Bremsstrahlung from Polarized Electrons*

W. R. JOHNSON AND J. D. ROZICS
University of Notre Dame, Notre Dame, Indiana
(Received April 27, 1962)

The production of unpolarized bremsstrahlung by polarized electrons in the Coulomb field of a nucleus is considered. An approximate analytical expression for the differential cross section summed over final electron spins is presented. This differential cross section is integrated numerically over angles of the emitted electron to determine the asymmetry in the photon angular distribution. Results of the numerical integration are given for incident electron velocities in the range $0.4c \leq v \leq 0.9c$.

I. INTRODUCTION

It is well known that transversely polarized electrons elastically scattered by the nuclear Coulomb field exhibit an azimuthal asymmetry about the direction of motion of the incident electron beam.[1,2] The existence of a similar asymmetry in the distribution of bremsstrahlung is therefore not surprising. Qualitatively, one expects the photon asymmetry to be an effect of the same order of magnitude as the electron scattering asymmetry; this is shown to be the case.

A calculation of the lowest order term in a series in the Coulomb parameter αZ of the photon asymmetry is presented. Since to lowest order the asymmetry is proportional to αZ, the bremsstrahlung cross section must be computed to one higher order than the Bethe-Heitler formula.[3] However, since we limit ourselves to consideration of the asymmetry, only terms in the αZ correction to the Bethe-Heitler cross section which contain the incident electron spin need be considered. These spin-dependent terms may be extracted rather easily from the S matrix.

To evaluate the cross-section differential in photon and electron angles and photon energy one must evaluate certain integrals which diverge if the external field is a Coulomb field. This difficulty is avoided by employing the device introduced by Dalitz[4] of replacing the Coulomb field by a "screened" field and allowing the screening to vanish after the cross section has been calculated.

The differential cross section, which is given in analytical form, is integrated numerically to determine the spin-dependent part of the photon angular distribution. The ratio of the spin-dependent part of the cross section to the Bethe-Heitler angular distribution gives the photon asymmetry.

The following general trends of the photon asymmetry are found: The asymmetry increases from zero at the soft photon limit of the spectrum to a maximum at the high-frequency limit; it is a maximum for incident electron velocities near $0.6c$; and it increases in magnitude from zero at 0° to a maximum near 130°, decreasing to zero at 180°.

II. CALCULATION OF THE DIFFERENTIAL CROSS SECTION

The differential cross section $d^6\sigma$ for production of bremsstrahlung is given by[5]

$$d^6\sigma = \frac{1}{(2\pi)^6}\frac{W_1}{p_1} d^3p_2 d^3k \sum_{\boldsymbol{\zeta}_2,\boldsymbol{\varepsilon}} w_{ba}, \tag{1}$$

where $p_i = (\mathbf{p}_i; iW_i)$, $i=1, 2$, represent the momentum four-vectors for the incident and final electron, respectively, and where $k = (\mathbf{k}; i\omega)$ represents the photon momentum four-vector. w_{ba} is the transition rate from the state $|a\rangle = |\mathbf{p}_1, \boldsymbol{\zeta}_1\rangle$ to the $|b\rangle = |\mathbf{p}_2, \boldsymbol{\zeta}_2; \mathbf{k}, \boldsymbol{\varepsilon}\rangle$, $\boldsymbol{\zeta}_1$ and $\boldsymbol{\zeta}_2$ being the spin vectors of the incident and final electron in the electron's rest system, and $\boldsymbol{\varepsilon}$ being the photon's polarization vector. The transition rate after summation over photon polarization and final electron spin is a linear function of the incident electron's spin,

$$\sum_{\boldsymbol{\zeta}_2,\boldsymbol{\varepsilon}} w_{ba} = w + \mathbf{w}\cdot\boldsymbol{\zeta}_1. \tag{2}$$

The contribution of w to the differential cross section gives to lowest order the Bethe-Heitler formula. We, therefore, only need consider the lowest order term in $\mathbf{w}\cdot\boldsymbol{\zeta}_1$.

Writing the S matrix as $S = I + iM$ and introducing $H = (M+M^*)/2$, $A = (M-M^*)/2$, one easily verifies that the spin-dependent part of the transition probability is given by

$$\mathbf{w}\cdot\boldsymbol{\zeta}_1 = 2\sum_{\boldsymbol{\zeta}_2,\boldsymbol{\varepsilon}} \langle b|A|a\rangle\langle a|H|b\rangle. \tag{3}$$

The matrix element of the anti-Hermitian term A can be written as a sum over a complete set of intermediate states $|n\rangle$,

$$\langle b|A|a\rangle = \tfrac{1}{2}i \sum_{|n\rangle} \langle b|M^*|n\rangle\langle n|M|a\rangle \tag{4}$$

in virtue of the unitarity of S. Denoting by $A_j{}^i$, $H_j{}^i$, $M_j{}^i$ the contributions to the corresponding matrices from Feynman diagrams with i vertices, j of which

* This work was supported in part by the U. S. Atomic Energy Commission.

