Propagation of Ultrasound in Ferromagnetic Metals at Low Temperatures*

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The theory of cyclotron resonance absorption of microwaves and of ultrasonic waves in ordinary metals is extended to ferromagnetic metals. First some arguments are given about the value of the force on the conduction electrons in a ferromagnet. If this force is known, the calculation of the current can be carried out in the same way as in ordinary metals. The elastic waves are coupled to the ferromagnetic resonance through the magnetostriction (magnetoacoustic resonance), and the absorption of elastic waves arises from two entirely different sources but depends in both cases essentially on the conductivity, that is, eddy currents are created by the motion of the magnetization in addition to the effects as in ordinary metals (viscosity of the electron gas, etc.). Formulas for velocity and attenuation of elastic waves are derived. For the case of microwave absorption, the surface impedance in the anomalous skin-effect region, including ferromagnetic resonance, is calculated.

INTRODUCTION

RECENTLY cyclotron resonance in metals has found much interest, since it may give direct information about the Fermi surface. This resonance can be seen either in the absorption of microwaves or in the attenuation of elastic waves due to the motion of conduction electrons under "anomalous conditions." These conditions occur when the mean free path or the time between two collisions of the electrons is comparable to other characteristic quantities of the experimental situation, such as skin depth, wavelength, or frequency.

In the case of microwaves, the penetration depth of the electromagnetic wave is very small, and in an external magnetic field the electrons may describe orbits leading through regions of very different amplitudes of the field. The most important situation is given by a magnetic field parallel or nearly parallel to the plane of surface of the sample and the electromagnetic field impinging perpendicular to the surface. If the mean free path is long enough, some of the conduction electrons will describe orbits, which almost touch the surface several times. The electric field is largest at the surface, and if the electron arrives at the surface each time in phase with the field, the absorption of the microwave will decrease since the current increases. The same effect occurs at each subharmonic of the resonance frequency, thus creating an oscillatory behavior of the absorption in dependence of the magnetic field. This effect has been investigated theoretically[1–4] and experimentally in tin,[5] zinc,[6] copper,[7] and aluminum.[8]

The behavior of an ultrasonic wave is somewhat different since such a wave is able to penetrate the crystal. It is very well known that a large part of the attenuation of ultrasound in metals is connected with the motion of the conduction electrons. Recent investigations are mostly concerned with the "collision-drag effect."[9,10] This effect provides an interaction between the elastic wave and the conduction electrons even when one assumes that the conduction electrons are "free" and thus will be affected neither by the crystal potential nor by the deformation of the crystal potential by the elastic wave. At low temperatures, which are necessary to get long mean free paths, scattering by impurities is dominant. The idea of collision drag is, now, that in this situation the equilibrium distribution of the electrons in velocity space is centered around the local velocity of the impurities which move with the lattice. This is an indirect coupling of the elastic wave to the conduction electrons. It is possible to understand many features of the attenuation of ultrasound under anomalous conditions with this model. But the collision-drag effect is not the only mechanism providing an interaction between lattice and free electrons. It is necessary to take into account that an observer moving with the center-of-mass system will see a current and thus an electromagnetic field, due to the fact that ions and electrons have different masses but equal and opposite charges. This field is especially important at high frequencies. A number of authors have published theoretical investigations on the subject. We mention only a few that have included the effect of an external magnetic field.[11–17] Some recent experimental investigations

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[13] M. J. Harrison, Phys. Rev. Letters **1**, 441 (1958).
[14] T. Kjeldaas, Phys. Rev. **113**, 1473 (1959).
[15] S. Rodriguez, Phys. Rev. **112**, 80 (1958); T. Kjeldaas and T. Holstein, Phys. Rev. Letters **2**, 340 (1959).
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[17] M. H. Cohen, M. J. Harrison, and W. A. Harrison, Phys. Rev. **117**, 937 (1960).

of ultrasonic absorption have been made on copper,[18] tin,[19] bismuth,[20] aluminum,[21] and gold and silver.[22]

