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Two-Particle Approximation for the Three-Pion Amplitude*

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A two-particle approximation is suggested for the $3\pi \rightarrow 3\pi$ amplitude: The amplitude is replaced by a $\rho+\pi \rightarrow \rho+\pi$ amplitude multiplied by 2π resonance factors. An approximate calculation of the $J=1$-, $T=0$ partial wave for the ρ-π amplitude is made, including only the "elastic" and one-pion-exchange cuts, but it seems to give no hint of a 3π resonance.

I. INTRODUCTION

It is becoming increasingly clear that in present theories of strong interactions some understanding of three-particle states is necessary if one wishes to do more than make qualitative predictions. The main obstacle preventing this understanding is the complicated nature of the corresponding amplitudes; one is forced to consider functions of five and eight variables. One might conjecture that the amplitudes can, in principle, be calculated using a version of the Mandelstam[1]-Landau[2]-Cutkosky[3] formalism, but even accepting this, the actual calculations would seem almost hopelessly complex.

The recent discovery of several two-particle resonances,[4–11] however, points to a possible simplifying approximation: Perhaps the three particles can be replaced by two, one of which is an unstable particle corresponding to a resonating state of two particles. This two-particle approximation has been recently discussed[12] in connection with $N\pi$-π states. In this paper we give some preliminary results obtained by treating the three-pion state as a state involving a $T=1$, $J=1-$, two-pion resonance (the ρ-meson[7]), and a pion.

An approximate treatment of the $3\pi \rightarrow 3\pi$ amplitude has previously been undertaken by Blankenbecler and Tarski[13] in an attempt to evaluate the isoscalar nucleon structure. Their treatment was similar in spirit to that of this paper in that they assumed factors corresponding to π-π interactions could be removed from the amplitude, leaving a reduced amplitude which is independent of the π-π energies. It differs, however, since they assume the complete amplitude can be represented by a single term: a product of three two-pion terms for both the initial and final states times a "reduced" amplitude which depends only upon the total energy. In this paper, on the other hand, we assume a sum of 9 terms, each allowing for the interaction of a single pair of pions in the initial and final states, with the reduced amplitude depending upon a momentum transfer in addition to the total energy. Since both approximations are quite drastic it is difficult to say which comes closer to the truth,[14] but the approximation of this paper has the advantage of allowing the one-pion-exchange contributions to be included exactly.

We begin with the unitarity relation for the connected part of the $3\pi \rightarrow 3\pi$ amplitude. We assume that this amplitude can be approximated as a sum over products of two-particle (ρ-π) amplitudes times 2π-resonance factors, the sum running over all nine possible pairings of two of the initial pions and two of the final pions into resonant states. In the sharp resonance limit, the

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* The results given in a preliminary report on this work [Bull. Am. Phys. Soc. 7, 56 (1962)] were incorrect because of an algebraic error and improper treatment of the one-pion-exchange cut.

† National Science Foundation Postdoctoral Fellow.

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three-particle unitarity relation then reduces to the usual two-particle unitarity relation for the ρ-π amplitude, except that, because of the mass-averaging, the ρ mass acquires a positive imaginary part proportional to the width of the 2π resonance.

We next discuss some of the kinematical details of ρ-π scattering, indicating how the partial wave amplitudes may be expressed in terms of four invariant amplitudes. At this point we specialize to the $T=0$, $J=0-$ partial wave amplitude, since it is in this channel that the[8] ω (and perhaps the[9] η) resonance occurs. Using an approximate N/D calculation we are able to construct this amplitude, ignoring all but the normal threshold and one-pion-exchange[15] singularities. The resulting amplitude is surely incorrect in the vicinity of the ignored singularities, but perhaps might give some indication of the influence of the one-pion-exchange contribution.

II. TWO-PARTICLE APPROXIMATION

We define A_3, the connected part of the three-pion scattering amplitude, by setting

$$\begin{aligned}\langle 1'2'3'_{\text{out}}|123_{\text{in}}\rangle=&\sum_{\text{perm}}\delta_{i'1}\delta_{j'2}\delta_{k'3}\\ &+i\sum_{\text{comb}}\delta_{i'i}(2\pi)^4\delta^4(p_{j'}+p_{k'}-p_j-p_k)(2^4\omega_{j'}\omega_{k'}\omega_j\omega_k)^{-1/2}A_2(j'k';jk)\\ &+i(2\pi)^4\delta^4(p_{1'}+p_{2'}+p_{3'}-p_1-p_2-p_3)(2^6\omega_{1'}\omega_{2'}\omega_{3'}\omega_1\omega_2\omega_3)^{-1/2}A_3(1'2'3';123),\end{aligned}\tag{1}$$

with A_2 the usual two-pion amplitude.

