addition, the low backgrounds observed in the majority of cases necessitated very small background corrections.

IV. RESULTS

The differential cross sections as a function of angle are shown in Fig. 1 for each (equivalent) deuteron laboratory energy. Since the 90° cross section is essentially constant with energy, each ordinate is offset in order to show the individual data points. The data at 10.3 MeV of Allred *et al.*[3] are plotted for comparison. The smooth curves were obtained from a least-squares fit to the data above 30° using Legendre polynomials of order 4. The Legendre coefficients as a function of energy appear in Fig. 2.

The differential cross sections in the center-of-mass system are tabulated with the rms error for each deuteron energy in Table II. Figure 3 displays a comparison with the 8-MeV data[2] of Burge *et al.* Since the differences between the two sets of data are often several times the standard deviations, a few points from the 7.5- and 8.5-MeV data are also shown. The latter tend to verify the self-consistency of the present results. Although it is seen that the positions of the maxima and minima agree very well between the three sets of data near 8 MeV, no constant multiplying factor can be found to bring them into agreement. The 13.7-MeV data[4] of Freemantle *et al.* are compared with the present measurements in Fig. 4. Here the agreement is fair, except at large angles.

ACKNOWLEDGMENTS

We are indebted to the cyclotron group for providing the appropriate beams for these experiments. We wish, also, to express our appreciation to the nuclear plate group for the large amount of plate reading which was accomplished, and, in particular, to Mrs. Rexine Booth who supervised much of the plate analysis work and data reduction. Finally, we wish to acknowledge the contribution of Mrs. Opal Milligan in processing the hundreds of emulsions utilized in the present experiments.

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Recoil Study of the Reaction $C^{12}(p,pn)C^{11}$†

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Recoil ranges of C^{11} from the reaction $C^{12}(p,pn)C^{11}$ are presented for incident proton energies from 0.25 to 6.2 GeV. From these data it is concluded that a neutron evaporation mechanism is probably not the major mechanism. The results are consistent with a fast reaction consisting of a single proton-neutron collision. Assuming this mechanism, an average kinetic energy of approximately 19 MeV can be deduced for the struck neutron (before the collision) in the C^{12} nucleus.

I. INTRODUCTION

THE usual theoretical approach to high-energy nuclear reactions rests on considerations of nucleon-nucleon collisions inside nuclei.[1,2] Calculations of most experimental observables involve the consideration of a complex spectrum of various kinds of collisions. One of the most direct studies of these collisions is the observation of products of the so-called simple reactions, (p,pn), $(p,2p)$, $(p,p\pi^+)$, etc. These reactions involve only a small number of collisions, and result in residual nuclei with small energies of excitation. Therefore the complexities of the interactions are minimized. These simple reactions are, however, sensitive to the individual properties of the target nuclei. Nuclear shell structure, for example, appears to have a significant effect on cross sections for (p,pn) reactions.[3]

At present, the experimental information concerning simple reactions consists mainly of excitation-function measurements for (p,pn) reactions. A few studies of $(p,2p)$ and $(p,p\pi^-)$ reactions have been made. In order to gain a more detailed picture of the kinematics of these reactions, measurements of angular and energy distributions are needed. It is very difficult to obtain velocity measurements for protons and neutrons ejected in these simple reactions, because of the occurrence of many reactions that are more complex. However, radiochemical techniques are suitable for observations of the recoil properties of the heavy residual nuclei.

Many different kinds of recoil measurements can be made—each having its own particular experimental

† Work done under the auspices of the U. S. Atomic Energy Commission.

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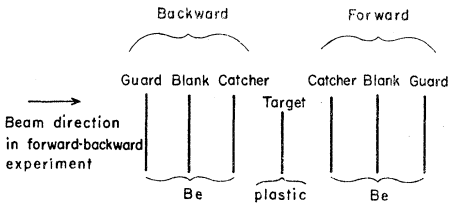

FIG. 1. Target diagram.

difficulties.[4] Most of the experimental difficulties arise from the fact that the recoil energies and ranges of (p,pn) products are expected to be very small (region of keV to a few MeV). We have chosen the very simple thick-target integral-range technique in order to get an initial survey of some features of the recoil properties of (p,pn) reactions.[5,6] The reaction $C^{12}(p,pn)C^{11}$ has been selected because a very simple experimental method is possible for this case.

The experimental method consists of irradiating a foil stack of thick plastic targets and thick Be catcher foils. The fraction of the C^{11} atoms that recoil from the target into the Be catchers was measured by direct observation of the beta radiation from target and catcher foils. From these measurements we obtain the quantities related to the average projection of the recoil ranges: (a) along the beam direction, (b) opposite to the beam direction, and (c) perpendicular to the beam direction. These measurements are sensitive to the combined effects of the angular and energy distributions of the C^{11} products. Quantitative conclusions can be reached only with the aid of a detailed theory of the (p,pn) reaction. Nevertheless, several important qualitative conclusions can be obtained from these initial experiments.

In the course of this study we have performed auxiliary experiments to test the experimental method and establish the range-energy relationship. The effect of diffusion of C^{11} from the plastic targets has been investigated. It has been established that this diffusion effect is very small for the polystyrene targets. In order to establish the range-energy relationship we have measured the range of N^{13} formed in the $C^{13}(p,n)N^{13}$ reaction. The kinematics of this reaction have been determined by other workers,[7] and our measured average range has been correlated with the known distribution of recoil velocities.

