Charge Structure of the Nucleon*

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It is suggested that F_{1n}, the charge form factor of the neutron, is negative at low transferred momentum, and that the charge distribution of the proton is such that the bare proton is surrounded not only by a positively charged layer but also by a negatively charged layer because of the strong vacuum polarization due to the three-pion resonance ω.

INTRODUCTION

As is well known, there is an ambiguity in the sign of F_{1n}, the charge form factor of the neutron,[1] obtained from inelastic electron-scattering cross sections in the deuteron by use of the Rosenbluth formula[2] and the method of intersecting ellipses.[3] Since it has been commonly accepted that the neutron is surrounded by the cloud of a negative pion in its outer region,[4] Herman and Hofstadter[5] reported the experimental values of F_{1n} with negative sign in 1960. Last year the Stanford[6] and Cornell[7] groups reported the other values with positive sign. The latter are analyzed by the interesting theoretical idea proposed by Bergia *et al.*,[8] which was based on dispersion theory and on strong pion-pion interactions. The vector boson theory of Sakurai also supports this positive sign.[9] Recently, Hand *et al.*[10] reported experimental values of G_{ch} and G_{mag}[1] which correspond to negative F_{1n}. It is possible, in principle, to determine the sign of F_{1n} by making use of elastic electron-scattering cross sections in the deuteron. The present experimental evidence[11] is not definitive on the sign problem.[12,13]

It is the purpose of this paper (i) to stress the usefulness of three known theorems obtained from γ_5-meson theory in the analyses of the electron-scattering cross sections, (ii) to suggest which sign of F_{1n} is favored from γ_5-meson-theoretical and from the dispersion-theoretical points of view of the influence of strong pion-pion interactions, (iii) to present a model of the physical proton (the bare proton surrounded not only by a positively charged layer but also by a negatively charged layer because of a strong vacuum polarization due to the three-pion resonance ω), and (iv) to explain the smallness of the rms radius $\langle r^2\rangle_{1n}^{\frac{1}{2}}$ of the neutron in terms of the strong vacuum polarization. Recently several pion resonances have been reported. In the present situation with respect to the resonances,[14] it will be shown that the negative sign of F_{1n} is favored.

THEOREMS

We shall first summarize three useful theorems obtained from meson theory, which hold for the electromagnetic form factors of the nucleon.

Theorem I. The total charge contributed from three-pion intermediate states is zero, even under the influence of strong pion-pion interactions. Figure 1(a) shows the most general diagram for three-pion intermediate states. The parts A, B, and C denote the rescattering corrections, two- and/or three-pion resonances, and the closed loop with any radiative corrections, respectively. The four-momentum q is that of the external photon. In part C there are three external pseudoscalar pions. This means that there remains one γ_5 in the spur calculation. When $q=0$, there remain two independent momenta, say p_1 and p_2, after all integrations with respect to internal momenta. Therefore, for part C one obtains

$$\mathrm{Spur}[\gamma_5\gamma_\mu(\gamma\cdot p_1)(\gamma\cdot p_2)]=0, \text{ etc.,} \quad \text{when} \quad q=0. \qquad (1)$$

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[1] In the strict sense, the quantities

$$G_{\mathrm{ch}}(q^2)=F_1(q^2)-(q^2/2M)F_2(q^2),$$

and

$$G_{\mathrm{mag}}(q^2)=(1/2M)F_1(q^2)+F_2(q^2)$$

should be called the charge and magnetic form factors of the nucleon, respectively, where q^2 is the square of the invariant transferred momentum and M is the nucleon mass. Throughout this paper, however, we shall by convention call F_1 and F_2 the charge and magnetic form factors. See F. J. Ernst, R. G. Sachs, and K. C. Wali, Phys. Rev. **119**, 1105 (1960).

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[9] J. J. Sakurai, Ann. Phys. (New York) **11**, 1 (1960); Nuovo cimento **16**, 388 (1960); Phys. Rev. Letters **7**, 355 (1961).

[10] L. N. Hand, D. G. Miller, and R. R. Wilson, Phys. Rev. Letters **8**, 110 (1962).

[11] J. A. McIntyre and G. R. Burleson, Phys. Rev. **112**, 2077 (1958); J. I. Friedman, H. W. Kendall, and P. A. M. Gram III, *ibid.* **120**, 992 (1960).

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[13] N. K. Glendenning and G. Kramer, Phys. Rev. Letters **7**, 471 (1961).