Unitarity then gives the discontinuity in A_3:

$$\begin{aligned}&(2i)^{-1}[A_3^{(+)}(1'2'3';123)-A_3^{(-)}(1'2'3';123)]\\ &=\sum_{\text{comb}}\frac{1}{2}\sum_{p_{1''}}(2\pi)^4\delta^4(p_{i'}+p_{1''}+p_i-p_1-p_2-p_3)A_2^{(-)}(j'k';1''i)A_2^{(+)}(1''i';jk)\\ &+\sum_{i'}\frac{1}{2}\frac{1}{2!}\sum_{p_{1''}}\sum_{p_{2''}}(2\pi)^4\delta^4(p_{1''}+p_{2''}+p_{i'}-p_1-p_2-p_3)A_2^{(-)}(j'k';1''2'')A_3^{(+)}(1''2''i';123)\\ &+\sum_{i}\frac{1}{2}\frac{1}{2!}\sum_{p_{1''}}\sum_{p_{2''}}(2\pi)^4\delta^4(p_{1'}+p_{2'}+p_{3'}-p_i-p_{1''}-p_{2''})A_3^{(-)}(1'2'3';i1''2'')A_2^{(+)}(1''2'';jk)\\ &+\frac{1}{2}\frac{1}{3!}\sum_{p_{1''}}\sum_{p_{2''}}\sum_{p_{3''}}(2\pi)^4\delta^4(p_{1''}+p_{2''}+p_{3''}-p_1-p_2-p_3)A_3^{(-)}(1'2'3';1''2''3'')A_3^{(+)}(1''2''3'';123).\end{aligned}\tag{2}$$

The corresponding diagram is shown in Fig. 1. In (2)

$$\sum_p\rightarrow\frac{1}{(2\pi)^3}\int d^4p\,\delta(p^2-\mu^2)\sum_\alpha,\tag{3}$$

where α is the isospin index.

We assume that the two-pion amplitude is dominated by the $J=1-$, $T=1$ resonance. We can then write

$$A_2(j'k';jk)\approx\sum_\alpha\sum_\lambda|C|^2\epsilon(\alpha_{j'}\alpha_{k'}\alpha)\,e(\lambda P_{j'k'})\cdot q_{j'k'}\epsilon(\alpha_j\alpha_k\alpha)\,e(\lambda P_{jk})\cdot q_{jk}(M_R^2-i\Delta-s_{jk})^{-1},\tag{4}$$

where

$$\begin{aligned}q_{jk}&=\tfrac{1}{2}(p_j-p_k),\\ P_{jk}&=p_j+p_k,\\ s_{jk}&=P_{jk}^2,\end{aligned}\tag{5}$$

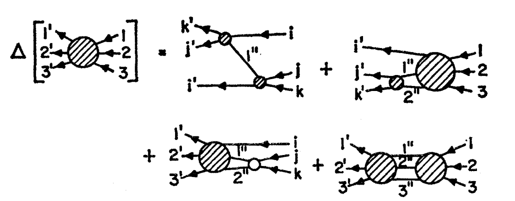


FIG. 1. The unitarity relation for the connected part of the three-pion scattering amplitude.

$\epsilon(\alpha\beta\gamma)$ is the totally antisymmetric tensor, and the $e(\lambda P)$ are the usual polarization 4-vectors for spin-one particles with momentum P and polarization λ. In (4), M_R is the mass of the ρ resonance; its width is $\Gamma=\Delta M_R^{-1}$. The dimensionless "coupling-constant" C is related to M_R and Δ through the unitarity relation

$$|C|^2=24\pi(\tfrac{1}{4}M_R^2-\mu^2)^{-3/2}M_R\Delta.\tag{6}$$

For $M_R^2\approx 29\mu^2$, $|C|^2$ is roughly the width of the resonance measured in MeV.

[15] The one-pion-exchange pole contribution to the ρ-π amplitude has previously been considered by M. Nauenberg and A. Pais, Phys. Rev. Letters **8**, 82 (1962) and by P. Carruthers, Nuovo cimento **22**, 867 (1961).

