$Bi^{209}(d,p)Bi^{210}$ Reaction at Low Bombarding Energies and with High Resolution*

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The $Bi^{209}(d,p)Bi^{210}$ reaction has been studied with a multiple-gap magnetic spectrograph and an 8-MeV electrostatic accelerator. Forty energy levels were observed up to an excitation energy of 3.2 MeV. The ground-state Q value was measured to be 2.369 ± 0.010 MeV. The angular distribution of each group was peaked at an observation angle of 180 deg because of the large Coulomb barrier and relatively low bombarding energy, and the width of the backward peak depends weakly on the l value of the captured neutron. There is good agreement concerning the shape of the angular distribution between the experimental data and calculations using distorted-wave, deuteron-stripping theory. The ground-state group belongs to a cluster of nine groups, one of which is probably two unresolved groups. These nine groups probably correspond to levels in the Bi^{210} ground-state configuration $(h_{9/2}g_{9/2})$. The intensities of these groups appear to be proportional to the statistical factor $(2J+1)$.

I. INTRODUCTION

FEW high-resolution studies of the energy levels of heavy nuclei have been made with charged-particle reactions. The low yield of these reactions at the bombarding energies of available electrostatic accelerators has been an important factor in this history. Recently, a multiple-gap magnetic spectrograph has been built at MIT for use with the ONR-Van de Graaff accelerator. This new instrument is especially suited for the study of reactions with low cross sections. We have found that the yield from the $Bi^{209}(d,p)Bi^{210}$ reaction at a bombarding energy of 8 MeV is adequate for a detailed study of this reaction. Two aspects of this work are of special interest: (1) the forty energy levels observed in Bi^{210} and (2) the unusual deuteron-stripping mechanism involved, since in this reaction the Coulomb barrier is almost twice the magnitude of the bombarding energy.

In terms of the shell model, Bi^{210} has a simple structure, consisting of a doubly magic core of Pb^{208}, with an extra neutron and an extra proton. Early experimental support for this description was obtained by Harvey,[1] who studied the $Bi^{209}(d,p)Bi^{210}$ reaction and the (d,p) reactions of the lead isotopes with a 14-MeV deuteron beam from a cyclotron. He found four groups from the $Bi^{209}(d,p)Bi^{210}$ reaction, which he interpreted as resulting from neutron capture in the $g_{9/2}$, $d_{5/2}$, $g_{7/2}$, and $d_{3/2}$ shell-model states. Holm, Burwell, and Miller[2] studied the same reaction with better resolution (about 90 keV), using an 11-MeV cyclotron beam. They found several additional groups that they attributed to neutron capture into the $i_{11/2}$ and $s_{1/2}$ shell-model states. They also obtained angular distributions whose shapes supported their suggested identification of these states.

Stokes[3] has measured the angular distributions of (d,p) reactions on very heavy elements at bombarding energies of about 8 MeV and found that the differential cross section is peaked at a 180-deg scattering angle. In the present work at similar bombarding energies, this same backward peaking of the angular distributions is observed.

This backward peaking in these deuteron-stripping reactions is a consequence of the large Coulomb barrier. The following is a classical model of the Coulomb stripping process which suggests why this backward peaking occurs. The Coulomb barrier for deuterons approaching the bismuth nucleus is about 14 MeV. At a bombarding energy of 7.5 MeV, the closest distance of approach of the center of mass of the deuteron to the nuclear surface is only about 8 F. Under these conditions of a large Coulomb barrier and low bombarding energy, a (d,p) reaction may be visualized as Rutherford scattering. The deuteron follows a classical orbit in a Coulomb field as it approaches the nucleus. The deuteron is electrically polarized by the Coulomb field, and this allows the neutron to get closer to the nucleus than the center of mass of the deuteron. At the closest point of approach of the deuteron, the neutron is captured by the nucleus, and the free proton continues on. The further the deuteron approaches towards the nucleus, the more likely will be the capture of the neutron by the nucleus. Zero impact parameter collisions are most likely to result in the neutron's being stripped from the deuteron, since it is for these collisions that the deuteron gets closest to the nucleus. In classical Rutherford scattering, particles with zero impact parameters are scattered through 180 deg. This description of the stripping process suggests why the angular distribution is peaked in a backward direction.

Oppenheimer and Phillips[4] made the first theoretical study of (d,p) reactions at low bombarding energies and postulated the deuteron-stripping process. However, their work was not concerned with the angular distribution, but only with the excitation function of the reaction. Theoretical calculations of the angular

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distribution for stripping reactions on heavy nuclei at low bombarding energy were first made by Ter-Martirosian,[5] and by Biedenharn, Boyer, and Goldstein.[6] These studies predicted that the cross section would be peaked at a scattering angle of 180 deg. Recently, high-speed computers have been used to calculate angular distributions and cross sections of deuteron-stripping reactions where the Coulomb effects are very strong. Barfield, Bacon, and Biedenharn[7] have prepared a computer program that calculates the shape of the angular distribution. Their calculations predict that the shape of the distributions should have some dependence on the l value of the captured neutron. The distorted-wave deuteron-stripping program of Tobocman[8] has been used at MIT by Gibson[9] to calculate angular distributions for the condition of the bismuth (d,p) reaction in the present work. (This code was originally prepared by Tobocman and modified by L. S. Rodberg, B. K. Swartz, and S. Hall at Los Alamos.) A comparison between the MIT computer calculations and the experimental data will be given below.

