

T_2 was used to calculate again the value of Q using Eq. (4). In general, this new value of Q differs slightly from the original value because the asymmetry of the $\sigma_r(\phi)$ function contributes more particles from one side of the angular acceptance range than from the other side.

The apparent shift in Q values from this cause was calculated for a number of cases and found to be largest where ϕ is near 90° . However, in the particular angular range of our measurements, this error in Q was found to be less than 1%.

Second-Order Effects of Nuclear Magnetic Fields*

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Using the Dirac equation, we consider the contributions to atomic levels of terms quadratic in \mathbf{A} . \mathbf{A} may be, for example, the nuclear magnetic dipole field. We show that a consistent quantum mechanical treatment cancels all terms arising from $[A_i, A_j] \neq 0$. This resolves a disagreement between hyperfine structure corrections calculated nonrelativistically and relativistically.

IN this note we consider the contributions to atomic levels of terms which are quadratic in \mathbf{A} , where \mathbf{A} is, for example, a nuclear magnetic dipole field. We will see that if A_i and A_j do not commute, a correct quantum mechanical treatment nevertheless cancels all terms arising from this noncommutativity. This result explains the disagreement between various hyperfine structure (hfs) calculations.

Consider an electron moving in the field of a fixed nucleus. Its motion is given by the Dirac equation,

$$[\beta m + \boldsymbol{\alpha} \cdot (\mathbf{p} + e\mathbf{A}) - e\phi]\psi = E\psi, \quad (1)$$

where ϕ is the nuclear Coulomb potential. Treating

$$V \equiv e\boldsymbol{\alpha} \cdot \mathbf{A}$$

as a perturbation, for a state i we obtain to order A^2

$$\Delta E_i = \langle i | V | i \rangle + \sum'_{n=+,-} \langle i | V | n \rangle \langle n | V | i \rangle (E_0 - E_n)^{-1}, \quad (2)$$

where n is summed over all positive and negative energy Coulomb states but i .

If i is a positive energy state, the Pauli approximation gives

$$\begin{aligned} \Delta E_i = & \langle i | V_+ | i \rangle + \sum'_{n=+} \langle i | V_+ | n \rangle \langle n | V_+ | i \rangle (E_0 - E_n)^{-1} \\ & + \sum'_{n=-} \langle i | V | n \rangle \langle n | V | i \rangle (E_0 - E_n)^{-1}, \end{aligned} \quad (3)$$

where

$$V_+ = (e/m)\mathbf{p} \cdot \mathbf{A} + (e/2m)\boldsymbol{\sigma} \cdot \mathbf{H}. \quad (4)$$

In the negative-energy sum we can replace $(E_0 - E_n)$ by $2m$ and then sum over all states, including the positive-energy levels. Thus, this term becomes

$$(e^2/2m)\langle i | V^2 | i \rangle, \text{ and}$$

$$\begin{aligned} \Delta E_i = & \langle i | V_+ | i \rangle + \sum'_{n=+} \langle i | V_+ | n \rangle \langle n | V_+ | i \rangle (E_0 - E_n)^{-1} \\ & + (e^2/2m)\langle i | A^2 + i\boldsymbol{\sigma} \cdot \mathbf{A} \times \mathbf{A} | i \rangle. \end{aligned} \quad (5)$$

All the terms in Eq. (5) are familiar from the usual Foldy-Wouthuysen¹ and Pauli² reductions except for the one proportional to $\boldsymbol{\sigma} \cdot \mathbf{A} \times \mathbf{A} \equiv \sigma_i A_j A_k \epsilon_{ijk}$. This vanishes if \mathbf{A} is a classical field, i.e., if A_j and A_k commute. However, if \mathbf{A} contains the nuclear spin operator, then

$$[A_j, A_k] \neq 0. \quad (6)$$

The point that we wish to make is that if (6) holds, \mathbf{A} is not a classical field despite the static nature of the source. A proper quantum mechanical treatment, in terms of Feynman graphs or old-fashioned non-covariant quantum electrodynamics, does not lead to Eq. (5).

Figure 1 shows the two Feynman graphs quadratic in \mathbf{A} . With $k, k', p \sim Z\alpha m$, the crossed diagram is smaller by $Z\alpha$ than the uncrossed diagram for positive energy states. However, for negative energy states these diagrams differ to lowest order only in the ordering of

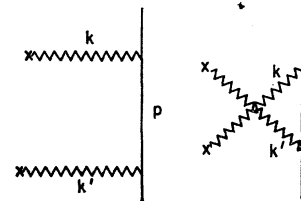


FIG. 1. The Feynman graphs quadratic in \mathbf{A} .

¹ L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950).

² H. A. Bethe and E. E. Salpeter, *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. XXXV, p. 140.