We now make our fundamental approximation: We assume that we can separate ρ-resonance factors from A_3, leaving a two-particle amplitude B. Explicitly, we set

$$A_3(1'2'3';123)\approx \sum_{\text{comb}}\sum_{\lambda'\lambda}\sum_{\alpha'\alpha} \frac{C^*\epsilon(\alpha_{j'}\alpha_{k'}\alpha')\, e(\lambda' P_{j'k'})\cdot q_{j'k'}}{s_{j'k'}-M_R{}^2-i\Delta} B^{\lambda'\lambda}(\alpha'\alpha_i';\alpha\alpha_i|P_{j'k'}p_{i'};P_{jk}p_i)\frac{C\epsilon(\alpha_j\alpha_k\alpha)\, e(\lambda P_{jk})\cdot q_{jk},}{s_{jk}-M_R{}^2+i\Delta}, \quad (7)$$

as pictured in Fig. 2.

Making this substitution in the unitarity relation, we find the usual two-particle unitarity relation for B:

$$(2i)^{-1}[B^{(+)\lambda'\lambda}(\alpha'\alpha_i';\alpha\alpha_i|P_{j'k'}p_{i'};P_{jk}p_i)-B^{(-)\lambda'\lambda}(\alpha'\alpha_i';\alpha\alpha_i|p_{j'k'}p_{i'};P_{jk}p_i)]$$
$$=\tfrac{1}{2}(2\pi)^{-2}(4s^{1/2})^{-1}Q(s)\int d\Omega''\sum_{\alpha''\beta}\sum_{\mu}B^{(-)\lambda'\mu}(\alpha'\alpha_i';\beta\alpha''|P_{j'k'}p_{i'};P''p'')$$
$$\times B^{(+)\mu\lambda}(\beta\alpha'';\alpha\alpha_i|P''p'';P_{jk}p_i)+\text{one-pion-exchange term}, \quad (8)$$

where

$$s=(P_{jk}+p_i)^2,$$
$$Q^2(s)=(4s)^{-1}[s-(M_R+\mu)^2][s-(M_R-\mu)^2], \quad (9)$$

and

$$(P'')^2=M_R{}^2.$$

In obtaining (8) from (2) and (7) we have made several approximations which are justified only if the width of the ρ resonance is small. In particular, in integrating over the mass of the intermediate ρ we have used an approximation equivalent to

$$\int_{4\mu^2}^{\infty} ds_{12}''F(s_{12}'')|s_{12}''-M_R{}^2+i\Delta|^{-2}\approx\pi\Delta^{-1}F(M_R{}^2). \quad (10)$$

The one-pion-exchange (OPE) term indicated on the right-hand side of (8) comes from the pole in the crossed channel pictured in Fig. 3. The corresponding contribution to B is

$$B_{\text{OPE}}{}^{\lambda'\lambda}(\beta'\alpha';\beta\alpha|P'p';Pp)$$
$$=\frac{|C|^2}{\mu^2-i\epsilon-(P-p')^2}(\delta_{\alpha\alpha'}\delta_{\beta\beta'}-\delta_{\alpha\beta}\delta_{\alpha'\beta'})$$
$$\times[e^*(\lambda'P')\cdot p][e(\lambda P)\cdot p']. \quad (11)$$

The denominator of B_{OPE} depends upon the masses of initial and final ρ mesons. When the average over the intermediate ρ mass is performed to obtain the unitarity relation, a careful analysis[12] shows that instead of $M_R{}^2$

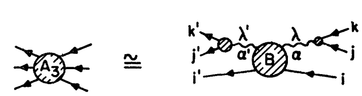


FIG. 2. The two-particle approximation. The wavy lines indicate ρ mesons.

we should use $M_R{}^2+i\Delta$ for the mass of the "average" intermediate ρ. The denominator in B_{OPE} must be similary modified when it is used in any unitarity relation (such as for the process $N+\bar{N}\to 3\pi$) or in computing a cross section.