II. EXPERIMENTAL METHOD

We have performed a number of thick-target recoil experiments with plastic targets and Be catcher foils. The basic target diagram is shown in Fig. 1. One or more of these stacks of foils were clamped together and exposed to proton beams from the Berkeley Bevatron, 184-in. cyclotron, and 60-in. cyclotron. After irradiation the foils were separated, and the relative β-activities of C^{11} (N^{13} in the 60-in.-cyclotron experiments) in the targets and catchers were determined by end-window proportional counters. The direction of the proton beam was parallel to the normal to the target plane for "forward-backward" experiments, and at an 80-deg angle for "perpendicular" experiments. The target holders have been described elsewhere.[8]

The targets were polystyrene and polyethylene foils of 2 to 3 mg/cm^2. Beryllium foils (from Brush Beryllium, Cleveland, Ohio) of ≈ 5 or ≈ 10 mg/cm^2 were used. Targets and catchers were cut to known areas by using stainless steel templates. For 60-in. cyclotron studies of the $C^{13}(p,n)N^{13}$ reaction, both target and catcher foils were cut to an area of 3.62 cm^2. In these studies a collimated external beam was used. For Bevatron and 184-in. cyclotron studies of the $C^{12}(p,pn)C^{11}$ reaction, targets and catchers were cut to areas of 2.33 and 3.00 cm^2, respectively. The larger areas of the catcher foils ensured that no recoils were lost from the target edges in these internal beam exposures. Before the irradiation, plastic foils were washed with methyl alcohol and distilled water. The Be foils were cleaned in various ways—always including washes with petroleum ether, distilled water, and acetone.

After irradiation, the blank, catcher, and target foils were rigidly mounted on Al plates for counting. Samples were fixed to the counting plate with double-faced adhesive tape, and covered with thin plastic (about 0.5 mg/cm^2). The most active areas of the foils were centered on the counting plates, and the side of the Be catcher foils that faced the target was mounted toward the counter. Simultaneous measurements were usually made on a group of end-window β proportional counters gated by a single off-on switch. In some experiments the samples were rotated from one counter to the next, but this was found to be unnecessary because the relative efficiencies of the various counters differed by less than 3%. In a few experiments γ counters were used.

The usefulness of these experiments as a measure of recoil properties of the nuclear reaction depends on a knowledge of the relative importance of thermal diffusion and recoil phenomena. It is known that some C^{11} diffuses out of plastic foils in the form of hydrocarbons., We will refer to loss by diffusion effects as hot-atom loss. It is essential to evaluate the following effects on these experiments: (a) hot-atom loss from the plastic targets, (b) retention of activity on Be catchers as a result of hot-atom loss from the plastics, (c) hot-atom loss from the Be catchers, and (d) the

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TABLE I. Measurements of hot-atom loss of C^{11} from plastic foils.

Experiment number	Type of measurement	Hot-atom loss (%)
	Polystyrene targets	
S-7	β(absolute)[a]	3.8
S-4	β(absolute)[a]	3.4
16[b]	β	0.9±2.0
16	γ	1.6±3.5
17	β	−0.7±2.5
17	γ	2.0±1.5
18	β	1.5±1.0
18	γ	3.2±1.0
Average		2.0
	Polyethylene targets	
16	β	8.7±2.1
16	γ	9.4±2.1
17	β	12.6
17	γ	12.7
18	β	11.0±4.3
18	γ	11.6±5.1
Average		11.0

[a] The C^{11} activity in the gas phase was observed.
[b] In experiments 16 to 18, C^{11} activity retained in plastic foils was measured relative to standard plastic foils.

dependence of the above effects on irradiation conditions such as beam intensity.

The hot-atom loss from polystyrene targets has been measured both by absolute measurement of gaseous C^{11} and by relative measurements of retained C^{11}. In two separate experiments a stack of polystyrene foils was mounted in an evacuated glass tube and exposed to neutrons produced from 48-MeV α bombardment of thick Be. After irradiation the gaseous activity of C^{11} was measured by sweeping it into a proportional counter with inactive methane carrier.[10] The C^{11} retained in the plastic foils was measured by end-window β proportional counters. An empty tube, not containing plastic, was simultaneously irradiated as a blank. The blank activity (gas phase) was about 10% that of the sample. The results of these experiments are given in Table I.

A group from the Brookhaven National Laboratory has made similar measurements of the hot-atom loss from polystyrene and polyethylene foils.[9] We have measured the specific activity of C^{11} activity retained in the plastic used in this work, relative to some plastic foils from the Brookhaven group. The polyethylene and polystyrene foils from Brookhaven were about 7 to 10 mg/cm² thick; in our work we used thinner foils. The dependence of counting efficiency on sample thickness was measured as described in the Appendix. The hot-atom loss from Brookhaven polystyrene was taken to be (3±1)%, and from the Brookhaven polyethylene was taken to be (12±3)%.[11] With these values as reference standards, the hot-atom loss from our plastics has been calculated from measurements of relative amounts of C^{11} activity retained in stacks of plastic foils exposed to 6.2-GeV proton beams. Various methods were used in the alignment of the different plastics and relative measurements of either β or γ activity were made.