[1] N. F. Mott, Proc Roy. Soc. (London) **A124**, 425 (1929).
[2] N. F. Mott, Proc. Roy. Soc. (London) **A135**, 429 (1932).
[3] H. Bethe and W. Heitler, Proc. Roy. Soc. (London) **A146**, 83 (1934).
[4] R. H. Dalitz, Proc. Roy. Soc. (London) **A206**, 509 (1951).
[5] J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Cambridge, Massachusetts, 1955), p. 390.

represent external field interactions, one finds by comparing powers of e and Z on both sides of Eq. (4)

$$\langle b|A_1^2|a\rangle=0, \tag{5a}$$

$$\begin{aligned}\langle b|A_2^3|a\rangle=&\tfrac{1}{2}i\sum_{\mathbf{p},\boldsymbol{\zeta}}\langle\mathbf{p},\boldsymbol{\zeta}|M_1^1|a\rangle\langle\mathbf{p},\boldsymbol{\zeta}|M_1^2|b\rangle^*\\&+\tfrac{1}{2}i\sum_{\mathbf{p},\boldsymbol{\zeta}}\sum_{\mathbf{k}',\boldsymbol{\varepsilon}'}\langle\mathbf{p},\boldsymbol{\zeta};\mathbf{k}'\boldsymbol{\varepsilon}'|M_1^2|a\rangle\\&\qquad\times\langle\mathbf{p},\boldsymbol{\zeta};\mathbf{k}'\boldsymbol{\varepsilon}'|M_1^1|b\rangle^*.\end{aligned} \tag{5b}$$

The lowest order contribution to the transition probability is therefore given by Eq. (3) with A replaced by A_2^3 and H replaced by H_1^2. Substituting the relevant matrix elements into Eq. (5b) one finds, after a simple reduction,

$$\langle b|A_2^3|a\rangle=-\frac{(4\pi)^{3/2}\alpha^{5/2}Z^2\delta(W_1-W_2-\omega)}{(2\omega)^{1/2}}\bar{u}_2O_\mu u_1, \tag{6}$$

with

$$O_\mu=\int d^3p\left(\frac{\delta(p_1-p)}{2p_1}\frac{N_\mu(m-ip)\gamma_4}{(q_2^2+\lambda^2)(q_{10}^2+\lambda^2)}+\frac{\delta(p_2-p)\gamma_4(m-ip)M_\mu}{2p_2(q_1^2+\lambda^2)(q_{20}^2+\lambda^2)}\right), \tag{7}$$

where

$$\mathbf{q}_1=\mathbf{p}_1-\mathbf{p}-\mathbf{k},\quad \mathbf{q}_2=\mathbf{p}-\mathbf{p}_2-\mathbf{k},\quad \mathbf{q}_{10}=\mathbf{p}_1-\mathbf{p},\quad \mathbf{q}_{20}=\mathbf{p}-\mathbf{p}_2,$$

and

$$\begin{aligned}N_\mu&=\frac{\gamma_\mu(-i(k+p_2)+m)\gamma_4}{(p_2+k)^2+m^2}+\frac{\gamma_4(-i(p-k)+m)\gamma}{(p-k)^2+m^2},\\ M_\mu&=\frac{\gamma_\mu(-i(k+p)+m)\gamma_4}{(p+k)^2+m^2}+\frac{\gamma_4(-i(p_1-k)+m)\gamma_\mu}{(p_1-k)^2+m^2}.\end{aligned} \tag{8}$$

The matrix element of the Hermitian term in Eq. (3) may be written

$$\langle a|H_2^2|b\rangle=\frac{i2\pi(4\pi)^{3/2}\alpha^{3/2}Z\delta(W_1-W_2-\omega)}{(2\omega)^{1/2}(q^2+\lambda^2)}(\bar{u}_1\bar{L}_\mu u_2), \tag{9}$$

where $\mathbf{q}=\mathbf{p}_1-\mathbf{p}_2-\mathbf{k}$ and

$$\bar{L}_\mu=\gamma_4L_\mu\gamma_4=\frac{\gamma_4(-i(k+p_2)+m)\gamma_\mu}{(p_2+k)^2+m^2}+\frac{\gamma_\mu(-i(p_1-k)+m)\gamma_4}{(p_1-k)^2+m^2}. \tag{10}$$

In writing Eqs. (7) and (9) we have assumed the external field to be a "screened" Coulomb field. The Fourier transform of the external potential is given by

$$a_\mu(s)=-2\pi i\delta(s_0)\frac{(4\pi\alpha)^{1/2}Z}{|s|^2+\lambda^2}\delta_{\mu4}. \tag{11}$$

From Eqs. (7) and (9) one obtains for the spin-dependent part of the differential cross section

$$d^5\sigma=2\frac{\alpha^4Z^3}{(2\pi)^3}\frac{p_2}{p_1}kdkd\Omega_{p_2}d\Omega_k\frac{1}{q^2}T, \tag{12}$$

with

$$T=\tfrac{1}{4}\operatorname{Tr}[O_\mu(m-ip_1)\gamma_5\sigma L_\mu(m-ip_2)]+\text{c.c.}, \tag{13}$$

where σ is a four-vector related to the electron's spin three-vector $\boldsymbol{\zeta}_1$ by

$$\sigma=\left(\boldsymbol{\zeta}_1+\frac{\mathbf{p}_1\cdot\boldsymbol{\zeta}_1}{m(W_1+m)}\mathbf{p}_1,\frac{i\mathbf{p}_1\cdot\boldsymbol{\zeta}_1}{m}\right).$$

