Ag^{107}(\alpha,n)$-In^{110} and $Ag^{107}(\alpha,2n)In^{109}$, just recently reported by Fukushima *et al.*,[23] are also plotted (as ×'s) in Fig. 4. An excitation function was calculated for comparison with these data, using the same $\sigma_c(E_b)$ curve, and the same $\bar{E}_j$'s as were used for In^{111} with $\mathcal{I}/\mathcal{I}_r=0.27$, and is shown as a dashed curve[31] in Fig. 4. This comparison is only meant to be qualitative, for there is no "internal" $\alpha,2n$ threshold measurement connected with these data; the literature[32] value of 15.6 MeV was used instead.

It is desirable to interpret τ in terms of a level density expression which has some theoretical justification, such as[8,18,33]

$$\bar{\omega}(E)\approx(\text{const})E^{-2}\exp[2(aE)^{1/2}]. \tag{21}$$

The "relationship" between τ and a is a little arbitrary since the shape of an excitation function calculated using expression 4 is slightly different from the shape obtained using Eq. (21). For the case $\nu=1$, this relationship was therefore defined to be

$$\frac{\int_{B+\Delta}^{E_{1/2}}(E_{1/2}-E)\exp(E/\tau)dE}{\int_0^{E_{1/2}}(E_{1/2}-E)\exp(E/\tau)dE}\equiv\frac{\int_{B+\Delta}^{E_{1/2}}(E_{1/2}-E)E^{-2}\exp[2\surd(aE)]dE}{\int_{1\text{ MeV}}^{E_{1/2}}(E_{1/2}-E)E^{-2}\exp[2\surd(aE)]dE}\equiv\frac{1}{2}, \tag{22}$$

where Δ is a constant which takes roughly into account the average "shift of threshold" associated with $\bar{E}_j$ (it was assumed that $B+\Delta=10$ Mev). The resulting locus of values $(\mathcal{I},a)$ which fit the data is shown in Fig. 5(b). Also shown, as two vertical lines, is the range encom-

[29] K. G. Porges, Phys. Rev. **101**, 225 (1956).

[30] J. R. Huizenga and G. Igo, Argonne National Laboratory, Argonne, Illinois, privately circulated report, 1961 (unpublished); see also J. R. Huizenga and G. Igo, Nuclear Phys. **29**, 462 (1962); a table of transmission coefficients for helium ions incident on silver, calculated using H and I's computer program, is given in reference 12.

[31] The difference between the solid curve for $\mathcal{I}=0.27\mathcal{I}_r$ and the dashed curve arises because the threshold for the $Ag^{107}(\alpha,2n)In^{109}$ reaction is higher than that for the $Ag^{109}(\alpha,2n)In^{111}$ reaction, so that higher values of $\sigma_c(E_b)$, and hence higher values of $J_m(E_b)$, are necessary for the Ag^{107} target than for the Ag^{109} target. Such a sensitivity of $\sigma_p/(\sigma_p+\sigma_f)$ to the total cross section for compound nucleus formation is not predicted if competitive gamma-ray emission is neglected.

[32] F. Everling, L. A. Konig, J. H. E. Mattauch, and A. H. Wapstra, Nuclear Phys. **18**, 529 (1960).

[33] For a more accurate calculation of $W(E,J)$, an expression such as (references 8, 18, 19)

$$\omega(E,J)\approx(\text{const})(2J+1)\exp[-(\hbar^2/2\mathcal{I})(a/E)^{1/2}J(J+1)]E^{-2}\times\exp[2(aE)^{1/2}]$$

could be used in the prescription given in I. However, Eq. (21) appears to be a plausible simplification to use here, subject to the condition $[\hbar^2/(2\mathcal{I})]J(J+1)\ll E$ in order that the energy-dependence of the spin-dependent term can be neglected, and provided that most particles are emitted with low angular momenta.

passed by 12 values of a calculated by Lang[8] from level spacings observed at the neutron binding energy in the mass region $A=105$ to 120. The same author's analysis of neutron spectra from four (n,n') reactions yields values of a from 10 MeV^{-1} to 22 MeV^{-1} in this mass region.

Since many workers use a level density expression of the form

$$\bar{\omega}(E) \propto \exp[2\sqrt{(a'E)}], \tag{23}$$

the conversion from τ to a' was also worked out, and the resulting a' scale is given at the top of Fig. 5(b).