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the operators. The uncrossed diagram gives

$$(e^2/4m)\alpha_i\alpha_j A_i A_j = (e^2/4m)(A^2 + i\boldsymbol{\sigma} \cdot \mathbf{A} \times \mathbf{A}), \quad (7)$$

and the crossed one gives

$$(e^2/4m)\alpha_i\alpha_j A_j A_i = (e^2/4m)(A^2 - i\boldsymbol{\sigma} \cdot \mathbf{A} \times \mathbf{A}). \quad (8)$$

Thus, we see that the $\boldsymbol{\sigma} \cdot \mathbf{A} \times \mathbf{A}$ term in Eq. (5) is incorrect and should be omitted.

It is precisely this term which is responsible for the disagreement between various hfs calculations. A nonrelativistic calculation has been published³ of the second-order perturbation-theory contributions to

$$R = (8\nu_2 - \nu_1)/\nu_1, \quad (9)$$

where ν_1 and ν_2 are the hfs of the 1s and 2s levels of a one-electron atom. The result is

$$\begin{aligned} R_s &= -[1 - (2/3) \ln 2], \\ R_d &= -(1/8)[17/16 - (5/3) \ln 2], \end{aligned} \quad (10)$$

in units of $g\alpha^2 m/M$ for s and d intermediate states,

³ C. Schwartz, *Ann. Phys. (New York)* **6**, 156 (1959).

respectively, where g is the nuclear g factor and M is the proton mass. Reference 3 also quotes a result based upon the Dirac equation,⁴

$$R_s' = -[17/12 - (4/3) \ln 2]. \quad (11a)$$

Another calculation⁵ using the Dirac equation has yielded this value for R_s' , and

$$R_d' = -[11/384 - (1/24) \ln 2]. \quad (11b)$$

It is easy to show that $R_s' + R_d'$ goes over into $R_s + R_d$ if one subtracts off the last term of Eq. (5).

We may remark that this cancellation is well known in other contexts, e.g., the Bethe-Salpeter equation⁶ and the Breit interaction.⁷

It is a pleasure to acknowledge valuable discussions with Professor N. Kroll and Professor L. Foldy.

⁴ L. L. Foldy has pointed out to me that Eq. (11a) is R_s' , not $R_s' + R_d'$.

⁵ D. A. Hockensmith, thesis, Case Institute of Technology, 1960 (unpublished).

⁶ See Sec. 4 of W. A. Newcomb and E. E. Salpeter, *Phys. Rev.* **97**, 1146 (1955); M. M. Sternheim (to be published).

⁷ See, for example, reference 2, p. 259.

Photoelectric K -Absorption Cross Section of Lithium*

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The photoelectric absorption cross section of metallic lithium has been measured in the spectral region extending from 80 to 280 Å. The wavelength dependence deduced from the present experiment is in general agreement with that reported by previous investigators. A more refined technique has made it possible to determine the absolute value of the cross section. Theoretical calculations of the atomic cross section have also been carried out and the results compare favorably with observations. All the measurements were obtained with the use of a grazing incidence spectrometer which was provided with a specially designed auxiliary vacuum system in which the reactive samples were prepared and then transferred to the spectrometer chamber. Thus, the absorption measurements were carried out with a minimum of contamination risk. The surface densities of the thin absorbing films used were determined by observing the change in the resonant frequency of a quartz crystal as lithium vapor was condensed onto it. Spectrograms were obtained for different values of the surface density. The internal agreement of these results provided an indication of the consistency of the crystal monitor. The cross section reached a maximum of 2.60 ± 0.3 mb at an incident photon wavelength of 193 Å. (The K edge of metallic lithium falls at 227 Å.)

I. INTRODUCTION

IN this paper we present measurements of the previously unknown absolute photoelectric absorption cross section of metallic lithium in the neighborhood of the K edge (227 Å). The spectral region investigated extended from 80 to 280 Å. Although the wavelength dependence of the cross section was established by O'Bryan¹ and Skinner and Johnston² in 1937, techniques

for determining, in a vacuum, the surface density of the absorbing layer of this highly reactive metal have become available only recently. In the present investigation it was possible to incorporate a device which did operate in a vacuum and which could monitor the small surface densities necessary for absorption work in the soft x-ray region. A grazing incidence spectrometer was used for comparing the incident and transmitted intensities and absorption spectra were recorded at a variety of surface densities, ranging from 11.2 to 22.6 $\mu\text{g}/\text{cm}^2$. The internal agreement of the observations provided an indication of the consistency of the monitoring by the crystal. The lithium films were pre-

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¹ H. M. O'Bryan, *Phys. Rev.* **49**, 944 (1936).

² H. W. B. Skinner and J. E. Johnston, *Proc. Roy. Soc. (London)* **A161**, 420 (1937).