The trace and polarization sum in Eq. (13) is carried out to give

$$T=T_1+T_2+\text{c.c.},$$

with

$$T_1=\int\frac{d^3p}{2p_1}\frac{\delta(p_1-p)}{(q_{10}^2+\lambda^2)q_2^2}\sum_{i=1}^4 v_i, \tag{14a}$$

$$T_2=\int\frac{d^3p}{2p_2}\frac{\delta(p-p_2)}{(q_{20}^2+\lambda^2)q_1^2}\sum_{i=1}^4 w_i. \tag{14b}$$

The functions v_i and w_i are listed in the Appendix. We have dropped the factor λ^2 in the denominator of Eq. (12) as well as in the q_1^2 and q_2^2 terms of Eqs. (14), since no divergences arise from these terms. The integrals in Eqs. (14) are carried out with the aid of the formulas for the basic integrals presented in the Appendix. After a great deal of algebraic manipulation, the resulting expression for T may be reduced to

$$T=2\pi m(\mathbf{p}_1\cdot\boldsymbol{\zeta}_1\gamma A+\nu B+\mu_1C+\mu_2D), \tag{15}$$

where

$$\mu_1=(\mathbf{k}\times\mathbf{p}_1)\cdot\boldsymbol{\sigma},\quad \mu_2=(\mathbf{k}\times\mathbf{p}_2)\cdot\boldsymbol{\sigma},$$
$$\gamma=(\mathbf{p}_2\times\mathbf{p}_1)\cdot\mathbf{k},\quad \nu=(\mathbf{p}_2\times\mathbf{p}_1)\cdot\boldsymbol{\sigma},$$

and where

$$A=\frac{1}{m}\left[\frac{W_1+W_2}{D_1D_2}\mathbf{q}\cdot\mathbf{R}+\frac{2}{D_2}\left(\frac{\beta_1+\beta_2}{\beta_1\beta_2}+W_2\frac{p_1+p_2}{p_1p_2}\right)Q\right], \tag{16a}$$

$$\begin{aligned}B=&\left(\frac{m^2}{D_2^2}+\frac{\mathbf{p}_1\cdot\mathbf{p}_2-(W_1+k)W_2}{D_1D_2}\right)\mathbf{q}\cdot\mathbf{R}-\left(\frac{m^2}{D_1^2}+\frac{\mathbf{p}_1\cdot\mathbf{p}_2-(W_1+k)W_2}{D_1D_2}\right)\mathbf{q}\cdot\mathbf{T}+\left(1+\frac{8m^2}{D_1D_2}+\frac{2W_2k}{D_2}\right)\frac{p_1+p_2}{p_1p_2}Q\\&+\left(\frac{1}{D_2}-\frac{1}{D_1}\right)\left(2W_1Q\frac{\beta_1+\beta_2}{\beta_1\beta_2}+\tfrac{1}{2}\mathbf{q}\cdot\mathbf{R}\right)+(\mathbf{k}\cdot\mathbf{U}+\mathbf{p}_1\cdot\mathbf{R}+\mathbf{p}_2\cdot\mathbf{T})/(2D_2)-(\mathbf{k}\cdot\mathbf{V}+\mathbf{s}\cdot\mathbf{R}+\mathbf{l}\cdot\mathbf{T})/(2D_1),\end{aligned} \tag{16b}$$

$$C=\left(\frac{m^2}{D_2^2}+\frac{\mathbf{p}_1\cdot\mathbf{p}_2-(W_1+k)W_2}{D_1D_2}\right)\mathbf{q}\cdot\mathbf{R}+\frac{D_1-D_2-2W_2k}{p_1D_1}Q+\left(\frac{1}{D_2}-\frac{1}{D_1}\right)\left(\frac{2(W_2k+m^2)}{p_1}Q+\tfrac{1}{2}\mathbf{q}\cdot\mathbf{R}\right)+\frac{2}{D_1D_2}P$$
$$+\frac{8W_2Q}{D_1}\left(\frac{\beta_1+\beta_2}{\beta_1\beta_2}\right)+\left(1-\frac{16m^2W_2k}{D_1D_2}-\frac{q^2}{D_1}\right)\frac{p_1+p_2}{p_1p_2}Q+((\mathbf{p}_1+\mathbf{s})\cdot\mathbf{R}-\mathbf{l}\cdot\mathbf{U})/(2D_2)-(\mathbf{k}\cdot\mathbf{U}-\mathbf{p}_2\cdot\mathbf{V})/(2D_1), \quad (16c)$$