[14] P. L. Bastien, J. P. Berge, O. I. Dahl, M. Ferro-Luzzi, D. H. Miller, J. J. Murray, A. H. Rosenfeld, and M. B. Watson, Phys. Rev. Letters **8**, 114 (1962).

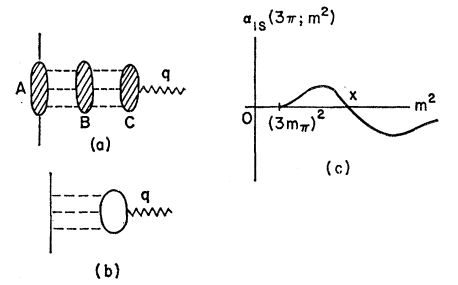

FIG. 1. (a) The most general diagram for three-pion intermediate states. The dashed lines and the wavy line denote pions and the photon, respectively. (b) One of the lowest order diagrams. (c) Schematic representation of $\alpha_{1s}(3\pi;\, m^2)$ obtained from the lowest order diagram, Fig. 1(b)—in arbitrary scale.

On the other hand, the current operator of the nucleon can be expressed as

$$\langle q_1 | j_\mu | q_2 \rangle = ie\bar{\Psi}(q_1)[\gamma_\mu F_1(q^2) + \sigma_{\mu\nu} q_\nu F_2(q^2)]\psi(q_2), \quad (2)$$

where $q=(q_1-q_2)$ and $\sigma_{\mu\nu}=\frac{1}{2}i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$. From Eqs. (1) and (2) it follows that

$$F_1(3\pi;\, q=0)=0, \quad (3)$$

where (3π) denotes the contribution from three-pion intermediate states. The result (3) does not depend on the structure of parts A and B. This theorem was proved in I[15] by use of the Ward identity.

Dispersion theory allows one to write the form factors in the form[16]

$$F_{1(s\text{ or }v)}(q^2)=\frac{Z_{2p}}{2}+\frac{1}{\pi}\int_{(nm_\pi)^2}^{\infty} dm^2 \frac{\alpha_{1(s\text{ or }v)}(m^2)}{q^2+m^2}, \quad (4)$$

where m_π is the pion mass, and s and v denote the isoscalar and isovector parts. As is well known, an even-odd rule[17] applies, that is, $2l$- and $(2l+1)$-pion intermediate states contribute only to isovector and isoscalar parts, respectively. Therefore, $n=2$ for the vector part and $n=3$ for the scalar part. Equation (3) means that

$$\int_{(3m_\pi)^2}^{\infty} dm^2 \frac{\alpha_{1s}(3\pi;\, m^2)}{m^2}=0.$$

It would be of interest to know whether or not the charge spectral function $\alpha_{1s}(3\pi;\, m^2)$ has a positive value near its threshold $3m_\pi$, and also how many times $\alpha_{1s}(3\pi;\, m^2)$ changes its sign and at what values of m these changes occur. We shall assume that the radiative corrections in parts A, B, and C do not change the answer drastically so that the qualitative and semi-qualitative answers to the above questions may be obtained from the lowest-order diagram, Fig. 1(b). We may feel that the strong pion-pion interactions at B should greatly change the absolute value of $\alpha_{1s}(3\pi;\, m^2)$ but would not change its sign. We may argue that the lowest-order perturbation calculation at A does not give the correct answer to these questions because of the s-wave effect, just as in the case of pion-nucleon scatterings. It has been shown in II[18] that the s-wave effect is negligibly small and has no misleading contributions.

It has been shown in I that $\alpha_{1s}(3\pi;\, m^2)$ obtained from Fig. 1(b) is positive near its threshold and changes its sign only once (at X) as is shown in Fig. 1(c). The position X depends on the mass of the particles which form the closed loop, i.e., $X=X(M')$. In order to evaluate X, we shall let the mass M' tend to infinity. Then the closed loop shrinks to a point and X tends to infinity. Therefore, in this limit $\alpha_{1s}(3\pi;\, m^2)$ becomes positive definite and its negative part shrinks to a negative δ function (as was shown in II). This observation would mean

$$X>M^2, \quad (5)$$

where M is the nucleon mass. This inequality was supported by the very rough numerical calculation in I.

