nuclei. Now, if we assume that this argument also holds true for those closed-shell nuclei which give rise to isomers, then we find that there should not be any decrease in the value of the multiplicity at the closed shell. This fact then leads us to the conclusion that the shell effects in the isomeric cross-section ratios are probably due to the spin falloff parameter σ and not to multiplicity changes.

ACKNOWLEDGMENTS

The author is thankful to Dr. M. Goldhaber for his kind interest. The author is also thankful to E. der Mateosian, G. T. Emery, G. Scharff-Goldhaber, and C. E. Porter for their helpful discussions, and to W. R. Kane and N. F. Fiebiger for discussion on the capture γ-ray spectra.

PHYSICAL REVIEW VOLUME 128, NUMBER 2 OCTOBER 15, 1962

Total Cross Sections for Fission of U^{238} Induced by He^4 and Heavy Ions*

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The total fission cross sections have been measured for bombardment of U^{238} with He^4, B^{11}, C^{12}, N^{14}, O^{16}, and Ne^{20} ions at energies up to 10.4 MeV/nucleon. Because of the high fissionability of these systems, it is assumed that the fission cross section is equal to the total reaction cross section for heavy-ion reactions. The data have been compared with the theoretical cross-section calculations of Thomas, assuming (1) a square-well nuclear potential, and (2) a parabolic approximation to the real part of the optical potential. At energies well above the Coulomb barrier, the data are well represented using a square-well potential and $r_0=1.50$F. Near the barrier, however, the agreement is poor. With the parabolic approximation, the entire excitation function can be generally reproduced except in the case of Ne^{20}. For the He^4 data, these calculations used a well depth $V_0=-67$ MeV, a nuclear radius $r_0=1.17$F, and a diffuseness parameter $d=0.574$F. These values for heavy ions were $V_0=-70$ MeV, $r_0=1.23$ to 1.26F, and $d=0.50$ to 0.44F, r_0 increasing and d decreasing as a function of increasing heavy-ion mass.

I. INTRODUCTION

THE measurement of total reaction cross sections provides a valuable means of investigating the basic characteristics of nuclear structure. From such information, one is able to derive a greater understanding of the range of the nuclear potential and its corresponding shape at the nuclear surface. In this paper we define the total reaction cross section to be the sum of all processes in which the incident particle is absorbed or scattered into a reaction channel other than the entrance channel; i.e., it includes all nuclear reactions except shape elastic scattering.[1]

Because of the many competing nuclear processes, total reaction cross sections are generally difficult to measure. Zucker has suggested that the low incident velocity of heavy ions should enhance the probability for compound-nucleus formation at the expense of direct interaction.[2] As a consequence, one would expect the determination of total reaction cross sections from heavy-ion bombardments to be simplified somewhat, in comparison with those involving lighter charged particles ($A\leqslant 4$).

Total reaction cross sections for heavy ions have been calculated from elastic-scattering data.[3,4] Experiments are currently in progress to measure σ_R directly by a beam-attenuation method.[5] The attenuation experiments, as well as several other studies,[6–8] have revealed that the compound-nucleus picture for heavy-ion reactions is much too simple. Instead, these reactions are quite complex—largely due to the occurrence of nuclear surface reactions. Surface reactions presumably take place among the high l-wave impact parameters that lie between those which lead to pure Coulomb scattering and those which lead to complete amalgamation of the target and projectile.[8] The projectile, although partially deflected by the Coulomb field, comes into approximate tangential contact with the target—resulting in inelastic scattering, nucleon transfer, or breakup of the projectile. These may occur in abundances representing as much as 45% of the total reaction cross section.[6–11]

* Work done under the auspices of the U. S. Atomic Energy Commission.
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If one considers only heavy-element target nuclei, however, this difficulty in the measurement of σ_R can be subverted. Because the residual nuclei formed from bombardment of U^{238} with heavy ions have low fission barriers (≈ 5 MeV) and high excitation energies, nearly every nuclear interaction will result in fission.[6] Hence, to a good approximation, the absolute fission cross section σ_f is equal to the total reaction cross section. This fact, plus the ease of detection of fission fragments, makes the U^{238}-heavy-ion systems quite favorable for the measurement of σ_R. Although the method is limited in its applicability to a few target masses, it can furnish a sensitive determination of the variation of σ_R with energy.