$$D=-\left(\frac{m^2}{D_1^2}+\frac{\mathbf{p}_1\cdot\mathbf{p}_2-(W_1+k)W_2}{D_1D_2}\right)\mathbf{q}\cdot\mathbf{T}+\frac{2(W_2^2+p_1p_2)}{p_2}\left(\frac{1}{D_2}-\frac{1}{D_1}\right)Q+\frac{8m^2(W_1+W_2)k}{D_1D_2}\left(\frac{p_1+p_2}{p_1p_2}\right)Q-\frac{2}{D_1D_2}P$$
$$-\frac{4(W_1+W_2)}{D_2}Q\left(\frac{\beta_1+\beta_2}{\beta_1\beta_2}\right)-\frac{2W_2k}{p_2D_1}Q+(\mathbf{p}_1\cdot\mathbf{U}-\mathbf{p}_1\cdot\mathbf{R}+\mathbf{p}_2\cdot\mathbf{T})/(2D_2)-(\mathbf{p}_2\cdot\mathbf{V}+\mathbf{p}_1\cdot\mathbf{R}+\mathbf{l}\cdot\mathbf{T})/(2D_1). \quad (16d)$$

In Eqs. (16) the following symbols are used:

$$\mathbf{R}=\frac{1}{[\mathbf{p}_1\times\mathbf{s}]^2}[\hat{p}_1L_4-\hat{s}L_6],\quad \mathbf{T}=\frac{1}{[\mathbf{p}_2\times\mathbf{l}]^2}[\hat{p}_2L_4'-\hat{l}L_6'],$$

$$\mathbf{U}=\frac{1}{[\mathbf{p}_1\times\mathbf{k}]^2}[\hat{p}_1(L_2+L_3)+\hat{k}(L_5'-L_5)-\hat{l}L_6'],$$

$$\mathbf{V}=\frac{1}{[\mathbf{p}_2\times\mathbf{k}]^2}[-\hat{p}_2(L_2+L_3')+\hat{k}(L_5'-L_5)+\hat{s}L_6],$$

$$Q=\frac{L_1+L_2}{D_3},\quad P=\frac{1}{2l}L_6'+\frac{1}{2P_2}L_4',$$

with

$$L_1=\ln\frac{4m^2k^2q^4}{D_1^2D_2^2},\qquad L_2=\ln\frac{W_1W_2+p_1p_2-m^2}{W_1W_2-p_1p_2-m^2},$$

$$L_3=\ln\frac{4m^2k^2}{D_1^2},\qquad L_3'=\ln\frac{4m^2k^2}{D_2^2},$$

$$L_4=\ln\frac{D_2^2}{q^4},\qquad L_4'=\ln\frac{D_1^2}{q^4},$$

$$L_5=\ln\frac{W_1+p_1}{W_1-p_1},\qquad L_5'=\ln\frac{W_2+p_2}{W_2-p_2},$$

$$L_6=\ln\left(\frac{p_1+s}{p_1-s}\right)^2,\quad L_6'=\ln\left(\frac{l+p_2}{l-p_2}\right)^2,$$

where

$$D_1=-2p_1\cdot k=2W_1k-2\mathbf{p}_1\cdot\mathbf{k},$$
$$D_2=-2p_2\cdot k=2W_2k-2\mathbf{p}_2\cdot\mathbf{k},$$
$$D_3=2q^2(W_1W_2+p_1p_2-m^2)-D_1D_2,$$
$$\mathbf{q}=\mathbf{p}_1-\mathbf{p}_2-\mathbf{k},\quad \mathbf{l}=\mathbf{p}_1-\mathbf{k},\quad \mathbf{s}=\mathbf{p}_2+\mathbf{k}.$$

A formula of such complexity requires some specific verification. We shall outline two checks which have been made. In the soft photon limit the bremsstrahlung cross section, when integrated over photon angles and a small range of photon energies, must be proportional to the elastic scattering cross section. One finds that in the limit $k\to 0$

$$T\to 16\pi\mathbf{n}\cdot\boldsymbol{\zeta}_1\frac{mp_1\sin\theta}{|\mathbf{p}_1+\mathbf{p}_2|^2}\times\ln\left(\frac{|\mathbf{p}_1-\mathbf{p}_2|}{2p_1}\right)\left\{\frac{m^2}{D_1^2}+\frac{m^2}{D_2^2}+\frac{2p_1\cdot p_2}{D_1D_2}\right\}, \quad (17)$$

where θ is the electron scattering angle and $\mathbf{n}$ is the unit normal to the plane of scattering. Integrating over photon energies and angles, one finds

$$d^2\sigma\to -F\mathbf{n}\cdot\boldsymbol{\zeta}_1\frac{4\alpha^3Z^3mp_2\sin\theta}{|\mathbf{p}_1+\mathbf{p}_2|^2|\mathbf{p}_1-\mathbf{p}_2|^2}\times\ln\frac{|\mathbf{p}_1-\mathbf{p}_2|}{2p_1}d\Omega_{p_2}, \quad (18)$$

where

$$F=-\frac{\alpha}{\pi^2}\int\left(\frac{m^2}{D_1^2}+\frac{m^2}{D_2^2}+\frac{2p_1\cdot p_2}{D_1D_2}\right)kdkd\Omega_k \quad (19)$$

is the well known factor of proportionality. The coefficient of F is found to be the lowest order term in the spin-dependent part of the elastic scattering cross section.[6]