II. EXPERIMENTAL PROCEDURE

A detailed description of the multiple-gap magnetic spectrograph is in preparation and will be published elsewhere; a preliminary description has been given.[10] The instrument is similar to the single-gap broad-range magnetic spectrograph.[11] However, in the new instrument, there are 25 gaps, instead of one, that analyze and focus charged particles from the target onto the nuclear track plates. These gaps are located every 7.5 deg from 0 to 172.5 deg with respect to the beam direction.

The best data on the $Bi^{209}(d,p)Bi^{210}$ reaction were taken in two runs, one at 7.5- and the other at 8.0-MeV bombarding energies. For each run, all twelve plate holders in the backward quadrant were used. Aluminum foil was placed in front of the plates to stop elastically scattered deuterons. During the 7.5-MeV run, several of the plate holders in the forward quadrant were also used, so that the angular distributions could be followed down to forward angles where the yield becomes small. The exposures were made with about 4000 μC of deuterons.

The target was made from natural bismuth metal which is monoisotopic. Analytical reagent quality bismuth was evaporated onto a Formvar backing. No impurity groups were observed in an analysis of the target composition made with elastically scattered deuterons. The target thickness (about 100 μg per cm^2) was measured by assuming that the elastic-scattering cross section of 7.5-MeV deuterons follows the usual Rutherford scattering formula. This assumption has been checked experimentally with an accuracy of 5% by Elbek and Bockelman[12] for 7.0-MeV protons on gold. A further check on this assumption is found in the distorted-wave calculations made by Gibson[9] which show that the deviations from Rutherford scattering cross section are less than 2% for 7.5-MeV deuterons elastically scattered from bismuth. The target was rotated at about 260 rpm, and beam currents of up to 0.4 μA were used.

The Q values in the bismuth work were read from a table calculated by the MIT-IBM 709 computer. This table gives relativistically correct Q values corresponding to each millimeter of plate distance for a particular reaction, magnetic field, bombarding energy, and plate holder. This Q-value table was calculated using the bombarding energy obtained from the geometry and measured magnetic field of the beam deflector magnet. The uncertainties in the bombarding energy obtained in this way may be as much as 15 keV because the radius of curvature of the deuteron beam in the deflecting magnet had not been measured accurately. Further uncertainties in the numbers given by this Q-value table are introduced by uncertainties in the polonium alpha-particle calibration of the spectrograph. At the time the calibration was made, only one of the present five fluxmeters had been installed in the spectrograph. Unfortunately, the probe of this one fluxmeter was in a poor position, which made it unduly sensitive to differential hysteresis. To eliminate the above uncertainties and obtain more accurate Q-value measurements, proton groups from the $C^{12}(d,p)C^{13}$ and $C^{13}(d,p)C^{14}$ reactions were used as references. Groups from these reactions, whose Q values are known to within a few kilovolts,[13] appeared at the top and middle of the plate holders along with the bismuth groups. The bombarding energy, which gave the correct Q value, was found for each of these carbon groups. A target-thickness correction of 8 keV was included, since the carbon in the target was beneath a layer of bismuth.

In this way, corrections for the bismuth Q values were obtained from the differences between the bombarding energies given by the carbon groups and the bombarding energy which had been used to calculate the table of Q values. These corrections were then applied to each bismuth Q value which had been read from the table. On a given plate holder, this correction was found to differ by 10 keV in going from the highest to lowest excitation group. This correction was used only

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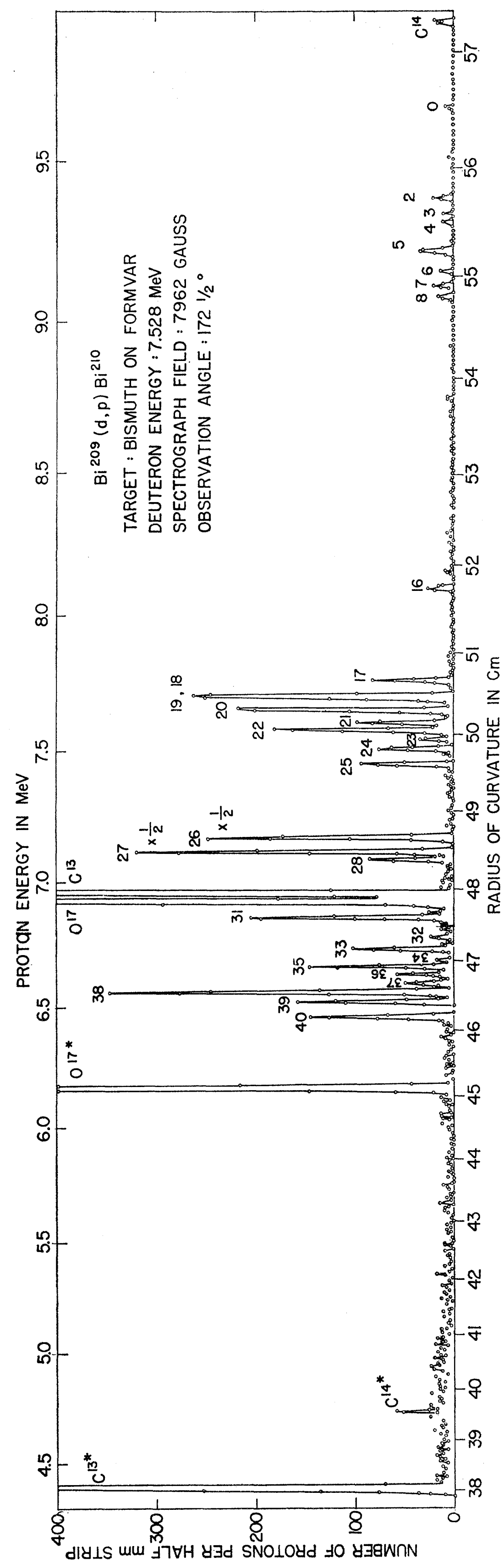

FIG. 1. Spectrum of protons observed at 172.5° from a bismuth target bombarded with 7.5-MeV deuterons.

on those plate holders where the carbon groups had been measured. For the 8.0- and 7.5-MeV runs, ground-state Q values of 2.373 and 2.365 MeV were obtained. The average of these, 2.369 MeV, is given as the best measurement of the ground-state Q value.