Inequality (5) may be applied as follows. Suppose the spin and the parity of the resonances η, ω, and ρ are 1^-. According to Sakurai's vector-boson theory, the relative sign of the coupling constants for the nucleon-η and the η-photon interactions on the one hand and between the nucleon-ω and the ω-photon interactions on the other is arbitrary.[9] He analyzed the experimental data for F_{1p} and obtained odd relative sign. This odd relative sign means that F_{1n} has positive sign at small values of q^2, because the mass M_η of η is very much smaller than the masses M_ω and M_ρ of ω and ρ. However, if we want to calculate the relative sign from the meson-theoretical point of view, we should get the even value for it; and not only the relative sign but also the sign itself is determined uniquely.

Theorem II. The Fourier transform of $F_{1n}(q^2)$ has a negative value, at least in the outermost region. We know that any spectral function in meson theory is zero at the threshold[19] and $\alpha_{1v}(2\pi;\, m^2)$ and $\alpha_{1s}(3\pi;\, m^2)$ are positive near their thresholds. Therefore, these spectral functions have the form

$$\alpha_1(n\pi;\, m^2) \propto \epsilon^{\lambda_n} \quad (\epsilon,\, \lambda_n>0) \quad (6)$$

near their thresholds, where $m=nm_\pi+\epsilon$. Simple calcu-

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[16] It was shown in I that

$$Z_{2p}=1-\frac{2}{\pi}\int_{(3m_\pi)^2}^{\infty} dm^2 \frac{\alpha_{1s}(m^2)}{m^2}=1-\frac{2}{\pi}\int_{(2m_\pi)^2}^{\infty} dm^2 \frac{\alpha_{1v}(m^2)}{m^2},$$

where Z_{2p} is the bare state probability of the proton (see Theorem III). This expression means that unsubtracted form (4) should hold when the present theory is a consistent theory and $1>Z_{2p}\geqslant 0$.

[17] G. F. Chew, R. Karplus, S. Gasiorowicz, and F. Zachariasen, Phys. Rev. **112**, 642 (1958).

[18] K. Hiida and N. Nakanishi, Progr. Theoret. Phys. (Kyoto) **22**, 863 (1959). We shall cite this paper as II.

[19] Quantum electrodynamics is the exceptional case. For example, the value of α_2 for the electron and the muon is infinity at their thresholds.

lation leads to

$$\rho_1(n\pi;r)\equiv\frac{1}{(2\pi)^3}\int d\mathbf{q}\,e^{i\mathbf{q}\cdot\mathbf{r}}F_1(n\pi;q^2)\propto\frac{e^{-nm_\pi r}}{r^{(\lambda_n+2)}} \quad (7)$$

at $r\to\infty$. Equation (7) means that the asymptotic behavior of $\rho_{1n}\equiv\rho_{1s}-\rho_{1v}$ is determined only by the behavior of α_{1v} near its threshold.

If there were no strong pion-pion interactions, then we might expect that ρ_{1n} is negative in the region

$$r\gtrsim 1/3m_\pi.$$

Even under the influence of the resonances ω and ρ, it would be possible that ρ_{1n} is negative in the region $r\gtrsim 1/m_\pi$ if the spin and the parity[14] of η are 0^-.

Theorem III. The proton has a charge $Z_{2p}e$ at its origin, where Z_{2p} is the bare-state probability of the proton and satisfies the relation[20]

$$1>Z_{2p}\geqslant 0.$$

On the other hand, the neutron cannot have any charge at its origin. This was shown in I and by Sachs.[21]

CHARGE FORM FACTORS OF THE NUCLEON

Our next task is to use the theorems and the idea given by Bergia *et al.*[8] to construct phenomenological form factors F_{1s} and F_{1v}, valid at relatively small q^2. We shall assume[14] that among two- and three-pion resonances, ω and ρ are the only resonances with spin and parity 1^-, and further that the masses of resonances above three pions are considerably larger than those of ω and ρ.

Theorem II should now be taken into consideration. It is clear that the charge distribution in the outer region of the nucleon should be determined by $\alpha_{1v}(2\pi;m^2)$ in the region $9m_\pi^2\gtrsim m^2\geqslant 4m_\pi^2$, because in this region $\alpha_{1s}=0$. We know that $\alpha_{1v}(2\pi;m^2)$ is positive at $m_\rho^2+c>m^2\geqslant 4m_\pi^2$ where $c>0$. The simplest form of the isovector charge form factor is given by

$$F_{1v}(q^2)=a_v+a_tm_t^2/(q^2+m_t^2)+a_\rho m_\rho^2/(q^2+m_\rho^2), \quad (8)$$

where a_t, $a_\rho>0$ from meson theory, and the term $a_tm_t^2/(q^2+m_t^2)$ represents the charge in the region $9m_\pi^2\gtrsim m^2\geqslant 4m_\pi^2$ and therefore $m_t^2\approx 7m_\pi^2$.