DISCUSSION

Probably the most important conclusions to be drawn from the reported calculation are (i) that the shape of the excitation function is governed as strongly by the distribution of E_j's as by the value of a, and (ii) that any plausible distribution of the E_j's which is assumed implies a corresponding value of a substantially larger than the value which results when competitive gamma-ray emission is neglected. Thus, that values of a' from most measurements of the spectra of emitted particles disagree with the values of "a'" from excitation functions which were analyzed neglecting competitive gamma-ray emission,[34] is not a discrepancy.

It is necessary to be cautious about assigning a meaning to the parameter $\mathscr{I}$. One concludes from Fig. 5(b), that $\mathscr{I}\approx 0.3$ to $0.5\mathscr{I}_r$ is most consistent with the results of the present calculations, combined with plausible values of a from other experiments. However, as can be seen from Fig. 2, this only means that the lowest levels of spin $J=11/2$ to $17/2$ in In^{111} ($J_m\approx 17/2$ when $\mathcal{E}=4$ MeV) tend on the average to have energies in the neighborhood of 2 to 2.5 MeV. Clearly, distributions of E_j other than that represented by $\bar{E}_j$ could be found which would allow "fits" to the data for $a=14$ to 18 MeV^{-1}, but any plausible distribution would nevertheless have to be such that the lowest spin 11/2 to 17/2 levels possess roughly those energies, on the average, as have already been found to be necessary if one uses $\bar{E}_j$.

Sperber's calculations[21] for the shell model (probably the most realistic of the four models he considered) suggest that the lowest spin 11/2 to 17/2 levels should occur at excitation energies in the neighborhood of 1.5 to 2 MeV on the average [corresponding roughly to $\mathscr{I}/\mathscr{I}_r\approx 0.5$ in Fig. 5(b)]. These energies are perhaps a little low; however, as can be seen from Fig. 5(b), considering the broad limits imposed by the experimental error together with the uncertainty in the assignment of a value for a, and especially since the simplified calculation used here seems to yield values of $\mathscr{I}/\mathscr{I}_r$ which are somewhat too small, it is seen that Sperber's result is consistent with the experimental data analyzed in this report.

[34] G. Igo and H. E. Wegner, Phys. Rev. **102**, 1364 (1956).

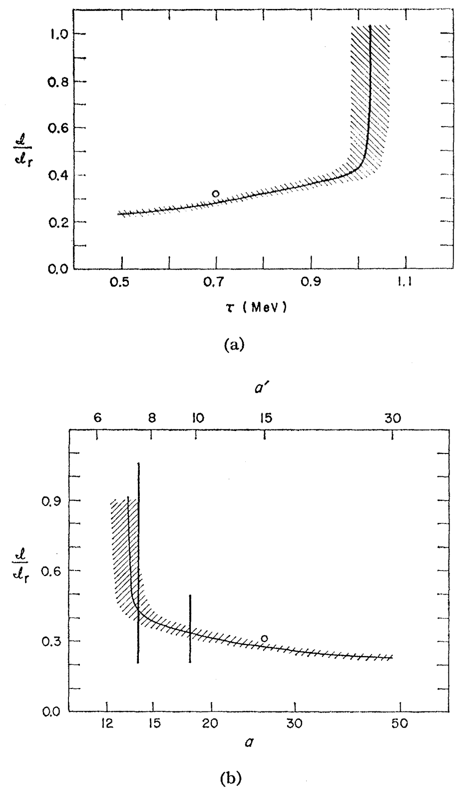

FIG. 5(a). The solid line is the locus of pairs of values of $\mathscr{I}$ and τ for which excitation functions can be calculated which "fit" the data of Bleuler *et al.*[22] (See text.) (b). The same as (a), except that the τ scale has been replaced by the scales for a (bottom) and a' (top) to which it is roughly equivalent. (See text.)

The influence on excitation functions of a distribution of E_j's even qualitatively similar to the surmise represented by Eqs. (16) and (18) should be relatively larger for those systems into which relatively broader distributions of angular momenta are introduced, e.g., in heavy-ion bombardments of sufficiently high incident energy. An attempt to discover such an effect in the neighborhood of $A\approx 100$ is represented by Table I, in which are compared values of "a'" obtained from $(X,2n)$ excitation functions which were analyzed neglecting competitive gamma ray emission (i.e., equivalent to setting $\bar{E}_j=0$ for all angular momenta in Eq. (2)]. Here, X can be a neutron or a helium ion. The values of "a'" obtained when X is a neutron are usually larger (closer to the values obtained from level counts and particle spectra) than the values obtained when X is a helium ion. This is as expected because in all of these cases the helium ion is capable on the average of bringing in more angular momentum than the neutron, so that if the E_j tend to increase with increasing high J the correction for gamma-ray emission should be relatively larger for the helium ion-induced reactions. A rough indication of the amount of angular momentum brought in for each case is provided by the values of l_m (evaluated at $\mathcal{E}=2.5$ MeV) in column three.