III. EXPERIMENTAL RESULTS

Plate-counting data from the $Bi^{209}(d,p)Bi^{210}$ reaction are shown in Fig. 1. These data come from the farthest backward gap at an observation angle of 172.5 deg, where the differential cross section is at a maximum. The number of tracks per half-millimeter counting strip is plotted vertically against the distance along the nuclear track plate. In this figure, about thirty groups can be seen which correspond to energy levels in Bi^{210}. Several contaminant groups from carbon and oxygen in the target are also present. The data taken at other angles are similar to those in Fig. 1, the principal difference being that the bismuth groups become weaker at more forward angles. The threshold of some process that raises the proton background may be seen in Fig. 1 at a proton energy of 5.3 MeV. This rise in background is more apparent in the other data that are not shown. The process that is responsible for this rise in background is probably the breakup of the incident deuteron in the electric field of the target nucleus, since the energy of the threshold is less than the bombarding energy by 2.2 MeV, which is the deuteron binding energy.

Some bismuth groups with differential cross sections too small to be seen in Fig. 1 appear in Fig. 2, where a special technique has been used to increase the "signal-to-noise" ratio. Shown in this figure are the superimposed data from four separate plate holders in the 8-MeV run. Before the data were superimposed, the plate distances in the 165-, 157.5-, and 150-deg data were shifted slightly so that all Q values came at the same plate distance as in the 172.5-deg data. On the far right in Fig. 2 is the proton group with the highest energy observed in the $Bi^{209}(d,p)Bi^{210}$ reaction. That this is the ground-state group is indicated by the presence of group No. 1 which lies 47 ± 3 keV above the ground-state group. This excitation energy is in good agreement with the energy of the 46.5-keV gamma ray, which is known[14] to proceed from the first excited state to the ground state of Bi^{210}.

The Q values and excitation energies of the forty energy levels observed in Bi^{210} are listed in Table I. The Q values are averages taken from data recorded by several plate holders on runs at 7.5- and 8.0-MeV bombarding energy. An estimate of the standard error of these Q values is 10 keV. The accuracy of the excitation energy of these levels is about 3 keV for levels Nos. 1 through 8 and 5 keV for the other levels. Levels Nos. 18 and 19 were unresolved, but the increased width

[14] *Nuclear Level Schemes*, compiled by K. Way, R. W. King, C. L. McGinnis, and R. van Lieshout, Atomic Energy Commission Report, TID-5300 (U. S. Government Printing Office, Washington 25, D. C., 1955 and continuing).