III. PARTIAL WAVE AMPLITUDES

For each set of isospin indices we can write[16]

$$B^{\lambda'\lambda}(P'p';Pp)=e_\mu{}^*(\lambda'P')[g^{\mu\nu}b_0(s,t)$$
$$+(p'^\mu p'^\nu+p^\mu p^\nu)b_1(s,t)+p'^\mu p^\nu b_2(s,t)$$
$$+p^\mu p'^\nu b_3(s,t)]e_\nu(\lambda P), \quad (12)$$

where

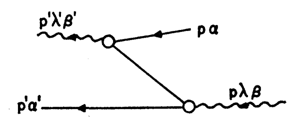
$$s=(P+p)^2=(P'+p')^2,$$

and

$$t=(P-P')^2=(p-p')^2. \quad (13)$$

We assume that the four invariant amplitudes, $b_i(s,t)$, have no kinematical singularities.[16] Because of the approximate nature of B there is some ambiguity as to the singularities which one should ascribe to the b_i. Presumably they will have all the singularities of A_3 arising from diagrams with four external vertices, two of which have a pair of the external pion lines emerging

FIG. 3. The one-pion-exchange diagram.



from them, the other two a single external pion line. If one wishes to retain the two-particle approximation, however, many of these singularities must be ignored, and the mass-averaging must be done carefully for the others. In this paper we avoid most of these difficulties by ignoring all singularities but the OPE pole and the "elastic threshold" at $s=(M_R+\mu)^2$.

In the center-of-mass system we define the helicity amplitudes[17]

$$f_{\lambda'\lambda}{}^{(J)}(s)=\int d\Omega\, \mathfrak{D}_{\lambda\lambda'}{}^{(J)*}(\phi,\theta,-\phi)$$
$$\times(2\pi)^{-2}(4s^{1/2})^{-1}Q(s)B^{\lambda'\lambda}(P'p';pP), \quad (14)$$

[16] A. C. Hearn, Nuovo cimento **21**, 333 (1961).
[17] M. Jacob and G. C. Wick, Ann. Phys. (New York) **7**, 404 (1959).

where the direction of $\mathbf{P}'$ relative to $\mathbf{P}$ is determined by the angles θ and ϕ. These helicity amplitudes have a discontinuity across the elastic cut,

$$(2i)^{-1}[f_{\lambda'\lambda}{}^{(J)}(s+i\epsilon)-f_{\lambda'\lambda}{}^{(J)}(s-i\epsilon)]$$
$$=\tfrac{1}{2}\sum_{\lambda''} f_{\lambda'\lambda''}{}^{(J)}(s-i\epsilon)f_{\lambda''\lambda}{}^{(J)}(s+i\epsilon). \quad (15)$$

Each b and f can be written in terms of three isospin amplitudes, corresponding to $I=0$, 1, and 2. If

$$b_{i;\,\beta'\alpha'\beta\alpha}=l_i\delta_{\alpha\beta}\delta_{\alpha'\beta'}+m_i\delta_{\alpha\alpha'}\delta_{\beta\beta'}+n_i\delta_{\alpha\beta'}\delta_{\beta\alpha'}, \quad (16)$$

then the isospin amplitudes are

$$b_i^{(0)}=3l_i+m_i+n_i,$$
$$b_i^{(1)}=\quad m_i-n_i,$$

and

$$b_i^{(2)}=\quad m_i+n_i, \quad (17)$$

with analogous relations for the f's.

For each J and I there are three amplitudes corresponding to parity $-(-1)^J$:

$$A_{0,0}{}^{(J,I)}=f_{0,0}{}^{(J,I)},$$
$$A_{+,+}{}^{(J,I)}=f_{1,1}{}^{(J,I)}+f_{1,-1}{}^{(J,I)},$$

and

$$A_{+,0}{}^{(J,I)}=f_{1,0}{}^{(J,I)}; \quad (18)$$

and one amplitude corresponding to parity $(-1)^J$:

$$A_{-,-}{}^{(J,I)}=f_{1,1}{}^{(J,I)}-f_{1,-1}{}^{(J,I)}. \quad (19)$$

The OPE pole contributes only to the invariant amplitudes $b_3{}^{(I)}$:

$$b_{3\,\mathrm{OPE}}{}^{(0)}(s,t)=-2b_{3\,\mathrm{OPE}}{}^{(1)}(s,t)=-2b_{3\,\mathrm{OPE}}{}^{(2)}(s,t)$$
$$=\frac{-2|C|^2}{\mu^2-2M_R{}^2-2i\Delta-2\mu^2-s-t}. \quad (20)$$

In the limit $\Delta\rightarrow 0$ this pole crosses the "physical region" for ρ-π scattering.