The extension of these theories from ordinary to ferromagnetic metals involves several complications, even when we circumvent the question of the complicated band structure of ferromagnetic metals and adopt a simple model with free conduction electrons and localized ferromagnetic electrons, which do not contribute to the current. The motion of the magnetization has a resonance which will be excited with microwaves (ferromagnetic resonance), as well as with elastic waves (magnetoacoustic resonance).[23,24] We will investigate these complications for the highly simplified model just described. That is, we want to calculate the absorption of microwaves and of elastic waves in a ferromagnetic metal due to the motion of conduction electrons under anomalous conditions, but we do this only for the simple case where current carriers and magnetization carriers are easily distinguished. Direct interactions as, for instance, the "*s-d*-exchange" interactions are neglected, and the conduction electrons are treated only as moving charges, but this does not mean that we can neglect the conduction-electron–ferromagnetic-electron interaction entirely. Since the magnetic moments of the ferromagnetic electrons are all aligned, the conduction electrons move not only in the electromagnetic potential of the external field, but also in the potential of all ferromagnetic electrons. This is particularly important if we consider the force exerted on a moving charge. This force enters in the Boltzmann transport equation for the calculation of the current and determines the cyclotron resonance frequency. This problem should be rigorously treated by methods of many-body physics. We do not carry through such an analysis and do not suggest a solution. We assume that the force on the conduction electron can be described by a Lorentz force,

$$\mathbf{F}=-e(\mathbf{E}+\mathbf{v}\times\mathbf{B}_{\text{eff}}), \tag{1}$$

with some effective induction $\mathbf{B}_{\text{eff}}$, and some arguments concerning it are given in Sec. I.

In Sec. II dispersion formulas for elastic waves for some special arrangements are derived. The analysis was done for plane waves and should be good for bulk materials. The free-electron model is used, and a formula results from which one can directly and easily calculate the velocity and the attenuation of the elastic wave. The arrangements treated are transverse waves traveling in the direction of the magnetization and perpendicular to it, with polarization either perpendicular or parallel to the magnetization. In the case of longitudinal waves there are no simple situations that would give anything new compared with ordinary metals. We start by setting up the equation of motion for a system of elastically bound, homogeneously magnetized ions plus an equal number of free electrons. The stress tensor includes terms depending on the magnetization and describing the magnetostriction. We are concerned only with the case where the magnetic field is large enough to saturate the crystal. We have even assumed that the magnetic crystal anisotropies can be neglected.

In Sec. III are added some considerations on the microwave absorption of a semi-infinite sample, magnetized parallel to its surface. Here, only the case where the electric field vector of the electromagnetic field is parallel to the external field is considered. The surface impedance now contains the permeability perpendicular to the external field and shows also the additional ferromagnetic resonance behavior.

In both sections wide use is made of the existing literature on the cyclotron effect in ordinary metals.

I. THE EFFECTIVE INDUCTION

In the theories on cyclotron resonance in metals a so-called "semiclassical" treatment has been used. The electrons are considered as classical particles with sharp position and velocity values. Their equilibrium distribution function, however, is supposed to be a Fermi function. The same function then is used in the classical Boltzmann transport equation. In order to solve this transport equation, we have to know the force exerted on the electron. In the Lorentz force the induction enters, and the question is now whether we can use the Lorentz force (1) with some effective induction for an electron moving through a ferromagnet.

A completely "free" electron has a uniform density distribution throughout the whole crystal. The effective induction for such an electron would be simply the macroscopic induction. Its ground state must be described by a Landau state. In reality, however, the crystal potential will modify the density distribution of the electron. This will average in its own way over the microscopic magnetization, and, in general, we will get another value for the effective induction. But we would not expect a change of an order of magnitude. If we take into account the electron-electron interaction, we get another new picture. The Coulomb interaction and the exchange interaction will affect the density distribution again. The latter creates the "Fermi hole" and will have the effect that the effective induction splits up for conduction electrons with different spins. So we would have to expect different resonance frequencies for conduction electrons with different spins. In this paper this effect will not be taken into account.

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[22] R. W. Morse, A. Myers, and C. T. Walker, Phys. Rev. Letters **4**, 605 (1960).

[23] C. Kittel, Phys. Rev. **110**, 836 (1958).

[24] A. I. Akhiezer, V. G. Bar'iakhtar, and S. V. Peletminski, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 228 (1958) [translation: Soviet Phys.—JETP **8**, 157 (1959)].

The Lorentz force (1) describes only the ponderomotive forces on the electrical charge of the electron. Dorfman[25] has suggested that the spin-orbit interaction $-\mathbf{v}\times\mathbf{M}\cdot\nabla V$, which gives a force on the magnetic moment of the electron, provides a very high effective field for cyclotron resonance. The spin-orbit interaction has been used by Argyres to develop a theory of the magnetic-optical effects.[26] Against Dorfman's proposal it has been stated by Kittel[27] that the spin-orbit interaction is invariant under the translational group of the lattice. The eigenfunctions of the electrons have to be rigorously of the Bloch form. But we should remember that the latter statement is only correct as long as we do not take into account the vector potential of all the aligned ferromagnetic electrons. On the other hand, it follows from the derivation of the spin-orbit interaction from Dirac's equation for the electron, that this term cannot act as a vector potential. We assume here that at the frequencies with which we are concerned the effect of the spin-orbit coupling on the current is still negligible. This may serve as justification to use the free-electron model, where the spin-orbit coupling vanishes anyway.