The results of all measurements of the hot-atom loss are shown in Table I. In the last column appears the measured hot-atom loss. Most measurements were for duplicate foils and the error shown is the standard error of these determinations. The hot-atom loss from our polystyrene was only about 2% but for polyethylene it was about 11%. Thus it is possible to correct the observed target activities for this effect.

The results of the high-energy (>250 MeV) recoil experiments show that the ratio of the observed C^{11} activity in the forward Be catcher divided by that in the backward catcher is the same for polyethylene and polystyrene targets (see Table III in Sec. III). The amount of C^{11} activity observed in the Be catchers was only about 5% of the total produced. It is clear that if any appreciable fraction of the hot-atom activity lost from the polyethylene or polystyrene targets was retained by the Be, then the observed forward-backward ratios would differ for the two materials. Thus, we conclude that essentially no C^{11} observed in the Be catchers is from hot-atom effects in the plastic targets.

The possibility exists that some C^{11} was lost from the Be catchers by hot-atom effects. Since no volatile compounds of C and Be are known, this possibility seems unlikely. It is known, however, that some beryllium oxide must be present on the surface of the Be foils. Thus, some possibility exists for hot-atom loss of CO or CO_2. We have made a preliminary search for C^{11} activity in the form of CO_2. From experiments using neutron irradiation of plastic targets and Be catchers, it has been possible to set an upper limit on the C^{11} as CO_2. Less than 30% of the C^{11} activity in the Be foils escaped as CO_2. This limit does not rule out the

TABLE II. Activation correction in a typical C^{11} recoil experiment.[a]

Sample	Type	Count rate after ≈15 min (counts/min)	Count rate after ≈7 h (counts/min)	First count rate corrected for activation (counts/min)
Blank	Be	3198	30	
Backward catcher	Be	7063	27	3916
Target	Polyethylene[b]	194 753	<3	194 753
Forward catcher	Be	12 654	26	9575
Blank	Be	3789		
Backward catcher	Be	7694	31	4047
Target	polystyrene[b]	189 273	<3	189 273
Forward catcher	Be	14 330	28	11 024
Blank	Be	2917	23	

[a] This particular experiment was for 6.2-GeV protons.
[b] The thicknesses of the polyethylene and polystyrene were 2.40 and 2.08 mg/cm², respectively.

[10] We are indebted to Dr. Lee Hyder for participating in these experiments.
[11] A. M. Poskanzer (private communication).

TABLE III. Thick-target recoil data for the $C^{12}(p,pn)C^{11}$ reaction.

Incident energy (GeV)	Number of experiments	Effective forward range (F_FW) (mg/cm^2)	Effective backward range (F_BW) (mg/cm^2)	Effective perpendicular range (F_PW) (mg/cm^2)	Forward backward ratio (F_F/F_B)
			Polystyrene targets[a]		
0.25	4	0.144±0.004[b]	0.0309±0.003		4.76±0.44
0.25	2			0.092±0.006	
0.40	4	0.141±0.002	0.0368±0.001		3.81±0.05
0.40	2			0.096±0.002	
0.70	3	0.132±0.006	0.0364±0.001		3.73±0.08
0.70	2			0.092±0.006	
3.0	1	0.118	0.0431		2.75
3.0	2			0.090±0.004	
6.2	4	0.115±0.004	0.0469±0.003		2.50±0.13
6.2	2			0.078±0.001	
			Polyethylene targets[a]		
0.25	2	0.135±0.008	0.0290±0.006		4.81±0.7
0.40	1	0.137	0.342		4.02
0.40	2			0.088±0.003	
0.70	1	0.142	0.043		3.3
0.70	1			0.080	
3.0	1	0.112	0.0382		2.88
3.0	2			0.081±0.001	
6.2	3	0.109±0.002	0.0421±0.002		2.60±0.08
6.2	3			0.077±0.005	

[a] These data have not been corrected for hot-atom loss from the plastic targets, or for counting efficiency. These combined effects are estimated to multiply the tabulated ranges by 1.00 for polystyrene and by 0.91 for polyethylene.
[b] The errors are the standard deviation of the mean.

possibility of significant loss of oxides of carbon. Nevertheless we have proceeded in analysis of the data with the assumption that this effect can be neglected.

At most bombarding energies, experiments were performed with quite different beam intensities. In every case the results were independent of beam intensity.

To summarize the effects of hot-atom loss, we conclude that polystyrene targets lose a very small fraction ($\approx 2\%$) of the C^{11} activity produced. This hot-atom activity is not retained by the Be catcher foils, and, therefore, does not appreciably affect the range measurements. All measurements have been found to be independent of beam intensity.

III. ANALYSIS OF EXPERIMENTAL RESULTS

Experimental observations for a typical high-energy (>250 MeV) irradiation are given in Table II. The first column gives the foil designation (see Fig. 1), and the second, the material. The third and fourth columns give the results of the first and last counting. These data show that the counting rates of the Be foils are almost equivalent after the C^{11} in the target has decayed away. This activity is attributed to activation of impurities in the Be foils. However, the first counting shows that the Be recoil catcher foils have significantly more activity than the blanks. We attribute this additional activity to recoil atoms of C^{11} from the target that have come to rest in the catchers. (See the discussion of hot-atom effects above.)