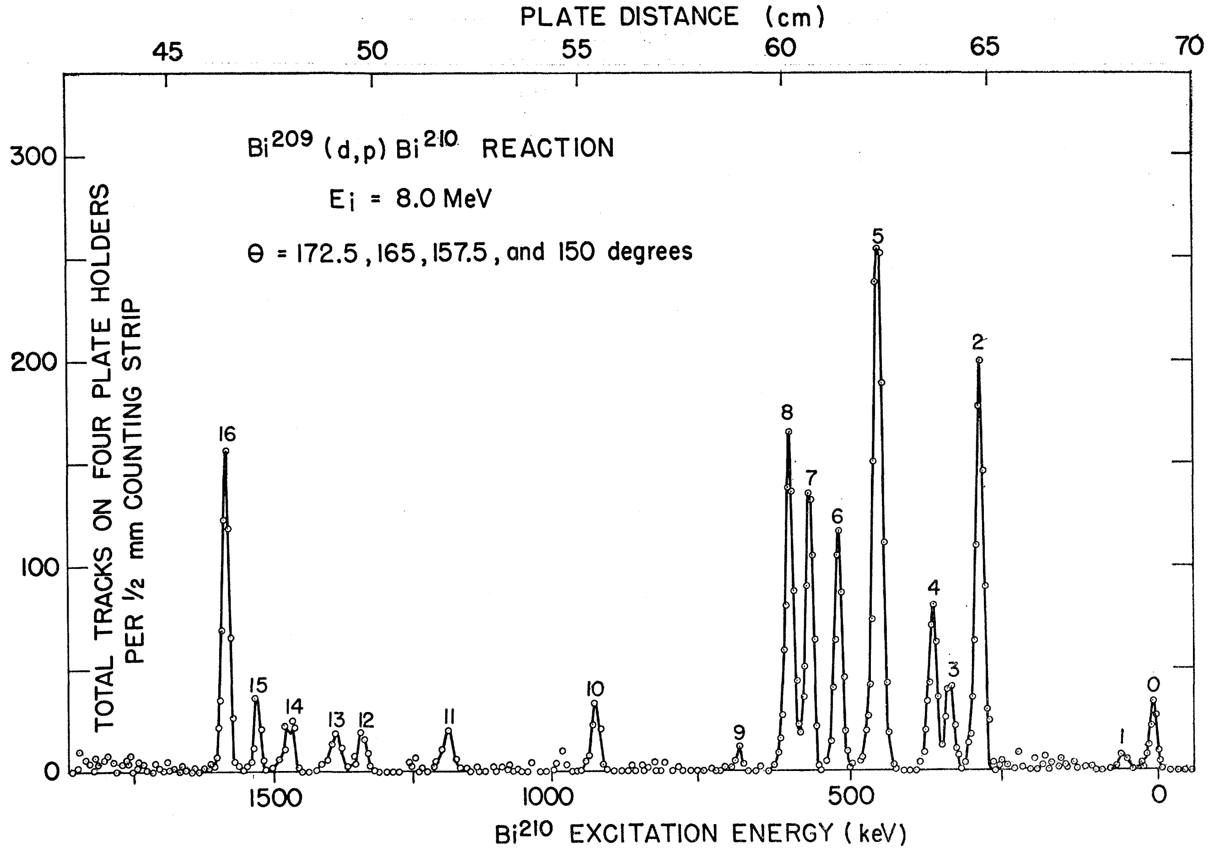

FIG 2. Detailed spectrum of protons near the ground-state group emitted from a bismuth target bombarded with 8-MeV deuterons. Data from the plate holders at 172.5, 165, 157.5, and 150 deg have been superimposed to increase the number of tracks in each group. The excitation energy of the excited states in Bi^{210}, corresponding to each group, can be read from the horizontal scale.

of the unresolved group corresponding to these levels indicated the presence of two groups separated by about 4 keV. Levels Nos. 5 and 38 are suspected on theoretical grounds (see below) to be composed of two closely spaced groups. Some increase in the width of group No. 5 relative to adjacent groups was observed. However, the increase was not large enough to establish definitely on the basis of width alone that group No. 5 is an unresolved doublet. No increase in width of group No. 38 relative to adjacent groups was observed. These width measurements place an upper limit of about 3 keV on the suspected doublet splitting of groups Nos. 5 and 38. No energy levels were observed above 3.2-MeV excitation. In particular, the level at 4.03-MeV excitation reported by Holm *et al.*[2] was not seen. The spectrograph magnetic field was set so that groups corresponding to energy levels up to 5.4-MeV excitation would have been seen if they had had sufficient yield. The rise in background from the electric breakup of the deuteron may have obscured levels above 4.6-MeV excitation. Also listed in Table I are the absolute differential cross sections measured for each level at 172.5-deg observation angle and 8-MeV bombarding energy. The estimated accuracy of these cross sections relative to each other is about 5% for the intense groups. As absolute measurements, these cross sections have an estimated accuracy of 20%.

The angular distributions of five groups observed in the $Bi^{209}(d,p)Bi^{210}$ reaction at 7.5-MeV bombarding energy are shown in Figs. 3 and 4. The angular distributions of three of these groups at 8-MeV bombarding energy are shown in Fig. 5. These particular groups were chosen, since they were of strong intensity and had widely differing Q values. All the other bismuth groups have similar angular distributions, the principal difference being in the width of the backward peak. The solid curves in Figs. 3, 4, and 5 are theoretical fits to the data using the angular distributions calculated with the distorted-wave stripping program of Tobocman.[8] The l values assumed in calculating the curves are shown in the figures. These l-value assignments are based on shell-model arguments given below. The differential cross sections of these calculated angular distributions have been multiplied by a constant factor to make the calculated curves fit the data points as well as possible. The theoretical calculations underestimate the absolute differential cross section by a factor of about 4.5 for the $Q=-0.203$-MeV group in the 8-MeV

TABLE I. Excitation energies, Q values, and differential cross sections of Bi^{210} levels formed through the $Bi^{209}(d,p)Bi^{210}$ reaction.

Level	E_x (MeV)[a]	Q[b] (MeV)	Differential cross sections[c] at $\theta=172.5°$ and $E_d=8.0$ MeV	Suggested J	Suggested configuration
0	0	2.369	1.4	1	$(h_{9/2}g_{9/2})$
1	0.047	2.322	0.4	0	
2	0.268	2.101	9.6	9	
3	0.320	2.049	2.3	2	
4	0.347	2.022	3.8	3	
5	0.433	1.936	14.9	5 and 8	
6	0.501	1.868	5.3	4	
7	0.547	1.822	7.0	6	
8	0.581	1.788	8.0	7	
9	0.672	1.697	0.4		$(h_{9/2}i_{11/2})$ or $(f_{7/2}g_{9/2})$
10	0.912	1.457	1.9		
11	1.172	1.197	1.2		
12	1.325	1.044	0.9		
13	1.372	0.997	1.0		
14	1.460	0.909	1.6		
15	1.517	0.852	1.4		
16	1.577	0.792	6.9	2	$(h_{9/2}d_{5/2})$
17	1.916	0.453	21.6		$(h_{9/2}d_{5/2})$ or $(p_{3/2}g_{9/2})$
18 and 19[d]	1.972	0.397	123		
20	2.027	0.342	58.9		
21	2.075	0.294	26.3		
22	2.102	0.267	51.4		
23	2.138	0.231	9.3		
24	2.173	0.196	24.9		
25	2.235	0.134	23.5		
26	2.517	−0.148	125	4	$(h_{9/2}s_{1/2})$
27	2.572	−0.203	181	5	
28	2.607	−0.238	24.7		$(h_{9/2}d_{3/2})$, $(h_{9/2}g_{7/2})$, or $(p_{1/2}g_{9/2})$
29	2.727	−0.358	14.0		
30	2.756	−0.387	19.3		
31	2.833	−0.464	61.1		
32	2.915	−0.546	3.6		
33	2.960	−0.591	31.4		
34	3.007	−0.638	6.7		
35	3.033	−0.664	37.8		
36	3.064	−0.695	14.1		
37	3.095	−0.726	17.4		
38	3.135	−0.766	111		
39	3.175	−0.806	46.2		
40	3.205	−0.836	41.7		