The assumption that $\sigma_R = \sigma_f$ for U^{238} is supported by several arguments. Studies of spallation products from reactions with heavy ions (HI) that can be written as (HI,*xn*), (HI,*pxn*), and (HI,*αxn*) have shown the maximum cross sections to be at most only a few hundred microbarns.[9,10] Reactions involving the transfer of an alpha particle—e.g., (C^{12},Be^8*xn*)—are known to have larger spallation cross sections, but are still less than 1% of the fission yield at any given energy.[12] The probability of neutron or proton transfer has been shown to be about 10–60 mb for light elements.[8] One can argue that fission will also be the most probable decay mode for these residual nuclei and thus will be included in σ_R.

Because of the low probability for re-emission of a heavy ion from such compound nuclei, the compound elastic-scattering cross section should be negligible, as should the inelastic scattering cross section for all but the highest *l*-wave incident particles. For nuclear surface reactions resulting in inelastic scattering with energy transfers less than 5–6 MeV, it is not possible to make a reliable estimate of the cross section. We have assumed this contribution to be small because of the large size of heavy-ion projectiles. Therefore, to within a few percent, $\sigma_f = \sigma_R$ for heavy-ion-induced fission of U^{238}.

To investigate the behavior of σ_R as a function of energy, the fission cross sections for bombardment of U^{238} with B^{11}, C^{12}, N^{14}, O^{16}, and Ne^{20} ions at energies up to 10.4 MeV/nucleon were chosen. Much of the data on the C^{12} system is based upon earlier work.[11] As an additional comparison, it was decided to study the fission excitation function for the system $He^4 + U^{238}$, and use previously determined spallation cross sections to obtain σ_R. These results are then compared with theoretical cross sections calculated by Thomas assuming (1) a square-well nuclear potential, and (2) a parabolic approximation to the diffuse well.[13]

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II. EXPERIMENTAL PROCEDURE

The fission chamber and electronics system used in these experiments have been described previously.[6] Two surface barrier silicon-diode crystals covered with about 50 μg/cm^2 of Au were used as detectors. One of these had a resistivity of 15 Ω-cm, and with a bias of 6 V could be used for detection of both fission fragments and elastically scattered beam particles with good resolution. The beam particles did not deposit their total energy in this crystal. A collimator 1.5×6 mm, aligned vertically and placed about 6 cm from the target, defined the solid angle accepted by the crystal. The second detector, of 1800 Ω-cm resistivity and with a bias of 200 V, served as a secondary energy calibration by measuring the pulse height of elastically scattered heavy ions at 30 deg to the beam in each measurement.

The detector pulses were amplified by a preamplifier in the bombarding area and then further amplified by a doubly-differentiating amplifier in the counting area. These signals for each of the individual spectra were then recorded on a Penco 100-channel analyzer. From the spectra, the total number of fission events or elastically scattered beam particles could be easily integrated.

The total cross section for binary fission is given by the expression

$$\sigma_f = 2\pi (d\sigma_f/d\Omega)_{90\text{ deg}} \int_0^\pi \frac{(d\sigma_f/d\Omega)_\theta}{(d\sigma_f/d\Omega)_{90\text{ deg}}} \sin\theta d\theta, \quad (1)$$

where θ refers to either the laboratory or the center-of-mass coordinate system. Here, $(d\sigma_f/d\Omega)_{90\text{ deg}}$ is the absolute differential cross section at 90 deg to the beam axis. The integral expression accounts for the angular distribution relative to 90 deg for the fission fragments. Thus, in order to determine accurate excitation functions, it was necessary to obtain: (a) the absolute value of $(d\sigma_f/d\Omega)_{90\text{ deg}}$ as a function of energy, (b) relative angular distributions at several energies, and (c) accurate knowledge of the bombarding energy.