The amount of C^{11} activity in the recoil catchers has been determined by correcting the observed counting rates for activation of impurities. The relative activities of the Be blank foils were essentially independent of decay time. The variation in the magnitude of these count rates is attributed to imperfect alignment of the Be foils, and to variations in the quantity of impurities.

The last observation was taken as a measure of the relative activities due to impurities. For each counting time, t, the activity of each Be-catcher foil due to impurity activation, $A_i(t)$, was taken to be the average blank activity $\langle B(t)\rangle$ normalized by the final counting rates:

$$A_i(t)=\langle B(t)\rangle A_i(\text{final counting})/\langle B(\text{final counting})\rangle.$$

The activity due to impurities, $A_i(t)$, was subtracted from the gross activity of each catcher for each measurement. After subtraction the 20-min decay period of C^{11} was observed in all but one experiment. This one experiment was rejected. The fractions (F) of the activity in each catcher were taken as the average result of the first several counts. The precision of the activation correction is reflected by the reproducibility of the measurements (see Table III).

The effective forward range is defined as the sum of the projection along the beam axis of the ranges of the nuclei recoiling forward divided by the total number of recoils. It is given by the product (F_FW), in which F_F is the fraction of the total C^{11} activity observed in the forward Be catcher, and W is the target thickness.[5,12] Similarly, the effective backward range is given by F_BW. We define the effective perpendicular range as (F_PW), in which F_P is the average fraction of the total

[12] L. Winsberg and J. M. Alexander, Phys. Rev. 121, 518 (1961).

C^{11} activity observed in the Be catchers for exposures with the target plane at 10 deg to the beam direction. The effective perpendicular range is one-half the average projection of the ranges on a direction perpendicular to the beam. It can be shown that this effective perpendicular range (F_PW) is $1/\pi$ times the average component of the range on a plane perpendicular to the beam.

A summary of the experimental data for the reaction $C^{12}(p,pn)C^{11}$ is shown in Table III. The first column shows the nominal beam energy. Beam energies of 0.25, 0.40, and 0.70 GeV were obtained from different radial positions in the 184-in. cyclotron. The other irradiations were performed at the Bevatron. The second column gives the number of experiments. The third through the fifth columns give the effective ranges, and the last column gives the ratio of the forward to backward ranges (or fractions), F_F/F_B. The quoted errors are the standard deviations of the mean (or standard error). Figure 2 shows the dependence of the measured effective ranges on proton energy.

A similar procedure was used for the analysis of 60-in. cyclotron irradiations. In these experiments C^{11} activity could not be produced because of the high threshold for the reaction $C^{12}(p,pn)C^{11}$. In these experiments 10 min. N^{13} was observed from the reactions, $C^{12}(p,\gamma)N^{13}$ and $C^{13}(p,n)N^{13}$. No studies of hot atom effects were performed; the assumption is made that these effects are negligible. In these reactions the momenta of the ejected neutrons or photons are such that all N^{13} recoil atoms must be directed forward in the laboratory system. Thus, the Be foil just behind the target should contain no N^{13} atoms that were produced in the target. This expectation is consistent with the counting data. In these experiments the proton-beam energy was changed significantly from one foil to the next. For this reason the decay curves for the various blank foils changed in a regular manner. The activation correction was estimated by plotting the blank foil activities from each measurement as a function of beam energy. This procedure is rather crude, and the resulting range values are probably in error by about 10%.

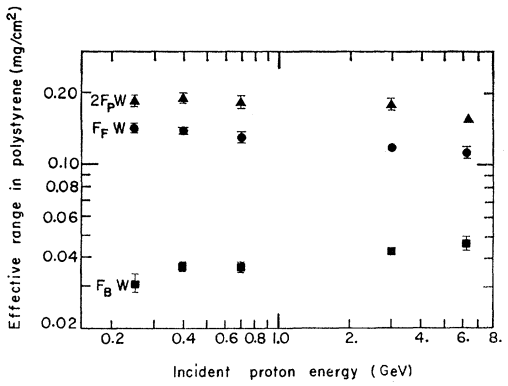

FIG. 2. Effective ranges in polystyrene as a function of bombarding energy. The effective forward range is F_FW, the effective backward range is F_BW, and the effective perpendicular range is F_PW.

TABLE IV. Thick-target recoil data for the $C^{13}(p,n)N^{13}$ and $C^{12}(p,\gamma)N^{13}$ reactions.

Incident energy (MeV)	Effective forward range (mg/cm^2)
4.86	0.14
4.86	0.12
5.65	0.14
6.54	0.14

The results of the N^{13} experiments are given in Table IV. The beam energies were calculated from range-energy tables of Sternheimer,[13] and the nominal maximum proton energy of 12.0 MeV.

IV. DISCUSSION

A. Scattering and the Range-Energy Relation

The keys to the interpretation of any recoil study are (a) the effects of scattering of the recoil atoms, and (b) the knowledge of the range-energy relationship.

Concerning scattering effects, very little quantitative information is available.[14,15] Some experiments with fission products have demonstrated that scattering effects cause errors of as much as 5% in effective ranges measured by the thick-target method.[15] These errors are caused by preferential scattering of recoils out of a target of heavy atoms (namely, U) and into a catcher of much lighter atoms (namely, Al). In our experiments scattering effects may be somewhat greater because the average recoil velocity is about $\frac{1}{2}$ that of the fission products. However, the numbers of recoils scattered into and out of the target foil should be very similar because the masses of the target and catcher atoms are similar. We have analyzed the results as if the recoils followed a straight path. However, the actual assumption made is that deviations from a straight path have similar effects in all experiments—range-energy experiments and nuclear reaction studies. The whole analysis depends only on the relative ranges measured in different experiments.