IV. THE $T=0$, $J=1-$ AMPLITUDE

In this section we restrict our attention to the $T=0$, $J=1-$ amplitude

$$A(s)\equiv[2\rho(s)]^{-1}f_{-,-}{}^{(1,0)}(s)$$
$$=-(16\pi Q^2)^{-1}\int_{-1}^{1} b_0(s,t)\cos\theta d\cos\theta$$
$$-(32\pi)^{-1}\int_{-1}^{1} b_3(s,t)\sin^2\theta d\cos\theta, \quad (21)$$

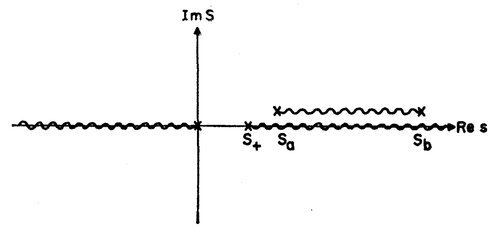


FIG. 4. The OPE and elastic cuts for the partial wave amplitudes.

where

$$\rho(s)=s^{-1/2}Q^3=(8s^2)^{-1}[(s-s_+)(s-s_-)]^{3/2}, \quad (22)$$

with

$$s_\pm=(M_R\pm\mu)^2.$$

The discontinuity across the elastic cut is

$$(2i)^{-1}(A^{(+)}-A^{(-)})=\rho A^{(-)}A^{(+)}, \quad (23)$$

and the OPE contribution is

$$A_{\mathrm{OPE}}(s)=\frac{|C|^2}{32\pi Q^2}\int_{-1}^{1}\frac{\sin^2\theta d\cos\theta}{\cos\theta-z_0(s)}, \quad (24)$$

with

$$z_0(s)=\tfrac{1}{2}(2M_R{}^2+2i\Delta+\mu^2-s+2Q^2)Q^{-2}.$$

The OPE pole produces two cuts in A; one running from $-\infty$ to 0, the other running slightly above the elastic cut from $s_a=2M_R{}^2+\mu^2$ to $s_b=\mu^{-2}(M_R{}^2-\mu^2)^2$. The discontinuities across these cuts are given by $+|C|^2F(s)$ and $-|C|^2F(s)$, respectively, where

$$F(s)=[32Q^2(s)]^{-1}[1-z_0{}^2(s)]$$
$$=\tfrac{1}{2}\mu^4(s-s_+)^{-3}(s-s_-)^{-3}s^2(s-s_a)(s_b-s). \quad (25)$$

The locations of the elastic and OPE cuts are shown in Fig. 4.

The cut above the elastic cut is a rather novel feature, and we investigate it in some detail. One should note to begin with that A cannot be a real function, since this would require a corresponding cut below the elastic cut. Ignoring the left-hand segment of the OPE cut we write, as usual,

$$A=D^{-1}N, \quad (26)$$

with D and N satisfying the integral equations

$$N(z)=-\frac{|C|^2}{\mu^2}\frac{1}{\pi}\int_{sa}^{sb}\frac{F(s)D^{(+)}(s)}{s-z}ds$$

and (27)

$$D(z)=D(s_0)-\frac{(z-s_0)}{\pi}\int_{s+}^{\infty}\frac{\rho(s)N^{(-)}(s)ds}{(s-s_0)(s-z)}.$$

Making use of the fact that $F(s)$ is sharply peaked about a point s_0 slightly greater than s_a, and taking the limit $\Delta\rightarrow 0$ to simplify the algebra, we find the approximate

solution

$$N(s)=-n(s)\mu^{-2}+iF(s)\mu^{-2}\theta(s-s_a)\theta(s_b-s), \tag{28}$$
$$\begin{aligned}D(s)=|C|^{-2}+(s-s_0)\mu^{-2}\{n_1(s)+(s-s_0)^{-1}\\ \times(\rho(s)F(s)\theta(s-s_a)\theta(s_b-s)-\rho(s_0)F(s_0))\\ +i[-F_1(s)+(s-s_0)^{-1}(\rho(s)n(s)\theta(s-s_+)\\ -\rho(s_0)n(s_0))]\},\end{aligned}$$

where

$$n(s)=\frac{P}{\pi}\int_{sa}^{sb}\frac{F(s')}{s'-s}ds',$$
$$n_1(s)=\frac{P}{\pi}\int_{s+}^{\infty}\frac{\rho(s')n(s')}{(s'-s_0)(s'-s)}ds', \tag{29}$$