Complementary evidence supporting the above view is provided by the work of Mollenauer,[35] who measured the production of the gamma rays themselves. He prepared the "compound nucleus" Cu^{63*} at an excitation energy of about 48 MeV in two ways (i) by the irradiation of V^{51} with C^{12} ions, (ii) by the irradiation of Co^{59} with helium ions. He calculates that the average angular momentum carried into the system by the carbon ions is $16.5\hbar$, while that carried in by the helium ions is $12.3\hbar$. He finds that the mean total de-excitation of the "compound nucleus" occurring by gamma-ray emission is 9.4 MeV for the $C^{12}+V^{51}$ system and 5.2 MeV for the $\alpha+Co^{59}$ system.

Houck and Miller[36] tried to estimate the importance of competitive gamma-ray emission in their analysis (which neglects angular momentum) of the ratio of the cross sections for the reactions $Fe^{54}(\alpha,n)Ni^{57}$ and $Fe^{54}(\alpha,p)Co^{57}$, by varying the values of the effective separation energies of the most loosely bound particles in Ni^{57} and Co^{57} (these energies are expressed by the symbol S_j' in their paper). They concluded that the pertinent features of their data depend more importantly on the specific values chosen for S_j' than they do on a', a result which adumbrates the conclusions of this paper and may provide some additional confirmation from a somewhat different aspect of excitation function data.

The distribution of the E_j's would be expected to play a strong role in the relative production of isomers in many nuclear reactions. Unfortunately the as yet poorly known characteristics of the complicated gamma-ray cascade (e.g., the number and multipolarity of the gamma rays) in the final de-excitation step are also rather important[12,37] in determining what the final isomer ratio should be, so that attempts to obtain information about the E_j distribution from such data, even if a were known, would give somewhat ambiguous results at present.

[35] J. F. Mollenauer, Phys. Rev. **127**, 867 (1962).

[36] F. S. Houck and J. M. Miller, Phys. Rev. **123**, 231 (1961).

[37] J. R. Huizenga and R. Vandenbosch, Phys. Rev. **120**, 1305 (1960).

TABLE I. Comparison of values of "a'" obtained, neglecting competitive gamma ray emission, from excitation functions for $(n,2n)$ and $(\alpha,2n)$ reactions

				"a'"		
Target	Projectile	l_m	J_t	α's incident	Nucleons incident	Reference
Zr^{90}	n	5.1	0		7.0	a
Mo^{92}	n	5.3	0		$\sim$5	b
Ag^{107}	α	8.3	1/2	3		23
				2		29
Ag^{109}	α	7.1	1/2	2.5		22
Sn^{112}	n	5.3	0		7.8	a
In^{115}	α	7.1	9/2	$\sim$2		c
Cd^{116}	n	4.8	0		$\sim$8	a
Sb^{123}	n	4.8	7/2		$\sim$9	a

[a] D. W. Barr, C. I. Browne, and J. S. Gilmore, Phys. Rev. **123**, 859 (1961). These authors interpreted the data of R. J. Prestwood and B. P. Bayhurst, *ibid.* **121**, 1438 (1961).

[b] J. E. Brolley, Jr., J. L. Fowler, and L. K. Schlacks, Phys. Rev. **88**, 618 (1952). The value of "a'" given by these authors is 3.1 MeV^{-1}, but there is an ambiguity due to the unknown contribution of Mo^{91m}, which was not measured. From "reanalyzing" the data this author prefers $\sim$5 MeV^{-1}.

[c] G. M. Temmer, Phys. Rev. **76**, 424 (1949). Igo and Wegner[34] report a value for a' of $\sim$2 MeV^{-1}, apparently based on these data.

Rather than trying to use excitation function data to extract a, it might be more appropriate to take reliable values of a from other experiments and use excitation functions to obtain information on the distribution of the E_j's, which is itself a very interesting problem. Perhaps the unknown distribution of E_j's could be at least crudely mapped out by the analysis of a Ghoshal experiment,[38] in which a given "compound nucleus" is made in as many different ways as possible, so as to give as many different population distributions in angular momenta as possible. The experiment should be carried out in such a way that all of the relevant reaction thresholds can be determined from the data.

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[38] S. N. Ghoshal, Phys. Rev. **80**, 939 (1950).