PHYSICAL REVIEW VOLUME 128, NUMBER 1 OCTOBER 1, 1962

Current Conservation and $|\Delta T|=\frac{1}{2}$ Rule in θ Decay*

MIRZA A. BAQI BÉG AND PAUL C. DECELLES
Brookhaven National Laboratory, Upton, New York
(Received May 17, 1962)

It is shown that, if the divergences of the vector currents of the weak interactions satisfy a spectral representation with at most one subtraction, the dominant term in the ratio of θ^+ to $\theta_1{}^0$ decay rates can be computed without explicit introduction of the electromagnetic field. Furthermore, a wide class of reasonable S-wave $\pi^+\pi^0$ scattering phase shifts can account for the exact θ^+/θ^0 branching ratio. In particular, if one accepts a $T=1$, 0^+ ζ meson, the final result is remarkably close to the experimental θ^+/θ^0 ratio.

I. INTRODUCTION

PERHAPS the most significant violation of the $|\Delta\mathbf{T}|=1/2$ rule is presented by the decay $K^+\to\pi^++\pi^0$. Previous attempts[1] to understand the θ^+/θ^0 branching ratio in terms of a fundamental $|\Delta\mathbf{T}|=1/2$ rule have consisted of (a) a qualitative assertion that electromagnetic effects reduce this rate by order α^2 (as compared to the $\theta_1{}^0$ decay rate) followed by (b) a calculation of the enhancement factor due to a strong final-state interaction in the $T=2$, $J=0$ state.

A perturbation calculation of the electromagnetic effects due to an internal photon line is a rather formidable task, the results of which may not be readily interpretable due to renormalization difficulties. Also, the assumption of a strong $T=2$, $J=0$ interaction seems at variance with available data on the $\pi\pi$ interaction.[2] Therefore, the θ^+ puzzle remains.

In the present note, we propose a resolution of the problem starting directly from the current$\times$current interaction. Our work is based essentially on the observation that the $|\Delta\mathbf{T}|=1/2$ rule emerges in the same limit in which the strangeness preserving vector current is exactly conserved.[3] This common limit is evidently the one in which the isotopic multiplets are fully mass degenerate.

II. DECAY AMPLITUDES AND FORM FACTORS

In this section we calculate the decay amplitude for θ^+ and θ^0 decays. We shall avoid any explicit introduction of the electromagnetic field; violations of isospin conservation are introduced through the inequality of charged and neutral pion masses as well as the inequality of the coupling constants associated with charged and neutral currents. Obviously in this approach, the pion states should no longer be enumerated as eigenstates of isotopic spin.

We write the interaction density in the form

$$\mathcal{H}(x)=G_CJ^\dagger(x)S(x)-G_N\frac{1}{\sqrt{2}}J_0{}^\dagger(x)S_0(x)+\text{H.c.} \tag{2.1}$$

$$=G\left[J^\dagger(x)S(x)-\frac{1}{\sqrt{2}}J_0{}^\dagger(x)S_0(x)\right]-\frac{\delta G}{\sqrt{2}}J_0{}^\dagger(x)S_0(x)+\text{H.c.} \tag{2.2}$$

$$\delta G=G_N-G_C,\quad G_C\equiv G,$$

where, in isospace, (S,S_0) and $(S_0{}^\dagger,S^\dagger)$ transform as spinors and $((J+J^\dagger)/\sqrt{2},\ (J-J^\dagger)/i\sqrt{2},\ J_0)$ transforms as a vector. We shall also make explicit use of the following (well-known) properties of the current

$$\partial_\mu J_0{}^\mu=0, \tag{2.3}$$

$$[S^\lambda,\mathcal{S}]=S^\lambda, \tag{2.4}$$

where $\mathcal{S}$ is the strangeness operator.[4] Furthermore, if one writes $\mathcal{H}=\mathcal{H}_++\mathcal{H}_-$, $\mathcal{H}_\pm=\pm P\mathcal{H}_\pm P^{-1}$, only $\mathcal{H}_-$ contributes in θ decay.