[a] The estimated uncertainty is 3 keV for levels Nos. 1 through 8 and 5 keV for the other levels.
[b] The estimated uncertainty is 10 keV.
[c] The numbers listed are absolute differential cross sections ×100 in millibarns per steradian with an estimated accuracy of 20%. The relative accuracy is about 5% for the groups with cross sections greater than 0.04 millibarn per steradian.
[d] The increase in width of this group compared to adjacent groups indicates a separation of about 4 keV for levels Nos. 18 and 19.

bombarding energy data and by a larger factor as the Q value increases. The calculated shapes of the angular distributions fit the data reasonably well, except perhaps from 160 to 180 deg, where the experimental angular distributions seem to rise more steeply than the calculated curves.

This agreement between measured and calculated shapes of the angular distributions is shown in greater detail in Table II. In this table, the widths of the experimental and theoretical angular distributions are compared. The parameter used to describe the width of the backward peak is the observation angle at which the differential cross section falls to one-third of its value at 180 deg. This one-third maximum point, rather than the half-maximum point, was chosen for experimental convenience, since uncertainties in the maximum cross section at 180 deg are less important at the one-third height than at the half-height. In addition, for these backward peaks the farther the reference point is from 180 deg, the more sensitive will this point be to changes in the width of the angular distribution. Also listed in Table II are the calculated one-third angles for each angular distribution listed. The l values of the captured neutron used in these calculations were obtained from the shell-model interpretation of the Bi^{210} energy levels, which will be discussed below. Additional calculated angles are given for the same Q and different l values to illustrate the effect of varying l. If one compared in Table II the experimental one-third angles and the one-third angles calculated for the l determined from shell-model arguments, the agreement is seen to be within the experi-

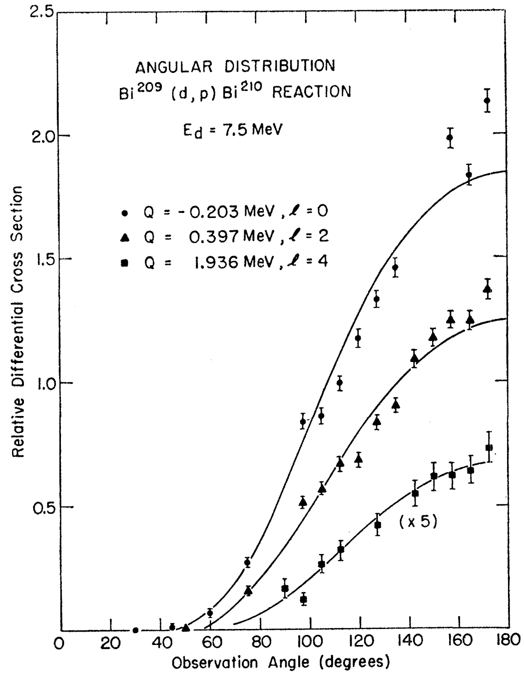

FIG. 3. Angular distributions of proton groups from the $Bi^{209}(d,p)Bi^{210}$ reaction at 7.5-MeV bombarding energy. The curves were calculated using the distorted-wave Born approximation theory of deuteron stripping.

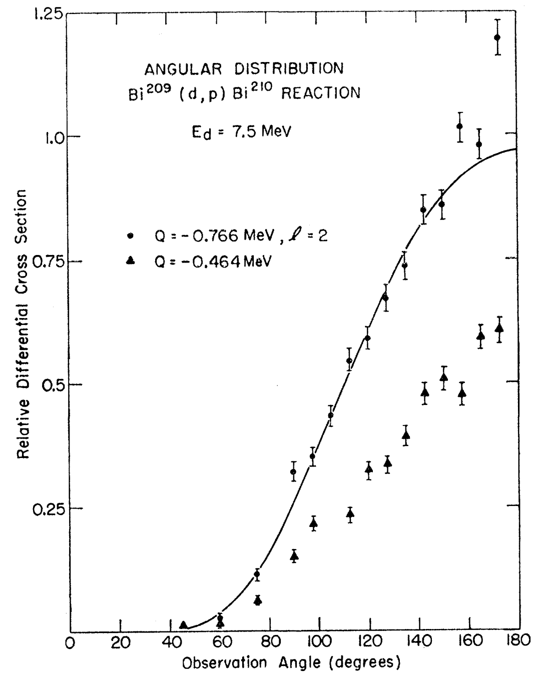

FIG. 4. Angular distributions of proton groups from the $Bi^{209}(d,p)Bi^{210}$ reaction at 7.5-MeV bombarding energy. The curve was calculated using the distorted-wave Born approximation theory of deuteron stripping.

mental error. This suggests that the distorted-wave stripping theory gives a fairly good description of the reaction mechanism. It would be interesting to turn this procedure around, assuming the theory is correct, and use the experimental angular distributions to determine the l values. Unfortunately, this could not be done, since the accuracy of the experimentally

TABLE II. A comparison of the experimental and calculated widths of the backward peaks in the angular distributions of the $Bi^{209}(d,p)Bi^{210}$ reaction. $\theta_{1/3}$ is the observation angle at which the differential cross section falls to 1/3 its value at 180 deg. The distorted-wave deuteron-stripping program developed by Tobocman was used to calculate the 1/3 angles. The experimental l values are obtained from the shell-model interpretation of the Bi^{210} energy levels. The l values in parentheses are uncertain.