A second check involving every term in B, C, and D is obtained at the high-frequency limit of the spectrum. If one takes only the leading terms in a series in p_2 in Eq. (13) and performs the resulting traces and integrals one obtains formulas for B, C, and D which agree with the corresponding limiting values in Eqs. (16). Since the asymmetry is maximum in this limit, we quote the result:

$$B\underset{p_2\to 0}{\longrightarrow}\frac{1}{p_2l}\left[\frac{4m^2}{l^4}-\frac{4m}{kl^2}+\frac{2}{mk}+\frac{1}{k^2}\right], \quad (20a)$$

[6] W. R. Johnson, T. A. Weber, and C. J. Mullin, Phys. Rev. **121**, 933 (1961).

$$C \underset{p_2\to 0}{\longrightarrow} \frac{4}{kl^4}\left[\frac{l}{m}\hat{p}_2\cdot\hat{l}-2\hat{p}_2\cdot\hat{k}\right]+\frac{l^2}{p_1m^3k^3}\left\{\frac{m^2k(k-m)}{l^4}+\frac{2m^3k^2(k-m)}{l^6}+\frac{p_1m^3}{2l^4}L_5+\left[\frac{m(m+k)}{l^2}-\frac{2m^2k(3k+m)}{l^4}\right]\right.$$
$$\left.\times\frac{mkp_1}{4}\mathbf{p}_1\cdot\mathbf{R}+\left[\frac{m(k-m)}{l^2}+\frac{2m^2k(2k+m)}{l^4}\right]\frac{mkp_1}{4}\mathbf{k}\cdot\mathbf{R}\right\}, \qquad (20b)$$

$$D \underset{p_2\to 0}{\longrightarrow} \frac{1}{p_2l^2}\left[\frac{4m^2}{l^4}+\frac{4m}{kl^2}-\frac{2}{mk}-\frac{3}{k^2}\right]. \qquad (20c)$$

III. PHOTON ASYMMETRY

The differential cross section obtained in Sec. II is now integrated over angles of the outgoing electron to give a photon cross section

$$d^3\sigma(k,\Omega_k,\boldsymbol{\zeta}_1)=-\mathbf{n}\cdot\boldsymbol{\zeta}_1\frac{r_0^2\alpha^2Z^3}{\pi}\frac{p_2}{p_1}\frac{dk}{k}d\Omega_kI(k,\theta), \qquad (21)$$

where θ is the photon production angle, $\mathbf{n}=(\mathbf{k}\times\mathbf{p}_1)/|\mathbf{k}\times\mathbf{p}_1|$ is the unit normal to the photon production plane, and

$$I(k,\theta)=\frac{k^2m^3}{2\pi}\int\frac{d\Omega_{p_2}}{q^2}[\mathbf{n}\cdot(\mathbf{p}_2\times\mathbf{p}_1)B$$
$$+\mathbf{n}\cdot(\mathbf{k}\times\mathbf{p}_1)C+\mathbf{n}\cdot(\mathbf{k}\times\mathbf{p}_2)D]. \qquad (22)$$

The Bethe-Heitler cross section, when integrated over angles of the outgoing electron, may be expressed similarly[7]:

$$d^3\sigma_{\mathrm{BH}}(k,\Omega_k)=\frac{r_0^2\alpha Z^2}{\pi}\frac{p_2}{p_1}\frac{dk}{k}d\Omega_kJ(k,\theta). \qquad (23)$$

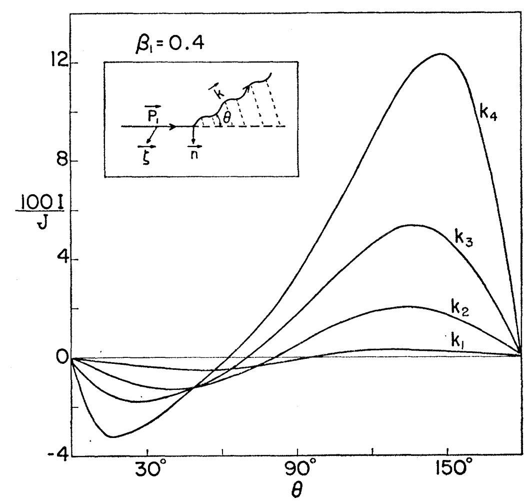

FIG. 1. The asymmetry ratio $100I/J$ for $\beta_1=0.4$. The asymmetry

$$P(k,\theta)=d^3\sigma(k,\Omega_k,\boldsymbol{\zeta}_1)/d^3\sigma_{\mathrm{BH}}(k,\Omega_k)=-\mathbf{n}\cdot\boldsymbol{\zeta}_1\alpha zI(k,\theta)/J(k,\theta),$$

where $\mathbf{n}$ is the unit normal to the production plane and $\boldsymbol{\zeta}_1$ is the incident electron spin. $k_1=10^{-4}m$, $k_2=0.25k_{\max}$, $k_3=0.50k_{\max}$, $k_4=0.75k_{\max}$, $k_{\max}=W_1-m$.