No quantitative theory of the stopping process is available for the energy region of interest here. However, some range-energy data for N^{14} are available in the literature, and these data are shown in Fig. 3.[16] These data can be adequately represented by an empirical relationship between average range, R, and energy E (or lab system velocity, V_L):

$$R=kE^{\alpha}=k'V_L^{2\alpha}. \quad (1)$$

The values of α are about 0.8 for all stopping materials,

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[16] D. Powers, and W. Whaling, Phys. Rev. **126**, 61 (1962).

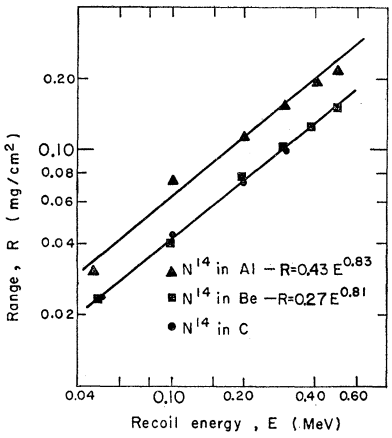

FIG. 3. The measured average ranges of N^{14} in various materials as a function of energy. The data are from reference 16.

and thus we assume that this value is appropriate for N^{13} and C^{11} atoms in plastic targets. It has been shown empirically that α is not extremely sensitive to velocity or stopping material.[12,14]

Some method of converting range data for a given recoil atom and stopping material to another recoil or stopper is needed. The Bohr theory of the very-low-energy stopping process predicts that the average range R_0 is proportional to energy for $A_R \gg A_S$:[17]

$$R_0 = BE, \tag{2}$$

where

$$B = 0.600 \frac{A_S(A_S+A_R)(Z_S^{2/3}+Z_R^{2/3})^{1/2}}{A_R Z_S Z_R}. \tag{3}$$

Here, Z and A are atomic and mass numbers, with subscripts S for stopping atoms and R for recoiling atoms. In a previous paper it has been shown empirically that this relationship gives reasonably accurate conversions of range-energy data.[12] This observation held true for $A_R \approx A_S$ and for recoil energies greater than those appropriate to the equation. Thus we will assume for conversion purposes that k values in Eq. (1) are proportional to the B values in Eq. (3).

The purpose of the range measurements for N^{13} was to get a calibration measurement for the range-energy relationship. It was necessary to determine the main reaction responsible for N^{13} production—$C^{12}(p,\gamma)N^{13}$ or $C^{13}(p,n)N^{13}$.

The relative cross sections for N^{13} production in polystyrene was determined in a separate series of experiments. This excitation function follows closely the shape of the $C^{13}(p,n)N^{13}$ excitation function determined by neutron detection.[7] This is evidence that the $C^{12}(p,\gamma)N^{13}$ reaction does not contribute appreciably to the N^{13} radioactivity we observed. The excitation function for N^{13} that we observed showed a shift in the energy scale from experiment to experiment. We attribute this variation to fluctuations in the initial energy of the proton beam. From the fluctuations, and from the comparison of our excitation function to that of Dagley *et al.*,[7] we can estimate errors for the incident energies in Table IV to be about ± 0.3 MeV.

As previously stated, it is likely that the 10-min N^{13} activity that we observed is mainly from the $C^{13}(p,n)N^{13}$ reaction. The kinematics of this reaction are known from experimental data. There are no excited states of N^{13} that decay by photon emission to the ground state. Thus the kinetic energies of the emitted neutrons as a function of angle are specified by the Q value of the reaction. Dagley *et al.* have measured the angular distribution of the emitted neutrons for many incident proton energies.[7] From these data, the energy and angular distributions of the N^{13} recoil atoms can be calculated.

Consistent with the data in Fig. 3, we assume that the recoil distance is proportional to the initial energy, E_L, in the laboratory system to the power 0.8. In these experiments we have measured the effective forward range or the average $\langle R_F \rangle$ of the projection of the ranges along the beam direction. The average quantity $\langle E_L^{0.8} \cos\theta_L \rangle$, where $\cos\theta_L$ is the laboratory angle of recoil with respect to the beam, can be evaluated from the kinematics of the reaction. From the relationship

$$\langle R_F \rangle = k \langle E_L^{0.8} \cos\theta_L \rangle \tag{4}$$

we have determined the value of 0.176 (mg cm^{-2} MeV$^{-0.8}$) for k of N^{13} recoils in polystyrene. (If 20% of the N^{13} observed was produced by $C^{12}(p,\gamma)N^{13}$ reaction, the value of k would be changed by less than 10%.) This determination was made only for the range datum at 5.65 MeV, because the neutron angular distribution is not sharply dependent on proton energy in this region.[7] From the value of 0.176 for k of N^{13} in polystyrene a value of 0.213 for k of C^{11} recoils in polystyrene was obtained by the conversion procedure previously described. Thus, the range-energy relationship used for C^{11} in polystyrene is $R = 0.213E^{0.8}$, with range expressed in mg/cm^2 and energy in MeV. This relationship has been obtained from recoils of up to $\sim 1/2$ MeV. We use the relationship for recoils of ~ 2 MeV.