II. PROPAGATION OF ELASTIC WAVES

We start with setting up the equation of motion for our system. This consists of N positive ions per unit volume with mass M_i, which are elastically bound and homogeneously magnetized, and an equal number of electrons with mass m. In the center-of-mass system the equation of balance for the average momentum per unit volume is given by

$$(\partial/\partial t)(m+M_i)\langle[\mathbf{v}_i+\gamma_e(\mathbf{v}_e-\mathbf{v}_i)]f\rangle = \mathrm{Div}(\mathbf{P}-\langle\mathbf{p}f\rangle)+\mathbf{j}\times\mathbf{B}_{\mathrm{eff}}. \quad (2)$$

The brackets $\langle\cdots\rangle$ denote the operation of integrating in velocity space over all electron velocities. The distribution function f for the electrons is so normalized that $\langle f\rangle=N$. Furthermore, $\mathbf{v}_i$ is the ion velocity, $\gamma_e=m/(m+M_i)$ the partial mass fraction of the electrons, and $\mathbf{j}$ the current density,

$$\mathbf{j}=Ne\mathbf{v}_i-e\langle\mathbf{v}_e f\rangle=Ne(\mathbf{v}_i-\mathbf{v}_{e\ \mathrm{av}}), \quad (3)$$

where $\mathbf{v}_{e\ \mathrm{av}}$ is the average electron velocity. The tensor divergence on the right side of (2) is applied on the deviation of the stresses from their equilibrium value, i.e., $\mathbf{P}$ is the elastic stress tensor and $-\langle\mathbf{p}f\rangle$ gives the deviation of the electron gas pressure from its equilibrium value. The distribution function f can be found as a solution of the Boltzmann transport equation

$$\partial f/\partial t+\mathbf{v}_e\cdot\nabla_r f+(1/m)\mathbf{F}\cdot\nabla_v f=-(1/\tau)(f-f_s), \quad (4)$$

where the ponderomotive force $\mathbf{F}$ is given by the Lorentz force (1). According to the concept of "collision drag" the distribution function f relaxes toward an equilibrium distribution function, which is centered around the ion velocity,

$$f_s=f_0(\mathbf{v}_e-\mathbf{v}_i).$$

It is well known[17] how to solve the transport equation (4). We give only the solution for transverse waves. In this case the density of the particles N is constant, and we get, after expanding f_s in deviations from the equilibrium function $f_0(\mathbf{v}_e)$ for a resting observer,

$$f=f_0(\mathbf{v}_e)+f_1=f_0+e(df_0/d\epsilon)\mathbf{J}\cdot[\mathbf{E}-(m/e\tau)\mathbf{v}_i];$$

$$\mathbf{J}=\int_0^\infty \mathbf{v}_e(t-t')\exp[i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')-i\omega(t-t')+(t-t')/\tau]\times d(t-t'). \quad (5)$$

In principle we are able now to perform the integration over all electron velocities in the equation of motion (2). The left side is

$$(\partial/\partial t)(m+M_i)\langle[\mathbf{v}_i+\gamma_e(\mathbf{v}_e-\mathbf{v}_i)]f\rangle = (\partial/\partial t)[\rho\mathbf{v}_i+Nm(\mathbf{v}_{e\ \mathrm{av}}-\mathbf{v}_i)],$$

where ρ is the specific density. To express the rate of change of the average relative velocity $\mathbf{v}_{r\ \mathrm{av}}=\mathbf{v}_{e\ \mathrm{av}}-\mathbf{v}_i$ we multiply the transport equation (4) with $\mathbf{v}_e$ and integrate over all electron velocities. Except for the physically insignificant terms $-\partial\mathbf{v}_i/\partial t$ and $(e/m)\mathbf{v}_i\times\mathbf{B}_{\mathrm{eff}}$, we get

$$(\partial/\partial t)\mathbf{v}_{r\ \mathrm{av}}=-(1/\tau)\mathbf{v}_{r\ \mathrm{av}} -(e/m)(\mathbf{E}+\mathbf{v}_{r\ \mathrm{av}}\times\mathbf{B}_{\mathrm{eff}})-(1/Nm)\,\mathrm{div}\langle\mathbf{p}f\rangle. \quad (6)$$