A. Absolute Differential Cross Sections

Relative values for $(d\sigma_f/d\Omega)_{90\text{ deg}}$ were obtained by measuring the number of fissions per number of incident beam particles as a function of energy. These values are proportional to the detector geometry G and target thickness T, according to the relation $(d\sigma_f/d\Omega)_{90\text{ deg}}{}^{\text{rel}} = GT(d\sigma_f/d\Omega)_{90\text{ deg}}{}^{\text{abs}}$. The product GT was then established by measurement of the relative differential cross section for elastically scattered heavy ions, using the same target and geometry. For this reaction we can write, as above, $[d\sigma(\theta)/d\Omega]_{\text{el}}{}^{\text{rel}} = GT[d\sigma(\theta)/d\Omega]_{\text{el}}{}^{\text{abs}}$. The absolute value for $[d\sigma(\theta)/d\Omega]_{\text{el}}{}^{\text{abs}}$ is given by Rutherford's formula for pure elastic scattering of two-point charges,

$$[d\sigma(\theta)/d\Omega]_{\text{el}} = \left(\frac{Z_1 Z_2 e^2}{4E}\right)^2 \Big/ \sin^4(\theta/2), \quad (2)$$

where Z is the nuclear charge, E the center-of-mass energy, and θ the center-of-mass angle of scattering.

The ratio of the experimental value of the elastic scattering differential cross section to the value predicted by Eq. (2) is characterized by a flat portion at small angles. This is followed by a 20 to 30% rise before a sharp drop off at larger angles.[3,4] Over the flat portion of the curve it is assumed that the absolute value for the differential cross section is given by the Rutherford formula.

For each heavy-ion-U^{238} system, we measured $[d\sigma(\theta)/d\Omega]_{el}^{rel}$ at three or more angles where Eq. (2) should be valid. Then, using this value and that predicted by Eq. (2), we calculated the product GT. This comparison was also made at one or two lower energies where the flat portion of the ratio of experimental to theoretical differential cross section extends over a wider range of angles. The agreement between the values obtained established no systematic change in the Faraday-cup efficiency with energy and ion. Knowing the product GT, we were then able to calculate $(d\sigma_f/d\Omega)_{90\text{ deg}}$. To determine the He^4 cross sections, the fission counting rates were normalized to maximum-energy O^{16} and C^{12} results obtained for the same target and geometry during a single experiment.

A target consisting of 110-μg/cm^2 UF_4 vaporized onto a 110-μg/cm^2 Ni backing foil was used in all these experiments. Experiment has shown that no fragments are lost in a target of this thickness; it was oriented at 45 deg to the beam axis. Contribution to the elastic scattering from the Ni foil was corrected by examination of a Ni foil of similar thickness. This contribution was significant only at the lowest angles; hence, scattering from the fluorine in the target was assumed to be negligible.

The number of projectile ions striking the target was measured with the Faraday-cup arrangement discussed in reference 6. This value was corrected with the aid of values for the equilibrium charge distributions for heavy ions passing through matter.[14] For lighter projectiles, this correction is negligible; its magnitude for N^{14}, O^{16}, and Ne^{20} is indicated in Table I.

TABLE I. Values for the most probable charge distributions $\bar{q}$ for heavy ions passing through Al at selected energies.[a]

N^{14} E (MeV)	$\bar{q}$	O^{16} E (MeV)	$\bar{q}$	Ne^{20} E (MeV)	$\bar{q}$
145.5	6.99	166.1	7.95	208	9.955
108.2	6.96	116.6	7.93	148	9.87
77.1	6.88	87.7	7.85	103	9.675

[a] See reference 14.