This range-energy relationship can be tested by two comparisons: (a) The measured ratio of ranges of C^{11} in polyethylene (CH_2) to polystyrene (CH_1) is about 0.83 (see Table III). The calculated ratio, using Eqs. (2) and (3) to estimate the relative stoping effectiveness, is 0.84. (b) From the data for N^{14} in carbon,[16] and the conversion method previously described, a value of 0.25 for k of C^{11} in polystyrene has been calculated. This value differs from ours by only about 20%.

B. Discussion of Results

The analysis of any recoil experiment requires certain assumptions about the kinematics of the reaction.

[17] N. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. 18, No. 8 (1948).

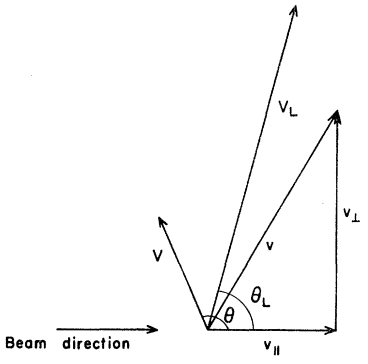

FIG. 4. Vector diagram. The vector **v** has component $v_{||}$ parallel to the beam and component $v_\perp$ perpendicular to the beam. The vector **V** is directed at an angle θ with respect to the beam. The resultant laboratory system velocity $\mathbf{V}_L$, the vector sum of **v** and **V**, is directed at an angle θ_L with respect to the beam.

These assumptions are best discussed in terms of a velocity vector diagram as shown in Fig. 4. First, we assume that the final resultant velocity, $\mathbf{V}_L$, of the C^{11} atom can be considered as the vector sum of two other vectors devoted **v** and **V**. Second, we assume that the velocity **v** can be resolved into a component parallel to the beam $v_{||}$ and a component perpendicular to the beam $v_\perp$. Third, we assume that the magnitude of the vector **V** is independent of its angle with respect to the beam, θ. Fourth, we assume a functional form for the angular distribution function $W(\theta)$. This function describes the probability that the vector **V** be directed at any angle θ. The velocities **v** and **V** are not taken to be unique but may have broad spectra. The analysis is in terms of average quantities involving v, V, and $W(\theta)$. A discussion follows for several simple descriptions of the $C^{12}(p,n)C^{11}$ reaction.

Evaporation. First let us suppose that the incident proton suffers a single elastic collision with a stationary nucleon of the C^{12} nucleus. After this collision the proton emerges from the nucleus. The energy and momentum of the struck nucleon are taken up by the whole nucleus and converted into excitation energy and recoil energy of the excited nucleus. Let the symbol E denote the energy imparted to the struck nucleon in this collision. After this collision imagine that a neutron is evaporated from the excited nucleus giving rise to a final product nucleus of C^{11}. Our inadequate knowledge of the kinematics of meson producing collisions preclude consideration of this possibility.

Now in terms of the velocity vector diagram we can denote as **V** the recoil velocity of the C^{11} nucleus due to the evaporation stage. Also we denote by **v** the recoil velocity due to the initial nucleon-nucleon collision. For simplicity let us assume that the angular distribution of the evaporated neutron is isotropic or $W(\theta)=1$.

A glance at the experimental results (See Table III or Fig. 2.) shows that the forward-backward ratios are not extremely large and that the effective perpendicular range is about equal to the average of the effective forward and backward ranges. These results imply that $V>v$. This need not necessarily be the case if the observed effective ranges are strongly affected by scattering during the stopping process. If V is indeed larger than v and $W(\theta)=1$ then the observed quantities are related to $v_{||}$, $v_\perp$, and V as follows:[18]

$$2W(F_F+F_B)=k'V^{2\alpha}[1+1.69(v_{||}/V)^2+0.195(v_\perp/V)^2] \quad (5)$$

$$W(F_F-F_B)=1.20k'V^{2\alpha}(v_{||}/V) \quad (6)$$

$$W(F_F+F_B-2F_P)=k'V^{2\alpha}[0.748(v_{||}/V)^2-0.374(v_\perp/V)^2] \quad (7)$$

In Table V we give the results of the calculations of $v_{||}$, $v_\perp$, and V (from the polystyrene experiments) for each bombarding energy. Now let us compare these results with predictions of the evaporation mechanism.

TABLE V. Values of $v_{||}$, $v_\perp$, and V for $W(\theta)=1$.[a]

Bombarding energy, E_b (GeV)	Impact velocity, $v_{||}$ (MeV/c) (amu)$^{-1}$	Impact velocity, $v_\perp$ (MeV/c) (amu)$^{-1}$	Isotropic velocity component, V (MeV/c) (amu)$^{-1}$	Kinetic energy (MeV)
0.25	5.2	8.5	15.4	15
0.40	4.6	8.8	15.9	17
0.70	4.3	8.5	15.4	15
3.0	3.4	7.9	15.9	17
6.2	3.0	0[a]	16.4	18

[a] This analysis has been made for the polystyrene experiments only. The polyethylene experiments give essentially the same results except for a higher value of the effective perpendicular range at 6.2 GeV.