and

$$F_1(s)=\frac{P}{\pi}\int_{sa}^{sb}\frac{\rho(s')F(s')}{(s'-s_0)(s'-s)}ds'.$$

Rough numerical calculation indicate that, for $\Gamma\lesssim100$ MeV, $D(s)$ is dominated by the constant term $|C|^{-2}$ although the remaining terms produce several interesting wiggles with amplitude $\lesssim(1/10)|C|^{-2}$. The amplitude $A(s)$ is thus in this approximation essentially the $J=1-$, $T=0$ projection of the OPE pole considered by Pais and Nauenberg.[13] It has a large peak just above $s_a\approx59\,\mu^2$; the left-hand end of the OPE cut, as shown in Fig. 5.

V. CONCLUSION

Since the $J=1-$, $T=0$ amplitude $A(s)$ seems to have a sharp peak at $s\approx65\,\mu^2$, it might at first glance seem strange that no corresponding peak has yet been seen in the 3π mass spectrum. The explanation follows from an analysis of Goebel[18]: To construct a production amplitude A_p from $A(s)$, we (schematically) perform the integration

$$A_p(z)=\frac{1}{\pi}\int_{s+}^{\infty}\frac{G(s')A(s')}{s'-z}ds'. \tag{30}$$

The peak in $A(s')$ is due to a singularity in the upper half plane above the elastic cut. Therefore, $A_p(z)$ will have a singularity at the same position, but only on unphysical sheets, far from the physical region $z=s+i\epsilon$ (but close to $z=s-i\epsilon$) on the physical sheet.

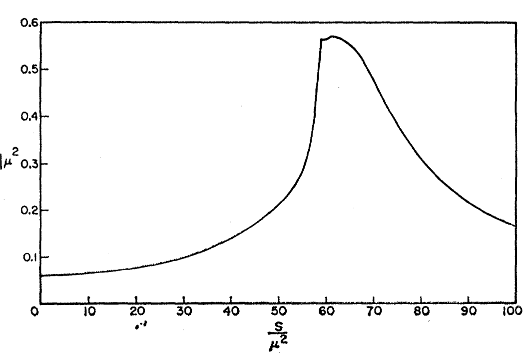


FIG. 5. The $T=0$, $J=1$-amplitude, including only OPE and elastic contributions.

[18] C. J. Goebel (to be published).

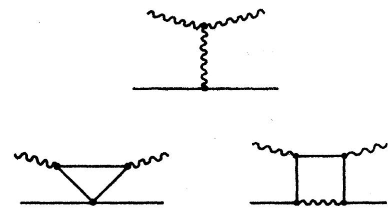


FIG. 6. Additional contributions included within the two-particle approximation.

Similar considerations show that if we take our two-particle approximation too literally we shall never be able to show that an ω resonance in a production amplitude follows from an ω resonance in $3\pi\rightarrow3\pi$ scattering. Assume, for example, that an exact calculation shows $A(s)$ to have a pole just below the real axis at $s=M_\omega^2\approx32\mu^2<s_+\approx41\mu^2$. Formula (30) then allows a corresponding pole of A_p only on an unphysical sheet, producing no resonance. The correct procedure would be to use $A(s)$ to construct the 3π amplitude using (7), and then to calculate the production amplitude without using the approximation indicated in (10). In this case the pole in A_p would again be on an unphysical sheet, but now quite close to the physical region since it can be reached by going down through the cut which begins at $s=9\mu^2$. Of course at the position of the ω resonance the 2π resonance factors cannot be at their maximum values, but for a reasonably broad ρ resonance they may still be large.

Our approximate calculation does not lead to a pole, or even a bump, in $A(s)$ near $s=M_\omega^2$. This does not necessarily mean, however, that the two-particle approximation is invalid. We have ignored many diagrams which may be significant, three of which are shown in Fig. 6. As mentioned at the beginning of Sec. III, the exact location of the corresponding singularities can be obtained only by a detailed study of the mass-averaging process. In addition to the diagrams of Fig. 6 one might expect an[9] η exchange diagram to be important, since it would produce a short cut in $A(s)$ just above the ρ-π threshold. If the η has positive G parity, however, as has been recently suggested,[19,20] then this diagram would be forbidden. If the ζ resonance[11] has the same quantum number as the ρ it may be necessary to include ζ-π amplitudes as well as ρ-π amplitudes in Eq. (7).