[7] H. W. Koch and J. W. Motz, Revs. Modern Phys. **31**, 924 (1959).

The resulting asymmetry is given by

$$P(k,\theta)=d^3\sigma(k,\Omega_k,\boldsymbol{\zeta}_1)/d^3\sigma_{\mathrm{BH}}(k,\Omega_k)$$
$$=-\mathbf{n}\cdot\boldsymbol{\zeta}_1\alpha ZI(k,\theta)/J(k,\theta). \qquad (24)$$

In the limiting case $p_2\to 0$ both the Bethe-Heitler cross section and the spin-dependent correction vanish but their ratio remains finite. In an exact calculation neither of these cross sections would vanish since a Coulomb normalization factor would replace the factor p_2/p_1, in Eqs. (21) and (23). The limiting value of Eq. (24) is, therefore, meaningful; it is given as the

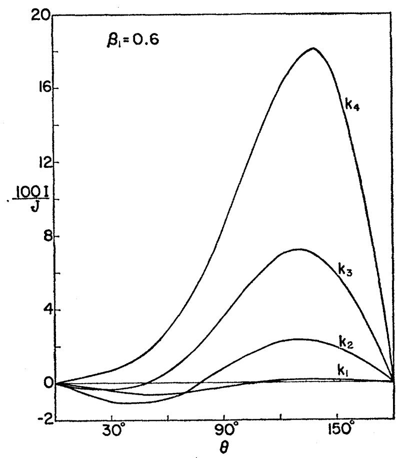

FIG. 2. Bremsstrahlung asymmetry for $\beta_1=0.6$.

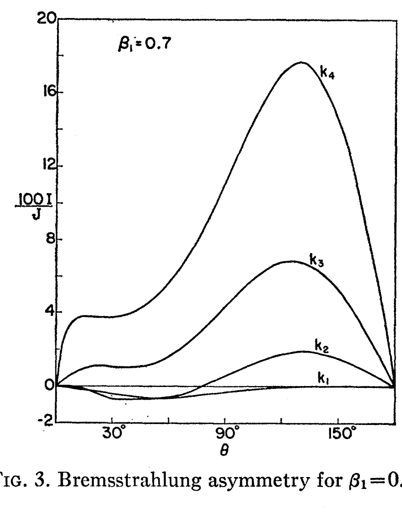

FIG. 3. Bremsstrahlung asymmetry for $\beta_1=0.7$.

ratio of $I_0(k,\theta)$ to $J_0(k,\theta)$, where

$$I_0(k,\theta)=2\sin\theta\left\{\left[\frac{mk(k-m)}{l^4}+\frac{2m^3k^2(k-m)}{l^6}\right]+\frac{p_1m^3}{2l^4}L_5+\left[\frac{m(m+k)}{l^2}-\frac{2m^2k(3k+m)}{l^4}\right]\frac{mkp_1}{4}\mathbf{p}_1\cdot\mathbf{R}\right.$$
$$\left.+\left[\frac{m(k-m)}{l^2}+\frac{2m^2k(2k+m)}{l^4}\right]\frac{mkp_1}{4}\mathbf{k}\cdot\mathbf{R}\right\}. \tag{25a}$$

$$J_0(k,\theta)=\left\{\frac{m(m-k)}{l^2}+\frac{4m(k^3-m^2k-m^3)}{l^4}-\frac{4m^3k(k^2-4m^2-5mk)}{l^6}-\frac{16m^6k^2}{l^8}\right\}. \tag{25b}$$

Since a factor of $\sin^2\theta$ occurs implicitly in Eq. (25b) the corresponding asymmetry will diverge at $\theta=0°$ and $\theta=180°$. For finite values of p_2 the asymmetry vanishes at $\theta=0°$ and $\theta=180°$.

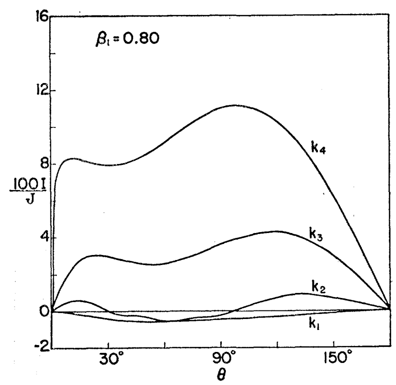

FIG. 4. Bremsstrahlung asymmetry for $\beta_1=0.8$.

The integral in Eq. (22) has been evaluated numerically for several values of θ and k, and for various incident electron velocities. The ratio $I(k,\theta)/J(k,\theta)$ is plotted in Figs. 1 to 5, where $k_1=10^{-4}m$, $k_2=0.25k_{\max}$, $k_3=0.50k_{\max}$, $k_4=0.75k_{\max}$, and $k_{\max}=W_1-m$. To obtain the asymmetry from these graphs one must multiply the given ratio by αZ.