Consider now the transition amplitude due to a specific term in $\mathcal{H}_-$, say $J_A{}^\dagger\cdot S_V$. Denoting by p_α, p_β, and k_λ the four-momenta and charges of the pions and K meson, respectively, we have[5]

$$\langle p_+p_0|J_A{}^\dagger(0)\cdot S_V(0)|k_+\rangle$$

$$=i\int d^4y\,\theta(y_0)\frac{e^{ip_0\cdot y}}{[(2\pi)^3(2p_0{}^0)]^{1/2}}\langle p_+|[j_0(y),J_A{}^\dagger S_V]|k_+\rangle+2\text{ equal-time commutator terms} \tag{2.5}$$

$$=i\sum_n\int d^4y\,\theta(y_0)\frac{e^{ip_0\cdot y}}{[(2\pi)^3(2p_0{}^0)]^{1/2}}[\langle p_+|[j_0(y),J_A{}^\dagger]|n\rangle\times\langle n|S_V|k_+\rangle+\langle p_+|J_A{}^\dagger|n\rangle\langle n|[j_0(y),S_V]|k_+\rangle]+\text{equal-time commutator terms.} \tag{2.6}$$

We now assume that the most important contribu-

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] M. L. Good and W. G. Holladay, Phys. Rev. Letters **4**, 138 (1960).

[2] J. Kirz, J. Schwarz, and R. D. Tripp, Phys. Rev. **126**, 763 (1962). For relevance to τ decay as well as earlier references see M. A. B. Bég and P. C. DeCelles, Phys. Rev. Letters **8**, 46 (1962).

[3] R. P. Feynmann and M. Gell-Mann, Phys. Rev. **109**, 193 (1958); S. S. Gershtein and J. B. Zeldovich, J. Exptl. Theoret. Phys. (U.S.S.R.) **2**, 576 (1957).

[4] T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960).

[5] See, for example, H. Lehmann, Suppl. Nuovo cimento **14**, 153 (1959).

tion to the sum in (2.6) comes from the vacuum state. This approximation is equivalent to the assumption that the strong interaction involves only a pair of particles at a time and that the pair effects may be additively superposed. Thus,

$$\langle p_+p_0|J_A{}^\dagger(0)S_V(0)|k_+\rangle \cong \langle p_+|J_A{}^\dagger(0)|0\rangle\langle p_0|S_V(0)|k_+\rangle. \quad (2.7)$$

In this manner we obtain

$$\langle p_+p_0|\mathcal{H}_-(0)|k_+\rangle = G\Big[\langle p_+|J_V{}^\dagger|-p_0\rangle\langle 0|S_A|k_+\rangle+\langle p_+|J_A{}^\dagger|0\rangle \times\langle p_0|S_V|k_+\rangle-\frac{1}{\sqrt{2}}\langle p_0|J_{0A}|0\rangle\langle p_+|S_V|k_+\rangle\Big] -\frac{\delta G}{\sqrt{2}}\langle p_0|J_{0A}|0\rangle\langle p_+|S_V|k_+\rangle, \quad (2.8)$$

and

$$\langle p_+p_-|\mathcal{H}_-|k_0\rangle=G\langle p_+|J_A{}^\dagger|0\rangle\langle p_-|S_V|k_0\rangle. \quad (2.9)$$

Using Lorentz invariance alone, the matrix elements occurring above may be written in terms of invariant form factors:

$$[(2\pi)^3 2k_+{}^0]^{1/2}\langle 0|S_A{}^\mu|k_+\rangle = k_+{}^\mu f_k(m_K{}^2), \quad (2.10)$$

$$[(2\pi)^3 2p_+{}^0]^{1/2}\langle 0|J_A{}^\mu|p_+\rangle = p_+{}^\mu f_{\pi+}(m_{\pi+}{}^2), \quad (2.11)$$

$$[(2\pi)^6 4p_+{}^0p_0{}^0]^{1/2}\langle p_+|J_V{}^{\mu\dagger}|-p_0\rangle = (p_+-p_0)^\mu F_+(s)+(p_++p_0)^\mu F_-(s), \quad (2.12)$$

$$[(2\pi)^6 4p_+{}^0k_+{}^0]^{1/2}\langle p_+|S_V{}^\mu|k_+\rangle = \{(p_++k_+)^\mu G_+(s)+(p_+-k_+)^\mu G_-(s)\}[-(\tfrac{2}{3})^{1/2}]. \quad (2.13)$$

The factor of $(\frac{2}{3})^{1/2}$ arises from isotopic spin considerations, since we consider only the $T=1/2$ state of the $K\pi$ system.