B. Angular Distributions

To account for the anisotropy of fission fragments from these reactions, the angular distributions of the fission fragments were measured at 10-deg intervals between 30 and 170 deg. These measurements were usually made at three widely differing energies for each system. The distributions were assumed to vary smoothly with energy.[11,15] The relative value of the integral in Eq. (1) varies between unity for an isotropic angular distribution and $\pi/2$ for the limiting theoretical angular distribution of $1/\sin\theta$.

For all the heavy-ion systems studied here, the behavior of the center-of-mass anisotropies as a function of energy were the same within the limits of error. The angular distributions were equivalent to those reported in reference 11. The integration factors ranged from 1.27 at maximum energy to about 1.17 near the barrier. This introduced, at most, a 2% error in the cross section. For He^4 bombardments, experimental anisotropies measured elsewhere were used to obtain the integration factor.[16] These values were between 1.15 and 1.10.

C. Bombarding Energy

All projectiles were obtained from the Berkeley heavy-ion linear accelerator, which accelerates ions to 10.4±0.2 MeV/nucleon.[17] Because the value of this experimental technique lies in the sensitivity of σ_R as a function of energy, extreme care was taken to obtain an accurate energy calibration. Lower energies were obtained by inserting carefully weighted aluminum foils into the beam.

The primary energy calibration for the heavy ions was based upon the range-energy relations of Northcliffe, assuming 10.4 MeV/amu.[18] The consistency of these calculations was checked by measuring the pulse height for elastically scattered projectiles with the 1800 Ω-cm crystal at 30 deg to the beam. In Fig. 1, the behavior of the pulse height vs the calculated energy is given for B^{11}. The maximum deviation between the calculated energy and a linear pulse-height behavior is about 0.5 MeV.

As an additional check on the absolute value of the energies, emulsions were exposed at the maximum, minimum, and usually one intermediate energy. The range curves in emulsions were found to be symmetric around a most probable track length. From comparison of the most probable track lengths with the results of range-energy relations in desiccated emulsions,[19] excel-

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TABLE II. Most probable bombarding energy and the full-width at half-maximum for the energy spread of heavy ions at selected energies.

Heavy ion	Average energy in emulsion (MeV)	Full-width at half-maximum (MeV)
B^{11}	114.8	1.7
	90.4	2.3
	52.6	2.3
C^{12}	110.9	3.1
	72.1	4.1
N^{14}	145.6	2.8
	75.0	4.2
O^{16}	165.9	2.0
	134.4	3.7
	85.9	4.3
Ne^{20}	207.8	4.4
	109.5	5.0

lent agreement with Northcliffe's results was observed at the two higher energies.

However, except for B^{11} and Ne^{20}, at the lowest bombarding energies the value obtained from the emulsion studies was 1 to 2 MeV lower than that calculated from the range-energy curves in Al. One possible source of this discrepancy is that any deviation of the plane of the degrading foil from 90 deg to the beam axis serves to increase the foil thickness seen by the beam. The energy values determined from the Al thickness are therefore upper limits. Whenever such deviations occurred, the energy values were based upon the average of the two, and appropriate error bars were assigned. The α-particle energy calculations were based upon the data of Bichsel.[20]

Because the slope of the excitation function is quite steep at the lowest energies, it was necessary to further correct the energies for the variation of the cross section due to the energy spread of the beam. To accomplish this, we graphically integrated the expression

$$\sigma(E')=\sigma_{\text{exp}}(E_0)$$
$$=\int_{E_0-\frac{1}{2}\Delta E}^{E_0+\frac{1}{2}\Delta E} P(E)(d\sigma/dE)dE \Big/ \int_{E_0-\frac{1}{2}\Delta E}^{E_0+\frac{1}{2}\Delta E} P(E)dE.$$

Here E' is the corrected energy, E_0 is the initial energy discussed above, and ΔE is the energy spread determined from the measured range straggling in emulsions. The function $P(E)$ was derived from the number of tracks of a given length in the emulsion, whereas $(d\sigma/dE)$ was interpolated from the data. Successive application of this correction usually increased the most probable energy of the lowest points from 0.5 to 1.0 MeV. The energy spreads (full width at half maximum) and energies found for the different ions are shown in Table II.