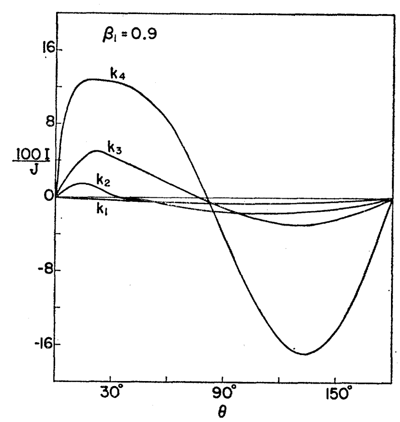

FIG. 5. Bremsstrahlung asymmetry for $\beta_1=0.9$.

ACKNOWLEDGMENTS

The authors wish to thank Professor C. J. Mullin for valuable conservations. Thanks are also due to Dr. R. F. O'Connell and J. J. Matese for checking various parts of the computation.

APPENDIX

The functions v_i and w_i of Eqs. (14) are given by

$$v_1=-\frac{4m}{D_2^2}[-m^2\Gamma+p_2\cdot k(\mathrm{K}+\Pi_1)], \tag{A1}$$

$$v_2=\frac{4m}{D_1D_2}\{p_1\cdot p_2\Gamma+p_2\cdot k\Pi_2-W_2\omega\Gamma+2W_1[(W_1+W_2)\mu_2-2W_2\mu_1+\omega\nu]-\sigma_0[(W_1+W_2)\Lambda_2-2W_2\Lambda_1+\omega\Delta]\}, \tag{A2}$$

$$v_3=\frac{4m}{D_2D}\{p_2\cdot p\Gamma+p_2\cdot k\Pi_2-W_2\omega\Gamma+2W_1\omega\Pi_2-4W_1W_2\mathrm{K}-2W_1[(W_1+W_2)\mu_2-2W_2\mu_1+\omega\nu]$$
$$+\sigma_0[(W_1+W_2)\Lambda_2-2W_2\Lambda_1+\omega\Delta]\}, \tag{A3}$$

$$v_4=-\frac{4m}{D_1D}[p_1\cdot p\Gamma+p_1\cdot k(\Pi_2-\Pi_1)+p_1\cdot k(\Pi_2-\Pi_1)+p_2\cdot k(\Pi_1+\mathrm{K})-p_1\cdot p_2\mathrm{K}-p\cdot p_2\mathrm{K}-m^2\mathrm{K}$$
$$+2\omega W_2\Pi_1-2W_2(W_1+W_2)\mathrm{K}], \tag{A4}$$

$$w_1=-\frac{4m}{D_2D}\{-p\cdot p_2\Gamma'+p_2\cdot k(\mathrm{K}'-\Pi_1')-(p\cdot k-p_2\cdot k)(\mu_2+\nu)+W_2\omega\pi_1'-\omega(2W_1+W_2)\Pi_2'+W_2[2(W_1+W_2)+\omega]\mathrm{K}'$$
$$-\sigma_0[(W_1+W_2)\Lambda_2'+\omega\Delta'-2W_2\Lambda_1']\}, \tag{A5}$$

$$w_2=\frac{4m}{D_1D}\{-p\cdot p_1\Gamma'+p\cdot k(\Pi_1'-\Pi_2')-p_2\cdot k\Pi_1'+(p_2\cdot p-p\cdot p_1+p_2\cdot p-p_2^2)(K'+\mu_2-\mu_1)$$
$$-(p\cdot k-p_2\cdot k)(\mu_1+\nu)+2W_2[(W_1+W_2)K'-\omega\Pi_1']+2W_2[(W_1+W_2)\mu_2-2W_2\mu_1+\omega\nu]\}, \quad (A6)$$

$$w_3=\frac{4m}{D_1D_2}\{-p_1\cdot p_2\Gamma'-p_2\cdot k\Pi_2'-(p\cdot k-p_2\cdot k)(\mu_1+\nu)+(p_2\cdot p-p_2^2+p_1\cdot p-p_1\cdot p_2)(\mu_1-\mu_2)$$
$$-2W_2[(W_1+W_2)\mu_2-2W_2\mu_1+\omega\nu]\}, \quad (A7)$$

$$w_4=\frac{4m}{D_1^2}[m^2\Gamma'-p_1\cdot k\Pi_2'-\sigma\cdot k(\Delta'+\Lambda_2')], \quad (A8)$$

with

$$\Pi_1=(\mathbf{p}_1\times\mathbf{q}_{10})\cdot\boldsymbol{\sigma},\quad \Pi_2=(\mathbf{p}_2\times\mathbf{q}_{10})\cdot\boldsymbol{\sigma},\quad K=(\mathbf{k}\times\mathbf{q}_{10})\cdot\boldsymbol{\sigma},$$
$$\Lambda_1=(\mathbf{p}_1\times\mathbf{q}_{10})\cdot\mathbf{k},\quad \Lambda_2=(\mathbf{p}_2\times\mathbf{q}_{10})\cdot\mathbf{k},\quad \Gamma=(\mathbf{q}\times\mathbf{q}_{10})\cdot\boldsymbol{\sigma},$$
$$\Delta=(\mathbf{p}_2\times\mathbf{p}_1)\cdot\mathbf{q}_{10},\quad D=-2p\cdot k=2Wk-2\mathbf{p}\cdot\mathbf{k}.$$

The primed quantities in Eqs. (A5) to (A8) are obtained from the corresponding unprimed expressions by replacing q_{10} by q_{20}.