The energy E imparted to the struck nucleon must be greater than the separation energy of a neutron for C^{12}, 18.3 MeV. But it must not be greater than about 30 MeV because the probability of evaporating only one neutron from such a highly excited nucleus is small. If the incident proton energy is very large ($E_b \gg 931$ MeV) then the proton is only deflected slightly from its original direction. The struck nucleon is directed almost perpendicular to the beam with a total momentum of $(2\times931\times E)^{1/2}$ MeV/c and a forward momentum component of about E MeV/c. Thus

$$v_{||}\approx E/12 \text{ (MeV/}c\text{)(amu)}^{-1} \quad (8)$$

and

$$v_\perp\approx(1/12)(2\times931\times E)^{1/2} \text{ (MeV/}c\text{)(amu)}^{-1}. \quad (9)$$

The maximum kinetic energy of an evaporated neutron would be $(11/12)(E-18.3)$ MeV, and the minimum kinetic energy approximately $(11/12)(E-8-18.3)$. (8 MeV is the separation energy plus the effective Coulomb barrier of an α particle of C^{11}.[3] The separation energy of a proton or neutron from C^{11} is larger.) Thus from this evaporation stage, momentum conservation requires that

$$V_{\max}=(1/11)[2\times931(E-18.3)(11/12)]^{1/2} \text{ (MeV/}c\text{)(amu)}^{-1} \quad (10)$$

and

$$V_{\min}=(1/11)[2\times931(E-8-18.3)(11/12)]^{1/2} \text{ (MeV/}c\text{)(amu)}^{-1}. \quad (11)$$

[18] These equations are modifications for $\alpha=0.8$ of more general relationships derived by N. Sugarman (private communication).

It is clear from Eqs. (8), (9), (10), and (11) that, for any distribution of values of E, the average value of $v_\perp$ must be greater than the average value of V and v_{11}. Similar considerations for $E_b \geq 250$ MeV give the same result. Also, the same qualitative result is reached if the struck nucleon is in motion with kinetic energy less than E. The analysis of the experiments as given in Table V is in strong disagreement with this prediction. For this reason we tentatively conclude that this evaporation mechanism does not account for the major part of the reactions. This conclusion is tentative because it is possible that the experimental observations are seriously affected by scattering during the stopping process. As discussed in the previous section we do not, however, consider this possibility to be very likely.

A single collision with a neutron having an isotropic momentum distribution. Consider a process in which the incident proton strikes a neutron in C^{12}, and both nucleons—along with all mesons created—escape from the nucleus with no further interactions. In this section we consider that the struck nucleon has an isotropic angular distribution (before the collision). This situation would result from the participation of neutrons of many quantum states. Later we consider the struck nucleon to have an anisotropic angular distribution. The residual nucleus is excited C^{11}. If the final product is to be 20-min C^{11}, then the excitation energy must be less than about 8 MeV, otherwise particle evaporation is expected.

We may visualize the reaction in terms of the independent-particle model. The nucleons of the target nucleus (C^{12}) are in motion with an average kinetic energy, K.E., inside a potential well of depth P.E. The separation energy of a neutron is the minimum difference between its P.E. and its K.E. (18.3 MeV for C^{12}). The incoming nucleon enters the nucleus, collides with a nucleon, and the collision partners escape. The residual nucleus recoils as the incident particle enters the well, as the particles collide (if the potential energy is velocity dependent), and as the particles leave the well.

Let us consider a case in which kinetic energies of the incident and emerging particles are much greater than their respective rest energies. The emerging nucleons and mesons have slightly less total energy than the incident projectile. The removal of the neutron requires an expenditure of 18.3 MeV, and residual excitation energy of the C^{11} nucleus can be as great as ≈ 8 MeV. Thus the emerging momentum is less than the incident momentum by ≈ 18.3 to ≈ 26.3 MeV/c. This momentum difference gives rise to a recoil velocity of the final product of approximately (1/11) (18.3 to 26.3) (MeV/c)(amu)$^{-1}$. In addition, there may be a small recoil velocity associated with the entrances and exists from the potential well. After this neutron removal, the C^{11} nucleus finds itself with a "momentum hole" corresponding to the momentum of the struck nucleon before the collision. This momentum hole gives rise to C^{11} recoil momentum equal in magnitude [$(2\times 931\times \text{K.E.})^{1/2}$ MeV/c] and opposite in direction to the momentum of the nucleon before the collision. In this section we consider that this momentum has an isotropic angular distribution [$W(\theta)=1$] and corresponds to V in the vector diagram (Fig. 4) of $V=(1/11)(2\times 931\times \text{K.E.})^{1/2}$ (MeV/c)(amu)$^{-1}$.

With this picture, Eqs. (5), (6), and (7) relate the observables to the various velocities and the results appear in Table V. We have identified V with the "hole velocity" or that corresponding to the momentum of the struck nucleon. We have identified v with the recoil velocity associated with the Q value of the reaction and the effects of the nuclear potential. The values of v for this mechanism are expected to be less than those of V. This qualitative result is borne out by the results as shown in Table V. Quantitative discussion of the values of v_{11} requires a more detailed model of the nucleus. The results in Table V indicate that the values of V are almost independent of incident energy, and correspond to about 17 MeV for the kinetic energy of the struck nucleon.