One additional piece of information obtained from the emulsions was the physical width of the beam after passing through the collimation system. This showed that the number of beam particles that failed to reach the Faraday cup because of scattering from the collimator or the target was negligible.

III. RESULTS

The measured cross sections and the corresponding most probable energies are listed in Table III. The

TABLE III. Measured values of the total fission cross section as a function of bombarding energy for He^4, B^{11}, C^{12}, N^{14}, O^{16}, and Ne^{20} incident upon U^{238}.

He^4		B^{11}		C^{12}		N^{14}		O^{16}		Ne^{20}	
E (MeV)	σ_f (mb)	E (MeV)	σ (mb)	E (MeV)	σ (mb)	E (MeV)	σ (mb)	E (MeV)	σ (mb)	E (MeV)	σ (mb)
41.6	1602±59	114.4	2228±82	124.0	2068±100	145.5	2132±90	166.6	2126±80	208	2340±88
				117.8	1915±95						
37.3	1366±50	111.6	2184±81	117.8	1858±95	140.2	2050±87	159.0	2142±81	202	2187±82
				110.6	1801±90						
34.4	1163±43	108.3	2114±78	110.6	1758±90	133.4	1900±81	159.0	2116±80	198	2158±81
32.0	1041±38	105.5	2056±76	104.5	1653±79	127.8	1770±76	150.4	1977±75	191	1999±76
29.8	845±31	102.5	1989±74	96.2	1311±68	121.9	1640±72	143.0	1849±70	184	1905±72
28.7	770±28	99.6	1941±72	90.0	1094±60	115.8	1500±67	135.2	1675±64	177	1818±69
27.6	660±25	96.0	1811±67	83.0	812±49	112.4	1420±63	127.8	1493±57	172.6	1788±68
26.75	563±21	92.7	1734±65	81.2	725±41	107.9	1254±57	116.6	1152±45	165.8	1615±62
25.5	445±17	89.6	1644±61	77.3	616±38	104.0	1200±55	111.0	971.5±38	160.2	1501±57
24.25	289±11	86.5	1562±58	74.3	479±33	101.3	1057±50	108.4	848.2±34	153.0	1352±52
22.9	162±6.6	82.2	1443±54	73.5	426±29	97.4	914±45	102.4	635.2±26	148.0	1228±48
										140.0	1024±40
21.6	61.2±2.9	79.3	1324±50	73.0	413±25	93.9	800±40	98.8	484.8±21	130.5	705±29
		76.7	1227±46	72.5	401±25	89.7	585±32	92.6	219.3±10.8	122.4	440.4±19
		73.5	1114±42	72.5	374±22	86.6	456±27	86.0	59.2±4.4	116.5	227±11
		68.4	891±34	70.0	280±20	82.0	244±18			107.0	38.1±3.3
		64.9	692±27	69.1	226±18	77.1	57.3±7.9			103.0	5.2±1.1
		62.6	520±21	67.5	102±14	77.1	50.0±7.3				
		60.6	391±16	66.0	39.2±5.0						
		58.4	257±11	65.0	35.6±5.0						
		56.6	132±7								
		54.0	44.0±3.0								

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errors in σ_f are standard deviations calculated from the statistical errors in $(d\sigma_f/d\Omega)_{\rm rel}$, from the error in geometry and target thickness of 3%, and from the 2% error in the knowledge of the integral in Eq. (1).