The last term and the velocity-dependent part of the Lorentz force will be compensated in the equation of motion, so that we finally get

$$NM_i(\partial/\partial t)\mathbf{v}_i=\mathrm{div}\mathbf{P}+Ne\mathbf{E}+(Nm/\tau)\mathbf{v}_{r\ \mathrm{av}}. \quad (7)$$

At this point we would like to add a remark on longitudinal waves. For simplicity we restrict ourselves for the moment to ordinary metals without external magnetic field. Pippard[10,11] has pointed out that as long as we neglect the displacement current, the convection current must vanish. This is due to the following fact: The periodic changes of density of the ultrasonic wave would create periodic space charges and therewith a "depolarizing" electric field. But this field is so large that the space charges never can build up. The electron current is identical to the ion current everywhere. Now, if we look at our equation of motion (2), we see that only one term remains that can cause an attenuation of the elastic wave. This term is the deviation of the electron gas pressure from its equilibrium value. An appropriate calculation leads to the same result as Pippard's. In the course of this calculation one has to account for the fact that the volume of the Fermi sphere varies periodically, since the electron density does so. To achieve this, Pippard did introduce an

[25] J. Dorfman, Doklady Akad. Nauk **110**, 201 (1956).
[26] P. N. Argyres, Phys. Rev. **97**, 334 (1955).
[27] C. Kittel, Phys. Rev. **108**, 1097 (1957).

"electric field" that keeps the radius of the Fermi sphere at its proper value. However, this field is not subject to Maxwell's equations, and it would be better to speak of a "Pauli force" since the force that appears here stems directly from the Pauli principle.[28]

We return now to the discussion of the equation of motion (7). In a ferromagnet the elastic stress tensor P contains terms that describe the magnetostriction and that depend on the magnetization. We choose the z axis as the direction of the magnetization. For small deviations of the magnetization from this direction we get in linear approximation for the stress tensor

$$\begin{aligned} P_{xx}&=2C_2\epsilon_{xx}+C_1(\epsilon_{xx}+\epsilon_{yy}+\epsilon_{zz}); \quad P_{xy}=2C_2\epsilon_{xy};\\ P_{yy}&=2C_2\epsilon_{yy}+C_1(\epsilon_{xx}+\epsilon_{yy}+\epsilon_{zz});\\ &\qquad P_{yz}=2C_2\epsilon_{yz}+(B/M_s)M_y; \quad (8)\\ P_{zz}&=2C_2\epsilon_{zz}+C_1(\epsilon_{xx}+\epsilon_{yy}+\epsilon_{zz});\\ &\qquad P_{zx}=2C_2\epsilon_{zx}+(B/M_s)M_x. \end{aligned}$$

Here ϵ_{xx}, ϵ_{yy}, $\cdots$ are the components of the strain tensor, C_1 and C_2 are elastic constants, B is the magnetoelastic coupling energy, and M_s the saturation magnetization.

The deviations M_x, M_y of the magnetization from the direction of saturation can be calculated from

$$(\partial/\partial t)\mathbf{M}=\gamma\mathbf{M}\times\mathbf{H}_{\text{tot}}+(\gamma A_{\text{ex}}/M_s^2)\mathbf{M}\times\Delta\mathbf{M} +\text{damping term.} \quad (9)$$

Here γ is the gyromagnetic ratio, and we have accounted for a spin-wave term with the exchange energy density A_{ex}. The total magnetic field $\mathbf{H}_{\text{tot}}$ consists of the external field $\mathbf{H}_0$, the induced field $\mathbf{H}$, and the "apparent" magnetic field of the elastic wave $\mathbf{H}_e$, which in our case is given by

$$\mathbf{H}_e=(B/M_s)(e_{xz},\epsilon_{yz},0).$$

We shall use Eq. (9) only in its linearized form.

The induced fields $\mathbf{E}$ and $\mathbf{H}$ are to be found from Maxwell's equations:

$$\begin{aligned} &\text{curl}\mathbf{H}=\mathbf{j}; && \text{div}(\mu_0\mathbf{H}+\mathbf{M})=0;\\ &\text{curl}\mathbf{E}=-(\partial/\partial t)(\mu_0\mathbf{H}+\mathbf{M}); && \text{div}\mathbf{E}=\rho_{\text{el}}/\epsilon_0. \end{aligned} \quad (10)$$

The equation of continuity for the current is needed only for longitudinal waves, it is automatically fulfilled for transverse waves. If we introduce the solution (5) of the distribution function into (3) and carry out the integration, we will get the current density in the form

$$\mathbf{j}=\boldsymbol{\sigma}\mathbf{E}+Ne[\mathsf{E}-(1/\sigma_0)\boldsymbol{\sigma}]\mathbf{v}_i, \quad (11)$$

where $\sigma_0=Ne^2\tau/m$, $\boldsymbol{\sigma}$ is the conductivity tensor, and E is the unit tensor. The deviation of the magnetization from the saturation can be written

$$\mathbf{M}=\boldsymbol{\chi}(\mathbf{H}+\mathbf{H}_e), \quad (12)$$

with a susceptibility tensor $\boldsymbol{\chi}$.