The next problem is to take Theorem I into consideration. The isoscalar charge form factor is given by

$$F_{1s}(q^2)=1/2+a_\omega m_\omega^2/(q^2+m_\omega^2)-a_\omega M_1^2/(q^2+M_1^2), \quad (9)$$

where $a_\omega>0$ because of (5), and the term $a_\omega M_1^2/(q^2+M_1^2)$ represents the negative charge contributed from three-pion intermediate states. We are interested mainly in the behavior of F_{1n} at relatively low values of q^2 because, if we once know the sign of F_{1n} at low values of q^2, we can also determine its sign at high values of q^2 from the electron-scattering experiments.

[20] H. Lehmann, Nuovo cimento **11**, 342 (1954).
[21] R. G. Sachs, Phys. Rev. **126**, 2256 (1962).

Therefore, the last term in Eq. (9) will be approximated by the constant term $(-a_\omega)$ because of (5) and

$$F_{1s}(q^2)=a_s+a_\omega m_\omega^2/(q^2+m_\omega^2) \quad (10)$$

is obtained. Our model is very rough so that we shall use $m_\omega=m_\rho$ in the following analysis.

Five parameters a_v, a_t, a_ρ, a_s, and a_ω (of which a_t, a_ρ, and a_ω must be positive from meson theory) are included in Eqs. (8) and (10) but these parameters must satisfy four constraints[22]:

$$a_v+a_t+a_\rho=a_s+a_\omega=1/2,$$
$$a_\omega/m_\rho^2=a_\rho/m_\rho^2+a_t/m_t^2=\langle r^2\rangle_{1p}/12, \quad (11)$$

where $\langle r^2\rangle_{1p}$ is the rms radius of the proton. There is, therefore, only one independent parameter. In this note we shall take a_t as the independent variable. However, a_t is not an independent variable in the strict sense of the term since it must satisfy the inequality

$$0<a_tm_t^2\ll a_\rho m_\rho^2. \quad (12)$$

The requirement leading to the upper bound comes from the assumption of strong pion-pion interactions. We shall see later that (12) is a useful relation. The solution of Eq. (11) for a_v, a_ρ, a_s, and a_ω in terms of a_t yields

$$\begin{aligned}
&a_\rho=(\langle r^2\rangle_{1p}/12)m_\rho^2-a_t(m_\rho^2/m_t^2),\\
&a_\omega=(\langle r^2\rangle_{1p}/12)m_\rho^2,\\
&a_v=1/2-(\langle r^2\rangle_{1p}/12)m_\rho^2+a_t[(m_\rho^2-m_t^2)/m_t^2],\\
&a_s=1/2-(\langle r^2\rangle_{1p}/12)m_\rho^2.
\end{aligned} \quad (13)$$

That is,

$$1/2>a_v>a_s. \quad (14)$$

From Eqs. (8), (10), and (13) one obtains

$$F_{1n}(q^2)=-[a_t(m_\rho^2-m_t^2)/m_\rho^2m_t^4]q^4+O(q^6); \quad (15)$$

that is, F_{1n} has a negative sign for small values of q^2.

STRONG VACUUM POLARIZATION DUE TO ω

The explanation for the inequality (14) and the smallness of the rms radius of the neutron, $\langle r^2\rangle_{1n}\approx 0$, is as follows. If we omit the contribution from three-pion intermediate states, there is the isoscalar charge $\frac{1}{2}e$ near the origin of the nucleon. The role of the three-pion intermediate states is to polarize the vacuum. In other words, in analogy with quantum electrodynamics it appears[23] that the positive charge $\frac{1}{2}e$ near the center of the nucleon attracts the negative charge $(-\delta e\equiv -a_\omega e)$ and repels the positive charge δe. The role of the three-pion resonance ω is to enhance the polarization very strongly. Thus, the isoscalar charge near the center of the nucleon is reduced from $\frac{1}{2}e$ to a_se. For a long time

[22] Because $\langle r^2\rangle_{1n}\approx 0$, the second equality holds only approximately. For simplicity we shall assume throughout this paper that the equality holds exactly.
[23] This analogy to quantum electrodynamics does not hold so strictly, because in quantum electrodynamics δe moves to infinity.