A qualitative comparison of the data at the maximum bombarding energies shows that the cross section increases regularly with increasing Z and A. As would be expected at these excitation energies, no noticeable effect of projectile spin is observed. The maximum B^{11} cross section is an exception to the regular variation with Z and A, but this result is not surprising because we are comparing it with projectiles having an equal number of neutrons and protons. The effect of adding a neutron while maintaining Z constant is twofold. First, because the Hilac accelerates ions to 10.4 MeV/nucleon, an additional neutron effectively adds 10.4 MeV to the bombarding energy. Second, it increases the nuclear radius, thus enhancing the probability for interaction, and slightly lowering the Coulomb barrier. If one chooses for comparison the B^{11} cross section at about 103 MeV, then the cross sections increase regularly as a function of increasing Z and A for all ions studied here.

Thomas has calculated heavy-ion cross sections as a function of energy on the basis of two simple nuclear potentials: (a) a square well and (b) a parabolic approximation to the optical-model real potential.[13] According to the model, if we represent the incoming projectile by a wave $\psi_i=\exp(-ikr)$, where k is the wave number, any particle that penetrates the barrier sufficiently to feel the nuclear force must be completely absorbed. Otherwise it continues with the same wave function.

In this model there is no provision for reactions in which the projectile is only partially absorbed; e.g., the nuclear surface reactions. However, one can interpret the calculations from a somewhat different point of view. The cross sections calculated by Thomas are derived from the probability of the projectile penetrating the barrier far enough to feel the attractive nuclear potential. For the nuclear surface reactions, it can be argued that the projectile must feel some part of the nuclear force because its wave function has some new form $\psi_f=\exp(-ik'r)$ after passing the target nucleus.

From this point of view, Thomas' calculations can be considered to be total reaction cross sections, to a good approximation.[21] This interpretation, then, suggests that the nuclear surface reactions occur at the expense of complete compound-nucleus formation in the high l-wave angular momentum states. These assumptions are justified in part by the results presented below.

A. Square-Well Model

The square-well calculations are based on the model presented by Blatt and Weiskopf.[22] The basic assumptions are: (a) The target and projectile nuclei are spheres having well-defined surfaces and radii, $R_i=r_0A_i^{1/3}$. (b) The effective potential energy for the system can be written as

$$V_l(r)=\frac{Z_1Z_2e^2}{r}+\frac{\hbar^2l(l+1)}{2\mu r^2},\quad r>R_1+R_2, \tag{3a}$$

$$V_l(r)=-V_0,\quad r<R_1+R_2, \tag{3b}$$

where r is the distance between the centers of the two nuclei, μ is the reduced mass of the system, and V_0 is a constant. (c) There is an interaction radius, $R=R_1+R_2$, such that for $r>R$ there is no nuclear interaction, and for $r<R$ there is a strong nuclear interaction causing the incident particle to be absorbed.

The comparison between the experimental data and σ_R predicted by this model is shown in Fig. 2 for C^{12}, N^{14}, O^{16}, and Ne^{20}—using $r_0=1.50$ F. This value for r_0 has been used by others to fit heavy-ion cross sections with a square-well model[9,23] and is the value commonly used to fit similar data from alpha-particle bombardments.[24] At energies of about 25 MeV or more above the classical Coulomb barrier, the square-well potential predicts the cross sections quite accurately. At lower energies the theoretical values are much too high.

B. Diffuse-Well Model

In the diffuse-well model, the real part of the effective optical-model potential proposed by Igo to fit alpha-particle data has been used[25,26]:

$$V_l(r)=\frac{Z_1Z_2e^2}{r}+\frac{\hbar^2l(l+1)}{2\mu r^2}-V_0\exp\left\{-\left[\frac{r-r_0(A_1^{1/3}+A_2^{1/3})}{d}\right]\right\}. \tag{4}$$