		Experimental		Calculated	
Q value (MeV)	E_d (MeV)	$\theta_{1/3}$ (deg)	l_n	$\theta_{1/3}$ (deg)	l_n
1.936	7.5	103±4	4	102.4 85	4 0
0.397	7.5	95±3	(2)	95.4 90	2 0
−0.203	7.5	94±3	0	91.3 105.7	0 4
−0.464	7.5	99±4	(2)	not calculated	
−0.766	7.5	99±4	(2)	95.4 92.6	2 0
1.936	8.0	103±3	4	99.2	4
0.397	8.0	91±3	(2)	91.3	2
−0.203	8.0	85±4	0	87.0	0

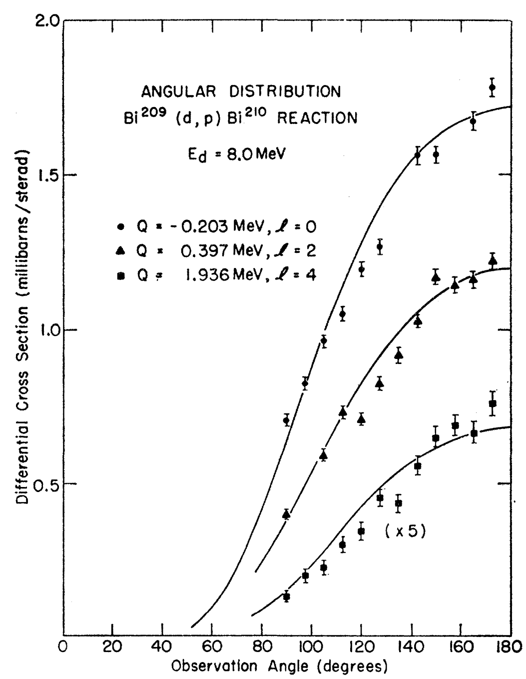

FIG. 5. Angular distributions of proton groups from the $Bi^{209}(d,p)Bi^{210}$ reaction at 8.0-MeV bombarding energy. The curves were calculated using the distorted-wave Born approximation theory of deuteron stripping.

measured one-third angles is not great enough to determine the l value uniquely.

IV. DISCUSSION

The forty energy levels observed in Bi^{210} are shown in Fig. 6. The absolute differential cross section of each group at 172.5 deg observation angle is plotted against the Q value and excitation energy. A prominent feature of this level structure is the pronounced clustering. Three well-separated clusters of levels are present. These are centered at $Q=2.0$, 0.35, and -0.18 MeV. Two more diffuse clusters fall at $Q=1.2$ and -0.65 MeV. These clusters correspond approximately to the groups seen by earlier workers at $Q=1.94$, 1.47, 1.0, 0.32, -0.21, -0.47, and -0.81 MeV, who used equipment of lower resolution. Each of these clusters apparently arises from one or two configurations of the extra neutron and proton in Bi^{210}. Holm *et al.*[2] have suggested that the configurations responsible for their groups were the following: The neutron is captured into the $g_{9/2}$, $i_{11/2}$, $d_{5/2}$, $s_{1/2}$, $g_{7/2}$, and $d_{3/2}$ shell-model states (these are listed in the order of increasing excitation energy), and the extra proton always remains in the $h_{9/2}$ state as in the Bi^{209} target.

The present high-resolution data, in which the individual levels from each configuration are observed, support many of the identifications given by Holm *et al.*[2] The residual interaction between the extra neutron and proton will split each configuration into $(2j+1)$ states where j is the total angular momentum of the extra proton or neutron, whichever is smaller. Thus, the ground-state cluster would show ten groups if these groups arise from the $(h_{9/2}g_{9/2})$ configuration. (In this notation, which describes the configuration of the proton and neutron outside the Pb^{208} core, the proton state is on the left and the neutron state is on the right.) This $(h_{9/2}g_{9/2})$ assignment is probably correct, since nine groups are observed in the present data, and one of these groups (No. 5) is probably an unresolved doublet, as suggested by its anomalously high yield compared to the other groups in the ground-state cluster. Further discussion of this point is given below.

The next group of levels (Nos. 9 through 15) all have small differential cross sections. Some of these levels may arise from the $(h_{9/2}i_{11/2})$ configuration, where the extra neutron is excited into the $i_{11/2}$ state. The weak intensity of these levels compared with levels in the ground-state cluster is probably due to the fact that the stripped neutron enters an $l=6$ state rather than an $l=4$ state. Not all of the levels Nos. 9 through 15, however, may be due to the $(h_{9/2}i_{11/2})$ configuration. Some of these groups may correspond to levels in the $(f_{7/2}g_{9/2})$ configuration which should occur at about 1.5-MeV excitation energy.