[28] This has been pointed out by W. Döring (private communication).

In principle the formulation of our problem is given by Eqs. (7) to (12). It could be solved directly, for example, in terms of plane waves for bulk material, but since we have two tensors describing the properties of matter here, namely, the conductivity and the susceptibility tensors, the resulting formulas would be much too involved. Therefore, we turn to the consideration of some simple representative cases.

1. Transverse Elastic Waves Traveling in the Direction of Magnetization

This case can be analyzed and solved rigorously with circularly polarized plane waves $u_\pm=u_x\pm iu_y$. The dispersion formulas of the two modes can be separated, and, after straightforward calculation, we obtain[29]

$$\begin{aligned} \frac{\rho\omega^2}{C_2k^2}=1&+\frac{1}{2}\frac{\mu_0B^2\chi_\pm}{M_s^2C_2}\frac{k^2+i\omega\mu_0\sigma_\pm}{k^2+i\omega\mu_0\sigma_\pm\mu_\pm}\\ &\mp\frac{iBm(1-\sigma_\pm/\sigma_0)}{2e\tau M_sC_2}\frac{i\omega\mu_0\chi_\pm\sigma_0}{k^2+i\omega\mu_0\sigma_\pm\mu_\pm}\\ &+\frac{i\omega m^2\sigma_0(1-\sigma_\pm/\sigma_0)}{e^2\tau^2C_2k^2}\frac{k^2+i\omega\mu_0\sigma_0\mu_\pm}{k^2+i\omega\mu_0\sigma_\pm\mu_\pm}. \end{aligned} \quad (13)$$

Here we have

$$\chi_\pm=\frac{\omega_0\pm\omega+A_{\text{ex}}M_sk^2}{(\omega_0+A_{\text{ex}}M_sk^2)^2+(i\omega-\Gamma)^2}\chi\cdot\omega_0,$$

and

$$\mu_\pm=1+\chi_\pm.$$

The expression for $\sigma_\pm$ is given in the Appendix. The symbols used are the rotational susceptibility $\chi=M_s/\mu_0H_0$, the ferromagnetic resonance frequency $\omega_0=\gamma H_0$, the reciprocal of the Bloch relaxation time Γ, the cyclotron frequency ω_c, the Fermi velocity v_F, and the propagation constant k.

From (13) we can easily get attenuation and velocity of the sound wave, if we take the frequency ω of the sound wave to be complex and calculate imaginary and real part of the right-hand side; but we should keep in mind that this is only correct as long as the right-hand side does not deviate too much from 1. Attenuation and velocity are connected with each other through the Kramers-Kronig relation, but it is much simpler to calculate both quantities directly.

2. Transverse Plane Wave Traveling Perpendicular to the Magnetization with Polarization Perpendicular to Magnetization

In this case the magnetoelastic coupling vanishes and we have only the direct-current effect as in ordinary

metals. For convenience we give the result[29]

$$\frac{\rho\omega^2}{C_2k^2}=1+\frac{i\omega Nm}{\tau C_2k^2}\left(1-\frac{\sigma_{22}}{\sigma_0}\right)\frac{k^2+i\omega\mu_0\sigma_0}{k^2+i\omega\mu_0\sigma_{22}}, \tag{14}$$

where the conductivity component σ_{22} is given in the appendix.

3. Transverse Plane Wave Traveling Perpendicular to the Magnetization with Polarization Parallel to the Magnetization

We proceed in the same manner and get[29]

$$\frac{\rho\omega^2}{C_2k^2}=1+\frac{1}{2}\frac{B^2\mu_0}{M_s^2C_2\mu}\frac{\chi'k^2+i\omega\mu_0(\chi'\mu-\chi''^2)\sigma_3}{k^2+i\omega\mu_0(\mu-\chi''^2/\mu)\sigma_3}+\frac{1}{2}\frac{mB}{e\tau M_sC_2\mu}\frac{\omega\mu_0\chi''(\sigma_3-\sigma_0)}{k^2+i\omega\mu_0(\mu-\chi''^2/\mu)\sigma_3}+\frac{i\omega Nm(1-\sigma_3/\sigma_0)}{\tau C_2k^2}\frac{k^2+i\omega\mu_0(\mu-\chi''^2/\mu)\sigma_0}{k^2+i\omega\mu_0(\mu-\chi''^2/\mu)\sigma_3}, \tag{15}$$

with

$$\chi'=\omega_0\chi\frac{\omega_0+A_{\mathrm{ex}}M_sk^2}{(\omega_0+A_{\mathrm{ex}}M_sk^2)^2+(i\omega-\Gamma)^2},$$

and

$$\chi''=\omega_0\chi\frac{i\omega-\Gamma}{(\omega_0+A_{\mathrm{ex}}M_sk^2)^2+(i\omega-\Gamma)^2}.$$