The integrals occuring in Eq. (14a) may be expressed in terms of the following basic integrals:

$$\int\frac{d^3p\mathbf{q}_{10}\delta(p-p_1)}{2p_1(q_{10}^2+\lambda^2)q_2^2}=\mathbf{p}_1A_V+\mathbf{s}B_V, \quad (A9)$$

$$\int\frac{d^3p\mathbf{q}_{10}\delta(p-p_1)}{2p_1(q_{10}^2+\lambda^2)D}=\mathbf{p}_1A_R+\mathbf{k}C_R, \quad (A10)$$

$$\int\frac{d^3p\mathbf{q}_{10}\delta(p-p_1)}{2p_1q_2^2D}=\mathbf{p}_1A_Q+\mathbf{s}B_Q+\mathbf{k}C_Q, \quad (A11)$$

$$\int\frac{d^3p\mathbf{q}_{10}\delta(p-p_1)}{2p_1q_2^2D(q_{10}^2+\lambda^2)}=\mathbf{p}_1A+\mathbf{s}B+\mathbf{k}C, \quad (A12)$$

with

$$A_V=\frac{\pi}{4}\frac{\mathbf{s}}{[\mathbf{p}_1\times\mathbf{s}]^2}\cdot[\hat{s}L_6-\hat{p}_1L_4],$$

$$B_V=\frac{\pi}{4}\frac{\mathbf{p}_1}{[\mathbf{p}_1\times\mathbf{s}]^2}\cdot[\hat{p}_1L_4-\hat{s}L_6],$$

$$A_R=\frac{\pi}{4}\frac{\mathbf{k}}{[\mathbf{p}_1\times\mathbf{k}]^2}\cdot[\hat{k}L_5-\hat{p}_1L_3],$$

$$C_R=\frac{\pi}{4}\frac{\mathbf{p}_1}{[\mathbf{p}_1\times\mathbf{k}]^2}\cdot[\hat{p}_1L_3-\hat{k}L_5],$$

$$A_Q=\frac{\pi}{2p_2D_2}L_2,$$

$$B_Q=\frac{\pi}{4}\frac{\mathbf{k}}{[\mathbf{p}_2\times\mathbf{k}]^2}\cdot\left[(\mathbf{s}H-\mathbf{k}G)\frac{L_2}{p_2D_2}+\hat{k}L_5-\hat{s}L_6\right],$$

$$C_Q=\frac{\pi}{4}\frac{\mathbf{s}}{[\mathbf{p}_2\times\mathbf{k}]^2}\cdot\left[(\mathbf{k}G-\mathbf{s}H)\frac{L_2}{p_2D_2}-\hat{k}L_5+\hat{s}L_6\right],$$

$$A=\frac{\pi}{4}\left[-\frac{N_A}{p_1\Delta}L_1+\frac{M_A}{p_2D_2\Delta}L_2\right],$$

$$B=\frac{\pi}{4}\left[-\frac{N_B}{p_1\Delta}L_1+\frac{M_B}{p_2D_2\Delta}L_2\right],$$

$$C=\frac{\pi}{4}\left[-\frac{N_C}{p_1\Delta}L_1+\frac{M_C}{p_2D_2\Delta}L_2\right],$$

where

$$H=2W_1k,\quad G=\mathbf{p}_1^2+s^2,$$
$$N_A=4\mathbf{p}_1\cdot[\mathbf{s}H-\mathbf{k}G],\quad N_B=-4\mathbf{p}_1^2D_1,\quad N_C=4\mathbf{p}_1^2q^2,$$
$$M_A=4(\mathbf{s}H-\mathbf{k}G)\cdot(\mathbf{k}q^2-\mathbf{s}D_1),$$
$$M_B=4H\mathbf{p}_1\cdot(\mathbf{s}H-\mathbf{k}G)+8\mathbf{p}_1^2(k^2q^2-\mathbf{s}\cdot\mathbf{k}D_1),$$
$$M_C=4\mathbf{p}_1\cdot\mathbf{k}(G^2-4\mathbf{p}_1^2s^2)-4\mathbf{p}_1\cdot\mathbf{s}(GH-4\mathbf{p}_1^2\mathbf{s}\cdot\mathbf{k})+8\mathbf{p}_1^2\mathbf{s}\cdot(\mathbf{s}H-\mathbf{k}G),$$
$$\Delta=D_3D_4,\quad D_4=2q^2(W_1W_2-p_1p_2-m^2)-D_1D_2.$$

The integrals in Eq. (14b) are determined from the above integrals by an appropriate interchange of vectors.