Even if these and other contributions are included it may turn out that the two-particle approximation does not give results in agreement with experiment: The two-particle contributions may be swamped by a "background" coming from diagrams which cannot be factored as required for Eq. (7). Since it would be

[19] P. L. Bastien, J. P. Barge, O. I. Dahl, M. Ferro-Luzzi, D. H. Miller, J. J. Murray, A. H. Rosenfeld, and M. B. Watson, Phys. Rev. Letters 8, 114 (1962).

[20] L. M. Brown and P. Singer, Phys. Rev. Letters 8, 155 (1962).

extremely convenient if three-particle states could be handled using the two-particle approximation, it seems important to continue experimental and theoretical work to check the validity of this approximation for all three-particle states in which two of the particles can resonate.

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Influence of $\bar{K}$-Nucleon Interactions on Pion-Hyperon Scattering*†

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Scattering amplitudes for coupled $\pi\Lambda$, $\pi\Sigma$, and $\bar{K}N$ channels are obtained by an extension of the method of Amati, Stanghellini, and Vitale. The method is shown to be essentially equivalent to the N/D method in the region of nonrelativistic baryon energies, under the assumption that the only important forces arise from the Born singularities. All possibilities for the $\Sigma\Lambda$ and $\bar{K}\Lambda$ relative parities are considered. The aim is to see to what extent earlier calculations, which neglect the $\bar{K}$-N interactions, are modified by its inclusion. If the $\bar{K}N$ coupling constants are as strong as the πN coupling, significant quantitative and qualitative modifications are obtained: an $I=1$, $J=3/2$ resonance with the properties of the Y_1^* may be obtained for $P(\Sigma\Lambda)=\pm1$; an $I=0$ resonance with the location and width of the Y_0^* may be obtained for $P(\Sigma\Lambda)=-1$, in the $P_{1/2}$ state, and for $P(\Sigma\Lambda)=+1$, in the $P_{3/2}$ state. If the $\bar{K}N$ couplings are significantly weaker than the πN coupling, a $P_{3/2}$ resonance with the properties of the Y_1^* is obtained only if $P(\Sigma\Lambda)=+1$ and if the $\Sigma\Sigma\pi$ coupling is very weak; in this case one obtains no $I=0$ resonance identifiable with the Y_0^*. An $I=0$, $P_{3/2}$ resonance at 1520 MeV may be obtained with a wide variety of couplings for $P(\Sigma\Lambda)=+1$; the predicted width of this resonance is very large ($\Gamma/2>50$ MeV). Resonances in other states, multichannel effects on resonance shapes, and KN elastic scattering are discussed.

I. INTRODUCTION

THE problem of π-Y and $\bar{K}$-N scattering has been studied by many authors.[1] The techniques used range from a completely relativistic approach using the Mandelstam representation,[2] through static model calculations,[3–7] to phenomenological scattering length calculations.[8,9] The major result of the first approach is the determination of the analyticity properties of the various scattering and reaction amplitudes. The low-energy $\bar{K}$-N data are fitted reasonably well by the scattering length approximation, suggesting that the low-energy s-wave interaction is determined mainly by rather distant singularities. (Inclusion of a π-π interaction in the momentum transfer channel modifies the details of the cross-section fit, but does not change the gross behavior.[10]) The static model, in many forms, has been used most extensively to predict which π-Y states will be resonant and to estimate the locations and widths of the resonances. The results of this technique which are most relevant to this paper may be briefly summarized as follows: Amati *et al.*,[3] assuming global symmetry and neglecting the $\bar{K}$-N interaction, predicted a resonance in the $I=1$, $P_{3/2}$ state with energy agreeing remarkably well with the experimental mass of the Y_1^*. In addition, they find an $I=2$, $P_{3/2}$ resonance about 160 MeV higher. They also point out that if the $\Sigma\Sigma\pi$ coupling is very weak, resonances may occur in other states in addition to these two, in particular, in the $I=0$, $P_{3/2}$ state. Assuming that the $\Sigma\Sigma\pi$ coupling is weak, Franklin[6] has tried to estimate the relative locations of these three resonances by estimating the effect on the cutoff integrals of the Σ-Λ mass difference and the crossed terms. Duimio and Wolters[4] applied the

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