For σ_3 see Appendix. In applying this formula we must be careful. If we are in the magnetoacoustic resonance, some terms on the right-hand side will be large and it is doubtful whether one can drive such a wave through the crystal without admixtures of other wave modes. This formula is only correct as long as the right side does not deviate too much from 1.

4. Longitudinal Waves Traveling Parallel and Perpendicular to the Magnetization

In both cases the magnetoelastic coupling vanishes, as one can see easily from (8). This coupling is different from zero only at intermediate angles of magnetic field to propagation direction. But then the calculation of the conductivity tensor is rather complicated. We shall not pursue this further.

Discussion

Obviously we must get the attenuation and sound velocity in ordinary metals if we put the magnetoelastic energy B equal to zero and if we neglect the deviations of the conductivity from the static value. Since both resonance phenomena arise from entirely different physical reasons, we get a mixture of both. The most striking difference is, of course, the temperature dependence. The magnetoacoustic resonance should be observable at room temperatures, whereas the cyclotron resonance is appreciable only in the region of the residual resistance. There is some experimental evidence supporting the mechanism of magnetoacoustical resonance, but the resonance itself has not yet been observed in bulk metals. Bömmel and Dransfeld[30] did find it with a specimen of YIG, but YIG is a good insulator. The resonance absorption in nickel is at least very much smaller than in YIG at 10^9 cps.[31] On the other hand, this coupling mechanism has been pretty well confirmed by Alers, Neighbours, and Sato[32] in the frequency region well below the resonance. In this case it is possible to calculate the dependence of the velocity and the attenuation on the angle between propagation direction and external field.[33] This dependence is different for both quantities. Experiment and theory are in good agreement except for the fact that the experimentally observed attenuation is a factor of 10 larger than that expected theoretically. To date there is no explanation for this.

There is no need to go into the discussion of the cyclotron resonance in ordinary metals. This is fairly well covered by the various papers on the subject. However, we may note that (13) and (15) include very different resonance phenomena, connected either with the frequency of the elastic wave (proper "cyclotron resonance") or with the radii of the electron orbits and the wavelength of the elastic wave ("geometric resonances"). First we look somewhat closer at (13), where we have no geometric resonances, since the circular orbits are perpendicular to the propagation direction of the elastic wave. It is sufficient for our purpose to consider only the second and the fourth term on the right side. We may note that only one of the two susceptibilities $\chi_\pm$ has a resonance denominator, i.e., only the elastic wave circularly polarized in the same sense as the spin wave will have a resonance. The behavior in metals is dictated by the frequency, where $k^2\approx\omega\mu_0\sigma$, i.e., where the wavelength and the electromagnetic skin depth are comparable, the wavelength being larger than the skin depth at lower frequencies. Usually this "relaxation" frequency is about 10^8 to 10^9 cps. Since the conductivity depends on the magnetic field in the anomalous region, this relaxation frequency may be shifted, in general, to lower values in the high-field limit. So if we consider the "low-frequency" case, $k^2\ll\omega\mu_0\sigma$, we would have to supplement a high-field limit, which is not done here, since we are not going into detail. There are arguments that the semiclassical treatment with the Boltzmann equation will lose validity at very high fields.[14,17]

[29] In Eqs. (13), (14), and (15) the ω and k of sound waves without coupling to spin waves and electromagnetic waves have to be used on the right-hand side.

[30] H. E. Bömmel and K. Dransfeld, Bull. Am. Phys. Soc. **5**, 357 (1960).

[31] K. Dransfeld (private communication).

[32] G. A. Alers, J. R. Neighbours, and H. Sato, J. Phys. Chem. Solids **9**, 21 (1958).

[33] G. Simon, Z. Naturforsch **13A**, 84 (1958).