With the exception of $F_-(s)$, all these form factors are well known and have been considered in the literature. Thus, f_k and f_π are related to the rates for $K_{\mu 2}$ and $\pi_{\mu 2}$ decays,[6] and $G_\pm$ are the form factors familiar from K_{l3} decays.[7,8] If one assumes that $G_+(s)$ as well as the divergence of S_V [viz., $(m_k{}^2-m_\pi{}^2)G_+(s)+sG_-(s)$] satisfy once subtracted spectral representations it can be shown that[7]

$$G_+(s)=e^{R_1(s)}G_+(0), \quad (2.14)$$

$$G_-(s)=[(m_K{}^2-m_\pi{}^2)/s]\{e^{R_0(s)}-e^{R_1(s)}\}G_+(0), \quad (2.15)$$

where

$$R_l(s)=\frac{s}{\pi}\int_{(m_K+m_\pi)^2}^\infty \frac{\Delta_l(s')ds'}{s'(s'-s-i\epsilon)},$$

$\Delta_l(s)$ being the phase shift for $K\pi$ scattering in the $T=\frac{1}{2}$ state, angular momentum l. In the same manner it can be shown that

$$F_+(s)=e^{\rho_1(s)}F_+(0), \quad (2.16)$$

$$F_-(s)=(\Delta m^2/s)[e^{\rho_0(s)}-e^{\rho_1(s)}]F_+(0). \quad (2.17)$$

with

$$\rho_l(s)=\frac{s}{\pi}\int_{4m_\pi{}^2}^\infty \frac{\delta_l(s')ds'}{s'(s'-s'i\epsilon)},$$

$\delta_l(s)$ being the phase shift for $\pi^+\pi^0$ scattering in angular momentum l, and $\Delta m^2=m_{\pi^+}{}^2-m_{\pi^0}{}^2$.

The ratio of θ^+ to $\theta_1{}^0$ decay rates can immediately be written in terms of these quantities. We have

$$\frac{R(K^+\to\pi^+\pi^0)}{R(K_1{}^0\to\pi^+\pi^-)}=\frac{1}{2}\frac{R(K^+\to\pi^+\pi^0)}{R(K_1{}^0\to\pi^+\pi^-)} =\frac{1}{2}\left[\frac{\Delta m^2\left\{e^{\rho_0(m_K{}^2)}f_K(m_K{}^2)F_+(0)-\frac{1}{\sqrt{3}}e^{R_0(m_\pi{}^2)}f_\pi(m_\pi{}^2)G_+(0)\right\}+\frac{\delta G}{G\sqrt{3}}f_\pi(m_\pi{}^2)G_+(0)(m_K{}^2-m_\pi{}^2)e^{R_0(m_\pi{}^2)}}{(\frac{2}{3})^{1/2}(m_K{}^2-m_\pi{}^2)e^{R_0(m_\pi{}^2)}f_\pi(m_\pi{}^2)G_+(0)}\right]. \quad (2.18)$$

The only unknown quantities in this expression are the measurable S wave $Q=1$, $\pi\pi$ and $T=1/2$, $K\pi$ scattering phase shifts. We now consider these in more detail and simplify Eq. (2.18).

III. APPROXIMATE RATIO OF PARTIAL WIDTHS

The enhancement factors occurring in Eq. (2.18) above are $e^{\rho_0(m_K{}^2)}$ and $e^{R_0(m_\pi{}^2)}$. Since the $K\pi$ threshold $(m_K+m_\pi)^2$ is extremely far removed from the pion mass, we can safely put $e^{R_0(m_\pi{}^2)}=1$. Since $\delta G/G\sim 0(\alpha)$, the above result can be approximated by

$$\frac{R(K^+\to\pi^+\pi^0)}{R(K_1{}^0\to\pi^+\pi^-)}\cong\frac{3}{4}\left(\frac{\Delta m^2}{m_K{}^2-m_\pi{}^2}\right)^2 \times\left|\frac{f_K(m_K{}^2)}{f_\pi(m_\pi{}^2)}\right|^2\left|e^{\rho_0(m_K{}^2)}\right|^2\left|\frac{F_+(0)}{G_+(0)}\right|^2. \quad (3.1)$$

[6] M. L. Goldberger and S. B. Treiman, Phys. Rev. **110**, 1178 (1958).
[7] S. W. MacDowell Phys., Rev., **116** 1047 (1959).
[8] J. Bernstein and S. Weinberg, Phys. Rev. Letters **5**, 481 (1960).