The third cluster of levels in the Bi^{210} level structure at $Q=0.35$ MeV does not exactly fit the $(h_{9/2}d_{5/2})$

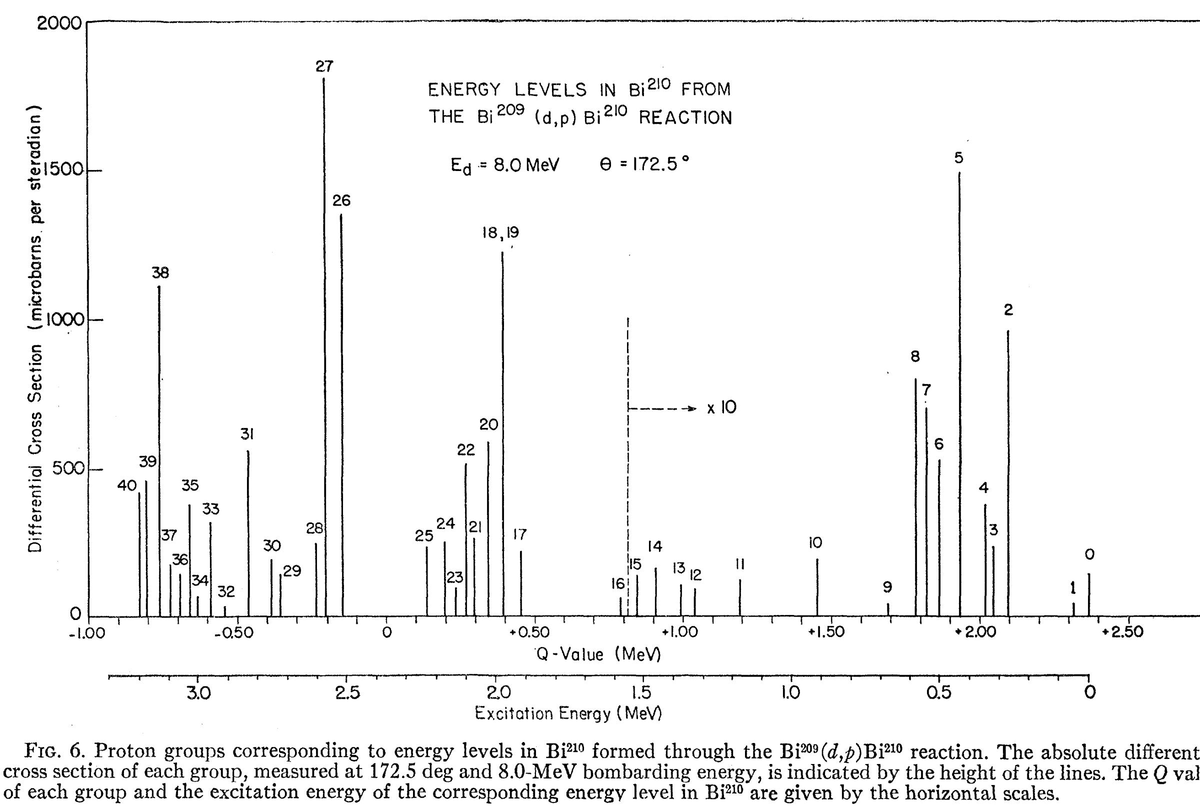

FIG. 6. Proton groups corresponding to energy levels in Bi^{210} formed through the $Bi^{209}(d,p)Bi^{210}$ reaction. The absolute differential cross section of each group, measured at 172.5 deg and 8.0-MeV bombarding energy, is indicated by the height of the lines. The Q value of each group and the excitation energy of the corresponding energy level in Bi^{210} are given by the horizontal scales.

identification given by earlier workers, since there are ten rather than six levels present. The high-resolution data[15] obtained in this laboratory with the $Pb^{208}(d,p)Pb^{209}$ reaction show that there is only one neutron state associated with the cluster at $Q=0.35$ MeV. Thus, the four extra levels are probably not single-particle neutron states. These four extra states are probably not core excitation states either, since the core-excited level in Pb^{208} at 2.615-MeV excitation energy has a very small cross section in the $Pb^{207}(d,p)Pb^{208}$ reaction data.[16] One possible explanation for these four extra levels in the $Q=0.35$ cluster is that they belong to the configuration $(p_{3/2}g_{9/2})$, where the extra proton has been excited into the $p_{3/2}$ state. Strong mixing between the $(h_{9/2}d_{5/2})$ and $(p_{3/2}g_{9/2})$ configurations might explain why the extra proton is strongly excited in this (d,p) reaction.

The pair of levels, Nos. 26 and 27, which have a 55-keV spacing, agrees nicely with the $(h_{9/2}s_{1/2})$ interpretation given by Holm *et al.*[2] for the group that they observed at $Q=-0.21$. The high yield of these two levels also supports the $(h_{9/2}s_{1/2})$ interpretation, since in a stripping reaction the capture of an $l=0$ neutron should have a large cross section.

Most of the levels in the diffuse cluster centered at $Q=-0.65$ probably belong to the $(h_{9/2}g_{7/2})$ and $(h_{9/2}d_{3/2})$ configurations. There should be 12 levels present from these two configurations; however, 13 groups were observed. This extra group (possibly level No. 38) may be an unresolved doublet from the $(p_{1/2}g_{9/2})$ configuration. Exact agreement in number of levels would probably be accidental, because there may be several weak levels near $Q=-0.65$ that have been obscured by the stronger groups.

Information about the total spins of the Bi^{210} energy levels is contained in the relative intensities of the various groups. One can readily observe in Figs. 1, 2, and 6 that the intensities of groups within one cluster usually have different values. In Bi^{210}, where the spread in energy levels from one configuration is small, the only quantity that changes significantly between the various groups from the same configuration is J, the total spin of the residual nucleus. The observed intensity is expected to be proportional to the statistical factor $(2J+1)$ in the absence of configuration mixing. This factor has been used by others[17–19] to extract information about the final state spin J.