For the attenuation in the low-frequency limit, the second and fourth terms in (13) are

$$\mathrm{Re}\left[\frac{1}{2}\left(\frac{B}{M_s}\right)^2\frac{k^2}{C_2\omega\sigma_\pm}\left(\frac{\chi_\pm}{\mu_\pm}\right)^2\right],$$

and

$$\frac{\omega m^2\sigma_0}{e^2\tau^2C_2k^2}\,\mathrm{Re}\left(\frac{\sigma_0}{\sigma_\pm}-1\right).$$

Kjeldaas has shown that $\mathrm{Re}[(\sigma_0/\sigma_\pm)-1]$ has a maximum at $\omega_c=-\omega$ and an edge at $\omega_c/\omega\approx v_F/c_s$ (c_s = velocity of sound).[14] Roughly speaking, the attenuation will fall off almost monotonically with increasing field with a strong decrease in the neighborhood of $\omega_c \sim kv_F$, so we can conclude that the first term, whose dependence on the conductivity can be written as $\mathrm{Re}[(\sigma_0/\sigma_\pm)-1+1]$, will do the same. The ferromagnetic resonance will be located on some point on this curve and can be shifted by the external field or the propagation constant of the elastic wave, but at very high fields the attenuation will rise again.[17]

In the high frequency limit, $k^2\gg\omega\mu_0\sigma$, the two terms are

$$(\mu_0B\chi_\pm/M_s)^2(\omega\sigma_\pm/C_2k^2)$$

and

$$(\omega m^2\sigma_0/e^2\tau^2C_2k^2)\;\mathrm{Re}(1-\sigma_\pm/\sigma_0).$$

The last term will reach a constant value with increasing field, whereas the first term will drop down except for the resonance behavior of one of the circularly polarized waves at the ferromagnetic resonance frequency.

A few words may be added to case 3. Here we have the geometric resonances, as already mentioned. The limiting cases of low and high frequencies can be written down in a similar way. However, a new feature is that the attenuation due to the magnetoelastic coupling is dependent on the off-diagonal element of the susceptibility tensor χ'', and thus will fall off rapidly outside the ferromagnetic resonance.

III. EXCITATION OF CYCLOTRON RESONANCE WITH MICROWAVES

In ordinary metals the theory of cyclotron resonance absorption in the microwave region is well known and has been found experimentally. The extension to ferromagnetic metals is easy, once we have assumed that the force on the conduction electrons is given by (1). The aim is to calculate the surface impedance of a semi-infinite sample, magnetized parallel to the surface. Since we use only the linear approximation in the ferromagnetic resonance, we can restrict ourselves to the case where the electric field vector of the microwave is parallel to the external field. In ordinary metals a resonance can be obtained even when the electric field vector is perpendicular to the external field, but this is not considered here.

We utilize a coordinate system with the z axis pointing into the sample and the x direction pointing into the direction of the external field. We can specify Maxwell's Eqs. (16)[34]:

$$\frac{\partial H_y}{\partial z}=-\frac{4\pi}{c}j_x;\quad \frac{\partial E_y}{\partial z}=\frac{i\omega}{c}H_x;$$

$$\frac{\partial H_x}{\partial z}=\frac{4\pi}{c}j_y;\quad \frac{\partial E_x}{\partial z}=-\frac{i\omega}{c}(H_y+4\pi M_y);$$

$$0=j_z;\qquad 0=H_z+4\pi M_z.$$

Again we have assumed that the magnetization has only small deviations from the x direction. We may use Eq. (9) for the motion of the magnetization, and we find

$$\begin{aligned}\frac{\partial^2E_x}{\partial z^2}&=\frac{4\pi i\omega}{c^2}\mu_rj_x,\\ \frac{\partial^2E_y}{\partial z^2}&=\frac{4\pi i\omega}{c^2}j_y.\end{aligned}\tag{16}$$

The expression for the permeability μ_r will be given later. The current is calculated with the Boltzmann transport Eq. (4), with the difference, of course, that f_s is the equilibrium distribution function itself. The analysis is difficult and somewhat lengthy. We may follow the paper by Azbel and Kaner[1] step by step, or use Rodriguez'[2] simpler treatment with Pippard's ineffectiveness concept. What is done is essentially to go to the Fourier transform of Eq. (16) and of the Boltzmann equation for the current, taking into account the boundary condition for the reflection of the electrons at the surface of the sample. If the conductivity tensor for the surface currents has no off-diagonal terms, we get for the surface impedance

$$Z=\frac{4\pi}{c}\frac{E_x(0)}{H_y(0)}=-\frac{4\pi i\omega\mu_r}{c^2}\frac{E_x(0)}{E_x'(0)}.\tag{17}$$