A single collision with a neutron having an angular distribution $W(\theta)=a+b\cos^2\theta$. These thick target experiments give us a measure of three effective ranges. Therefore we can determine only three quantities. In the previous sections we discussed the values of v_{11}, $v_\perp$, and V obtained with the assumption that V is isotropic or $W(\theta)=1$. Another possible assumption is that $W(\theta)$ is not isotropic but is described by some simple function such as $W(\theta)=a+b\cos^2\theta$. With this assumption we have four unknowns v_{11}, $v_\perp$, V, and b/a and therefore a unique solution is not possible from the measurements. We can get a qualitative feeling for the effects of $W(\theta)$ by assuming that $v_\perp$ is zero and solving for values of v_{11}, V and b/a. The equations relating the effective ranges to v_{11}, b/a, and K.E. or $V^2/2$ are as follows [to second order in (v_{11}/V)]:[18]

$$2W(F_F+F_B)=k\{(\text{K.E.})^{0.8}/[1+(b/3a)]\}\{[1+(b/2a)]+(v_{11}/V)^2[1.69+(0.31b/a)]\}, \quad (12)$$

$$W(F_F-F_B)=k\{(\text{K.E.})^{0.8}/[1+(b/3a)]\}\times(v_{11}/V)[1.20+(0.453b/a)], \quad (13)$$

and

$$W(F_F+F_B-2F_P)=k\{(\text{K.E.})^{0.8}/[1+b/3a)]\}\times\{(b/8a)+(v_{11}/V)^2[0.749+(0.144b/a)]\}. \quad (14)$$

The results of the calculation of v_{11}, K.E., and b/a are given in Table VI. It is clear by comparing Tables V and VI that the values of v_{11} and K.E. (or V) are not very sensitive to the inclusion of anisotropy as approximated here. We conclude that it is possible to infer the average value of V or, for this mechanism, the kinetic energy of the struck nucleon rather well, even though the angular distribution of the struck nucleon and the values of v cannot be untangled.

From these data we conclude that for incident energies greater than 0.25 GeV, low-energy-transfer

TABLE VI. Impact velocity, energy of struck nucleon, and anisotropy of struck nucleon.[a]

Bombarding energy, E_b (MeV)	Impact velocity, v (MeV/c) (amu)$^{-1}$	Kinetic energy of struck nucleon (MeV)	Anisotropy of struck nucleon $W(90)/W(0)$
0.25	5.2	20	2.9
0.40	4.6	21	2.5
0.70	4.6	20	2.5
3.0	3.4	19	2.2
6.2	3.0	18	1.0[a]

[a] See footnote for Table V.

processes followed by neutron evaporation are not the major mechanism of the $C^{12}(p,pn)C^{11}$ reaction. For the higher bombarding energies, a fast reaction consisting of a single proton-neutron collision is consistent with these data. Assuming that this is indeed the mechanism of the reaction, we estimate the kinetic energy of the struck nucleon to be ≈ 19 MeV, and thus its potential energy to be 37 to 45 MeV.

A more satisfactory comparison of these results with nuclear models could be made by calculations of the type performed by Winsberg and Clements.[19] They have treated collisions between incident nucleons and Fermi gas nucleons in a nucleus. Average components of the momentum of the struck nucleon were presented for elastic collisions and various kinds of inelastic collisions. These calculations are not directly applicable to our measurements. For comparison to our results only those collisions with nucleons near the surface of the Fermi sea should be considered. The effect of the entrance to and exit from the potential well should be considered. An appropriate average should be made for the various kinds of collisions (elastic, single meson production, double meson production, etc.). Momentum should be converted to range and the effective-range values calculated directly.

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[19] L. Winsberg and T. P. Clements, Phys. Rev. **122**, 1623 (1961).

APPENDIX

We have performed several experiments to determine the relative counting efficiency of C^{11} on the proportional counters as a function of sample thickness. This information was needed for the measurement of hot-atom loss of C^{11} from the targets (see Sec. II). Also, we needed an estimate of the relative counting efficiencies of Be catchers and plastic targets.

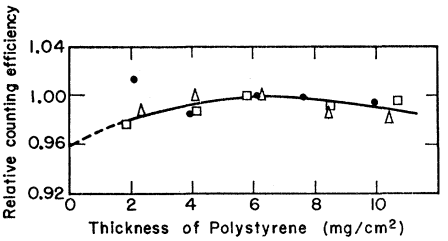

FIG. 5. Relative counting efficiency of C^{11} on the end-window proportional counters. These data were taken on Shelf 7 with different experiments denoted as follows: ○, experiment 10; □, experiment 12; and △, experiment 15.

Stacks of about 15 polystyrene foils (≈ 2 mg/cm^2) were irradiated, and the relative activity of each foil was measured. Then samples of one, two, three, four, and five foils, respectively, were mounted and counted. In experiment 15 the relative activities of these samples were measured by γ counting.

The results of these measurements for C^{11} are shown in Fig. 5. Similar measurements have been made for Na^{24} in Al extending to sample thicknesses of about 0.15 mg/cm^2.[8] The data for Na^{24} indicate that counting efficiency is not drastically dependent on sample thickness down to 0.15 mg/cm^2. Thus we have drawn the dashed line in Fig. 5, and estimate that the Be catcher foils have a counting efficiency about 2.5% less than the plastic targets.