A rough test of the assumption that the observed intensity is proportional to $(2J+1)$ is to compare the relative yields of the two levels in the $(h_{9/2}s_{1/2})$ doublet, levels Nos. 26 and 27. The two levels in this doublet must have $J=4$ and 5. The data show that the lower excitation member of the pair has the lower yield. Its spin should be $J=4$ if the differential cross section is proportional to $(2J+1)$ and configuration mixing is not too strong. The order of spins in this $(h_{9/2}s_{1/2})$ configuration is predicted by a rule given by de-Shalit and Walecka.[20] This rule states that, for configurations where the total angular momentum j of either the extra proton or neutron is 1/2, the lowest state has even or odd J, according to whether the configuration parity is odd or even. Since the parity of the $(h_{9/2}s_{1/2})$ configuration is odd, the rule predicts that the lowest state has spin 4. This agrees with the spin that is obtained, assuming that the observed intensity is proportional to $(2J+1)$.

Another situation that gives one confidence in the reliability of the $(2J+1)$ statistical factor concerns level No. 16, which on the basis of intensity belongs to the cluster at $Q=0.40$ MeV. Parts of this cluster are groups corresponding to the $(h_{9/2}d_{5/2})$ configuration, which should obey the Nordheim strong rule. This rule predicts that the state with the lowest J (i.e., $J=2$) will have the lowest excitation. Level No. 16 may be this $J=2$ state, since it has the lowest intensity of any level in the $Q=0.40$ cluster, as would be expected if the intensity is proportional to the statistical factor $(2J+1)$.

An application of greater interest is the extraction of spins of the ten energy levels in the $(h_{9/2}g_{9/2})$ ground-state configuration. Here, configuration mixing is probably small. This is indicated by work in progress, in which the positions and wave functions of Bi^{210} energy levels have been calculated using a computer code developed by True. The theoretical basis of these calculations is similar to that used by True and Ford[21] in their work on Pb^{206}. In the present calculations on Bi^{210}, the maximum impurity in the $(h_{9/2}g_{9/2})$ configuration is 1.5% in intensity. Consequently, it is reasonable to expect that configuration mixing in Bi^{210} is small enough so that the observed yield of the groups in the ground-state cluster is proportional to $(2J+1)$. Table III shows the results of this effort to find the level spins. The set of spins shown is that which gives the best agreement between the experimentally observed relative differential cross sections and the relative differential cross sections obtained by splitting up the total yield to the ten levels in proportion to $(2J+1)$. A few other sets of level spins are possible, although less probable. For example, interchanging the assignment of spin $J=7$ and $J=8$ does not seriously spoil the fit. Group No. 5 was assumed to contain two unresolved groups. If this assumption is not made, a good fit with the $(2J+1)$ factor cannot be obtained with the observed intensities because group No. 5 is too intense by 50%. This group contains 28.4% of the total in-

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TABLE III. Spins of Bi^{210} energy levels in the $(h_{9/2}g_{9/2})$ configuration extracted under the assumption that the differential cross section is proportional to $(2J+1)$.

Level	E_x (MeV)	Number of tracks in group	Experimental relative[a] $d\sigma/d\Omega$	Calculated relative[a] $d\sigma/d\Omega$	Probable spin J values
0	0	130	0.027±0.003	0.03	1
1	0.047	34	0.007±0.002	0.01	0
2	0.268	894	0.183±0.006	0.19	9
3	0.320	212	0.043±0.003	0.05	2
4	0.347	349	0.071±0.004	0.07	3
5[b]	0.433	1389	0.284±0.008	0.28	5 and 8
6	0.501	488	0.100±0.005	0.09	4
7	0.547	657	0.134±0.005	0.13	6
8	0.581	743	0.152±0.006	0.15	7

[a] The sum of the relative differential cross section is 1.
[b] The assumption has been made that this group is composed of two unresolved groups.

tensity of all groups in the ground-state cluster. The group that corresponds to the energy level with the largest spin $(J=9)$ in the $(h_{9/2}g_{9/2})$ configuration should have only 19% of the total intensity, assuming the intensities are proportional to $(2J+1)$.

The spins of $J=1$, 0, and 9 for the lowest three states in Bi^{210} extracted in this way agree with spin assignments made from beta- and alpha-decay measurements. The study of the beta decay of Bi^{210} and Pb^{210} shows that the ground state of Bi^{210} has a spin $J=1$, and the first excited state at 46.5 keV has a spin of $J=0$. Bi^{210} has an alpha-emitting state with a 2.6×10^6 yr half-life. This state is expected to have a very high spin, probably $J=9$. Suggestions have been made[2,22] that this isomeric state is about 0.28 MeV above the ground state. Level No. 2 in the present data at 268-keV excitation may correspond to the isomeric state.

A further check on this spin-extraction procedure is to calculate the spin sequence within the $(h_{9/2}g_{9/2})$ configuration, assuming some reasonable form of the effective residual interaction between the extra proton and neutron. This has been done, using the computer code of True mentioned above. The central interaction used was a finite range force of the same strength as that which was used in the Pb^{206} calculations of True and Ford.[21] The order of spins given by these calculations for the $(h_{9/2}g_{9/2})$ configuration is $J=0$, 1, 9, 2, 3, 7, 5, 4, 6, and 8, starting with the ground state. This same spin sequence is reported by Holm *et al.*[2] as having been obtained by Newby and Konopinski.[23] The set of spins that forms the best fit to the present experimental data is the following: $J=1$, 0, 9, 2, 3, 5 and 8, 4, 6, and 7. As mentioned above, interchanging spins 7 and 8 does not seriously spoil the fit. Except for the reversal of the $J=0$ and 1 states (a well-known problem which may be due to the neglect of tensor forces in the calculations), the agreement is remarkably good between the calculated spins and the spins extracted from the data using the $(2J+1)$ statistical factor.

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