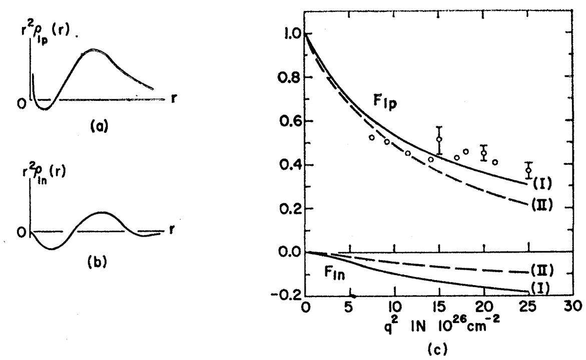

FIG. 2. (a) Schematic representation of our proton model and (b) of our neutron model—both in arbitrary scale. (c) Two examples of F_{1p} and F_{1n}. The points for F_{1p} with and without flags are Cornell [reference 7] and Stanford [reference 6] data, respectively.

the smallness of the rms radius of the neutron was a mystery.[4] Nowadays the mechanism to reduce the radius is clear. Because of the strong vacuum polarization due to ω, the positive charge $a_\omega e$ moves far away from the center of the nucleon and hence $\langle r^2\rangle_{1s}\approx\langle r^2\rangle_{1v}$.

CHARGE STRUCTURE OF THE PROTON AND THE NEUTRON

The next problem is to investigate the charge structure of the proton. One may ask whether the bare proton is surrounded by positive charge alone or by both positive and negative layers of charge. According to Theorem III, the proton has the charge $Z_{2p}e$ at its center. This bare charge is concealed by the clouds contributed from massive intermediate states, and the negative charge cloud caused by the vacuum polarization due to ω. Since the vacuum polarization is very strong, the bare charge should be concealed by the negative charge cloud. Further, this layer of negative charge is surrounded by a layer of positive charge. In our model of the proton [Eqs. (8), (10), and (13)], the charge $(a_v+a_s)e$ represents the sum of the bare charge $Z_{2p}e$, the polarized negative charge, and the charge contributed from higher massive states.

In order to prove the above statement, it is sufficient to show $a_v+a_s<0$, because $1>Z_{2p}\geqslant 0$. To prove this we shall assume the contrary, namely that $a_v+a_s\geqslant 0$. Then it follows from Eq. (13) that

$$1-(\langle r^2\rangle_{1p}/6)m_\rho^2+a_t[(m_\rho^2-m_t^2)/m_t^2]\geqslant 0. \quad (16)$$

Substituting $\langle r^2\rangle_{1p}\approx(0.8\times10^{-13}\text{ cm})^2$, $m_\rho^2\approx 30m_\pi^2$, and $m_t^2\approx 7m_\pi^2$ into Eq. (16) then yields $a_t\geqslant 0.182$. Using these numerical values then yields

$$a_t m_t^2>a_\rho m_\rho^2. \quad (17)$$

The relation (17) contradicts (12). Thus, $a_v+a_s<0$ must hold.

Our proton model is represented schematically in Fig. 2(a). This proton model coincides perfectly with that proposed in III.[24] The same type of proton model was adopted by Katayama *et al.*[25] to explain the neutron-proton mass difference.

Confirmation of the negative sign of F_{1n} at low values of q^2 and improved accuracy in the electron-scattering experiments will present one means of confirming our proton model. The author would like to stress here another method to confirm our proton model, namely, it is sufficient to show experimentally that there exists such a q^2 that

$$F_{1p}(q^2)<-1/3, \quad (18)$$

where the number three comes from the fact that we live in three-dimensional space. The 2-mile machine at Stanford may be able to confirm the existence of such a value of q^2. Inequality (18) was proved in III by use of Bochner's theorem.[26]

The neutron has no bare charge at the origin. In the innermost region, there is a negative charge mainly due to the vacuum polarization. The middle region is occupied by positive charge because the vacuum polarization is so strong that the resulting positive charge is greater than the negative charge caused by $\rho(a_\omega>a_\rho)$. The outer region is occupied by negative charge as was mentioned before. Our neutron model is represented schematically in Fig. 2(b). This neutron model, which coincides with those proposed in III, was first proposed by Schiff.[27]

In our model [Eqs. (8), (10), and (13)], we did not use Theorem III explicitly. If we want to analyze the electron-scattering cross sections up to very high energies, the theorem should be used explicitly. Even if the spin and the parity of the resonance ζ[28] are 1^-, our results still hold.