The result of the above-mentioned analysis connects $E_x(0)$ with its derivative $E_x'(0)$ in the form,

$$E_x(0)=-E_x'(0)\frac{2}{\pi}\int_0^\infty\frac{s}{s^3+a^3}ds,\tag{18}$$

with

$$a^3=\frac{3\pi^2i\omega\mu_r}{c^2}\frac{B_x}{1-\exp(-2\pi\bar{\gamma})},$$

and

$$B_x=\frac{8e^2}{3h^2}\int_0^{2\pi}\frac{u_x^2}{K}d\varphi.$$

Here it is assumed that we have a more general Fermi surface, but with revolutionary symmetry about the x axis. The surface is described by polar angles θ and φ

[34] In this section we use electromagnetic units in order to have an easier comparison to Azbel and Kaner's result.

with respect to the x axis, the Gaussian curvature K at $\theta=\pi/2$, and the x component of the normal vector u_x. The quantity $\bar{\gamma}$ is given by $\bar{\gamma}=[(1/\tau)+i\omega]/\omega_c$.

The evaluation of the integral yields finally

$$Z=\frac{8}{9}\left(\frac{\sqrt{3}\pi\omega^2\mu_r^2}{c^4B_x}\right)^{\frac{1}{3}}(1+i\sqrt{3})[1-\exp(-2\pi\bar{\gamma})]^{\frac{1}{3}}. \quad (19)$$

The result for ordinary metals is obtained by putting $\mu_r\equiv1$. If the ferromagnetic resonance is not damped, we have

$$\mu_r=(\omega_0^2\mu^2-\omega^2)/(\omega_0^2\mu-\omega^2), \quad \mu=1+4\pi M_s/H_0.$$

Then μ_r^2 is always real. If we take μ_r to be complex, $\mu_r=\mu'-i\mu''$, we get an additional phase shift in Z:

$$Z=\frac{8}{9}\left(\frac{\sqrt{3}\pi\omega^2(\mu'^2+\mu''^2)}{c^4B_x}\right)^{\frac{1}{3}}e^{i2\Phi/3}(1+i\sqrt{3})\times[1-\exp(-2\pi\bar{\gamma})]^{\frac{1}{3}},$$

with

$$\Phi=\tan^{-1}(\mu''/\mu').$$

We may note that the fundamental cyclotron resonance frequency is larger than the ferromagnetic resonance frequency, which is $\omega_0(\mu)^{\frac{1}{2}}$ here. Since at the subharmonics of the cyclotron resonance frequency a resonance absorption is also observable, we may ask at which place the ferromagnetic resonance occurs. The dependence of the cyclotron resonance frequency and the ferromagnetic resonance frequency on the external field is different, and the ferromagnetic resonance frequency may be shifted, for instance, to a point in between the fundamental and the first subharmonic cyclotron resonance by choosing the field $H_0/4\pi M_s>0.27$.

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APPENDIX

The conductivity in a magnetic field may be calculated with a solution of the Boltzmann equation due to Chambers.[35] This solution is

$$f=f_0-\frac{df_0}{d\mathcal{E}}\int_{-\infty}^{t}(-e)\mathbf{E}(\mathbf{r}',t')\cdot\mathbf{v}(\mathbf{r}',t')e^{-(t-t')/\tau}dt',$$

where the integration over t' has to be carried out along the trajectory of the electron. This trajectory is determined by the external field (or the effective induction in a ferromagnet). The analysis has been carried through by several authors,[15,17] and we quote the results here for convenience:

$$\frac{\sigma_\pm}{\sigma_0}=\frac{3}{4}\int_0^\pi\frac{\sin^3\theta d\theta}{1+i\tau(\pm\omega_c+kv_F\cos\theta-\omega)},$$

where ω_c is the cyclotron frequency,

$$\frac{\sigma_{22}}{\sigma_0}=3\sum_{u=-\infty}^{+\infty}\frac{s_u(kv_F/\omega_c)}{1+i\tau(u\omega_c-\omega)},$$

and

$$\frac{\sigma_3}{\sigma_0}=3\sum_{u=-\infty}^{+\infty}\frac{r_u(kv_F/\omega_c)}{1+i\tau(u\omega_c-\omega)},$$

with

$$s_u(X)=\int_0^{\pi/2}[J_u'(X\sin\theta)]^2\sin^3\theta d\theta,$$

$$r_u(X)=\int_0^{\pi/2}J_u^2(X\sin\theta)\cos^2\theta\sin\theta d\theta.$$

The J_u are Bessel functions. For more details see Cohen, Harrison, and Harrison.[17]

[35] See C. Kittel, *Elementary Statistical Physics* (John Wiley & Sons, Inc., New York, 1958), p. 194.