We next remark that any S-wave resonance in the $\pi^+\pi^0$ system will contribute to $e^{\rho_0(s)}$, however small its partial width for $\pi^+\pi^0$ decay may be. This is because $e^{\rho_0(s)}$ has been normalized to unity at $s=0$, and, for a sharp resonance is independent of the partial width. Thus, an S-wave multipion system [$(\text{mass})^2=S_R$] with $Q=1$, decaying even electromagnetically into a π^+ and π^0 will give a phase shift of the form

$$\delta_0(s)=\pi\theta(s-s_R), \tag{3.2}$$

and yield an enhancement factor

$$e^{\rho_0(m_K^2)}=s_R/(s_R-m_K^2). \tag{3.3}$$

For example, the recently discussed ζ meson,[9] if it has spin parity 0^+, would provide such an enhancement.[10]

IV. NUMERICAL ESTIMATE

It was noted earlier that $f_K(m_K^2)$ and $f_\pi(m_\pi^2)$ are expressible in terms of the rates for $K_{\mu2}$ and $\pi_{\mu2}$ decays. One obtains after an elementary calculation

$$\frac{\Gamma(K_{\mu2})}{\Gamma(\pi_{\mu2})}=\left|\frac{f_K(m_K^2)}{f_\pi(m_\pi^2)}\right|^2\left(\frac{m_\pi}{m_K}\right)^3\left(\frac{m_K^2-m_\mu^2}{m_\pi^2-m_\mu^2}\right)^2. \tag{4.1}$$

Using the known values for the decay rates,[11] we get

$$|f_K(m_K^2)/f_\pi(m_\pi^2)|^2\cong 0.076. \tag{4.2}$$

Equation (3.1) still contains the unknown parameter $|F_+(0)/G_+(0)|^2$. Bernstein and Feinberg[12] have estimated this parameter on the basis of the known rate for K_{e3} decay and on the assumption that $G_+(s)$ is constant over the available spectrum, an assumption that appears to be substantially correct. They obtain

$$|F_+(0)/G_+(0)|^2\cong 24. \tag{4.3}$$

Using this figure, and Eq. (3.3) with $s_R=16m_\pi^2$, one finds

$$R(K^+\to\pi^+\pi^0)/R(K_1^0\to\pi^+\pi^-)\cong 0.11\times 10^{-2}. \tag{4.4}$$

According to the latest experimental data on K^+ branching ratios,[11] provided that two-third of all K_1^0 decays go through the charged mode, the experimental ratio is $\sim 0.2\times 10^{-2}$. Our calculated ratio is, thus, in good agreement.

V. CONCLUSION

We have shown that, if the divergences of the vector currents satisfy a spectral representation with, at most, one subtraction,[13] the dominant term in the ratio of θ^+ to θ_1^0 decay rates can be computed without explicit introduction of the electromagnetic field. Furthermore, a wide class of reasonable S-wave $\pi^+\pi^0$ scattering phase shifts can account for the exact θ^+/θ^0 branching ratio. In particular, if one accepts a $T=1$, 0^+ ζ meson, the final result is remarkably close to the experimental θ^+/θ^0 ratio.

ACKNOWLEDGMENT

The authors would like to thank Dr. J. Bernstein for an interesting discussion.

[9] R. Barloutaud, J. Heughabaert, A. Leveque, J. Meyer, and R. Omnes, Phys. Rev. Letters **8**, 32 (1962); B. Sechi Zorn, Phys. Rev. Letters **8**, 282 (1962).

[10] The qualitative point has been made by G. Feinberg and A. Pais, Phys. Rev. Letters **8**, 341 (1962).

[11] B. P. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters **7**, 346 (1961).

[12] J. Bernstein and G. Feinberg, Phys. Rev. **125**, 1741 (1962).

[13] The validity of this hypothesis has been recently questioned in the literature [H. Chew, Phys. Rev. Letters **8**, 297 (1962); P. Dennery and H. Primakoff, *ibid.* **8**, 350 (1962)]. This criticism is essentially based on the observation that $K_{\mu3}$ decay data requires that $G_-(0)/G_+(0)\cong -9$, whereas the expression quoted above leads to $G_-(0)/G_+(0)=-(m_K^2-m_\pi^2)/m_{K^*}^2\cong -0.3$. However, H. Chew has pointed out (private communication from H. Primakoff) that this could well be due to the occurrence of $T=3/2$, $K\pi$ amplitudes in *leptonic* decays [R. P. Ely, W. M. Powell, H. White, M. Baldo-Cedin, *et al.*, Phys. Rev. Letters **8**, 132 (1962)].