As a numerical example, Fig. 2(c) shows $F_{1p}(q^2)$ and $F_{1n}(q^2)$, where $\langle r^2\rangle_{1p}=(0.75\times10^{-13}\text{ cm})^2$, $m_\rho^2=m_\omega^2=30m_\pi^2$, and $m_t^2=7m_\pi^2$ are used. To draw the curves (I) and (II), $a_t=0.1$ and $a_t=0.05$ were used. In these cases, the values of $a_t m_t^2/a_\rho m_\rho^2$ are 1/12 and 1/44, respectively.

Note added in proof. After this paper was submitted, the author was informed by Professor R. Hofstadter that their new experimental result seems to be in qualitative agreement with this paper. See C. de Vries, R. Hofstadter, and R. Herman, Phys. Rev. Letters **8**, 381 (1962).

ACKNOWLEDGMENTS

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Professor H. Yukawa, and Professor G. Takeda for many valuable discussions in Japan on the electromagnetic structure of the nucleon. Through these discussions, the author's understanding of the structure was obtained. He also would like to thank Dr. M. Hamermesh for his warm hospitality at the Argonne National Laboratory and Dr. F. E. Throw for careful reading of the manuscript.

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Two-Particle Approximation for the Three-Pion Amplitude*

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A two-particle approximation is suggested for the $3\pi \rightarrow 3\pi$ amplitude: The amplitude is replaced by a $\rho+\pi \rightarrow \rho+\pi$ amplitude multiplied by 2π resonance factors. An approximate calculation of the $J=1$-, $T=0$ partial wave for the ρ-π amplitude is made, including only the "elastic" and one-pion-exchange cuts, but it seems to give no hint of a 3π resonance.

I. INTRODUCTION

IT is becoming increasingly clear that in present theories of strong interactions some understanding of three-particle states is necessary if one wishes to do more than make qualitative predictions. The main obstacle preventing this understanding is the complicated nature of the corresponding amplitudes; one is forced to consider functions of five and eight variables. One might conjecture that the amplitudes can, in principle, be calculated using a version of the Mandelstam[1]-Landau[2]-Cutkosky[3] formalism, but even accepting this, the actual calculations would seem almost hopelessly complex.

The recent discovery of several two-particle resonances,[4–11] however, points to a possible simplifying approximation: Perhaps the three particles can be replaced by two, one of which is an unstable particle corresponding to a resonating state of two particles. This two-particle approximation has been recently discussed[12] in connection with $N\pi$-π states. In this paper we give some preliminary results obtained by treating the three-pion state as a state involving a $T=1$, $J=1-$, two-pion resonance (the ρ-meson[7]), and a pion.

An approximate treatment of the $3\pi \rightarrow 3\pi$ amplitude has previously been undertaken by Blankenbecler and Tarski[13] in an attempt to evaluate the isoscalar nucleon structure. Their treatment was similar in spirit to that of this paper in that they assumed factors corresponding to π-π interactions could be removed from the amplitude, leaving a reduced amplitude which is independent of the π-π energies. It differs, however, since they assume the complete amplitude can be represented by a single term: a product of three two-pion terms for both the initial and final states times a "reduced" amplitude which depends only upon the total energy. In this paper, on the other hand, we assume a sum of 9 terms, each allowing for the interaction of a single pair of pions in the initial and final states, with the reduced amplitude depending upon a momentum transfer in addition to the total energy. Since both approximations are quite drastic it is difficult to say which comes closer to the truth,[14] but the approximation of this paper has the advantage of allowing the one-pion-exchange contributions to be included exactly.

We begin with the unitarity relation for the connected part of the $3\pi \rightarrow 3\pi$ amplitude. We assume that this amplitude can be approximated as a sum over products of two-particle (ρ-π) amplitudes times 2π-resonance factors, the sum running over all nine possible pairings of two of the initial pions and two of the final pions into resonant states. In the sharp resonance limit, the

* The results given in a preliminary report on this work [Bull. Am. Phys. Soc. 7, 56 (1962)] were incorrect because of an algebraic error and improper treatment of the one-pion-exchange cut.
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