Here V_0, r_0, and d are the parameters in the real part of the Woods-Saxon optical potential.[27] The cross section can be expressed as

$$\sigma_R=\pi\lambda\!\!\!^{-2}\sum_{l=0}^{\infty}(2l+1)T_l, \tag{5}$$

where $\lambda\!\!\!^{-}$ is the de Broglie wavelength of the projectile, and T_l is the transmission coefficient for the lth partial wave. The values of T_l are calculated by assuming that the potential given by Eq. (4) can be approximated by a parabola. Hill and Wheeler[28] have shown that for a

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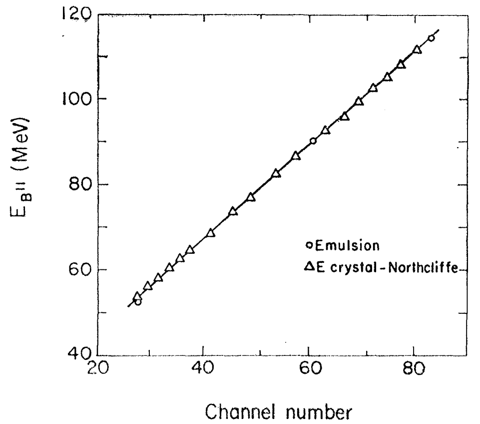

FIG. 1. (△)-Energy vs pulse height of scattered B^{11} from 1800-Ω cm crystal; energy calculated from weighed aluminum thickness, using Northcliffe's range-energy curves[18]; (○)-energy obtained from emulsion data, with a degrader thickness giving the corresponding pulse height.

parabolic barrier

$$T_l=\{1+\exp[2\pi(B-E)/\hbar\omega]\}^{-1}, \tag{6}$$

where B is the barrier height, E is the energy of the system, and

$$\hbar\omega=(\hbar/\mu)(\partial^2 V/\partial r^2)^{1/2}, \tag{7}$$

evaluated at the point where $V(r)$ is a maximum, V and r being defined in Eq. (4).

Using the optical model, Igo was able to fit the data from alpha-particle bombardments, using $r_0=1.17$ F, $V_0=-67$ MeV, and $d=0.574$ F for the real part of the potential.[25] In Fig. 3, we have used the same parameters to fit σ_R for the He^4+U^{238} system with the parabolic approximation. Within the limits of error, the agreement is satisfactory. Here, σ_R represents the sum of our measured fission cross sections plus interpolated cross sections for the (α,xn) reactions,[29] and for the (α,pxn) and $(\alpha,\alpha'n)$ reactions.[24] Our data for the U^{238} (α,f) cross sections are in good agreement with previous observations.[24,30,31]

When these same values for the diffuse-well parameters were used, it was not possible to fit the data for σ_R from the heavy-ion reactions. By minimal variation of r_0, V_0, and d, best fits to the data were obtained; they are shown in Fig. 4. These calculations were performed on an IBM-650 computer, using a program written by Thomas.[13] For all systems except $Ne^{20}+U^{238}$, reasonably good agreement at all energies was obtained with the parameters given in Table IV. The B^{11} results agreed with either of the two sets of values listed, although that with $r_0=1.23$ F was slightly better.

We found that V_0 was the least sensitive of the parameters in the calculation of σ_R, and, therefore, it was held constant. Huizenga and Igo have shown that increasing the depth of V_0 in the optical-model potential has only a small effect on the predicted cross section.[26] This result arises because absorption of heavy projectiles occurs at the nuclear surface, before the projectile can feel the depth of the potential. Hence, the shape of the nuclear potential at the surface—fixed primarily by the radius r_0 and the diffuseness parameter d—has a much greater influence on the reaction cross section. It is observed that as the mass of the projectile increases, it is necessary to use larger values of r_0 and smaller values of d in order to fit the data. This in effect reduces the diffuseness of the nuclear surface; i.e., the nuclear potential becomes more like that of a square well.

Although other values of r_0, V_0, and d, or the use of an energy-dependent V_0, may provide as good a fit to the data as that shown here, at present no attempt is being made to explore these values. The data will be analyzed further by using an optical-model potential with an imaginary part and a finite nuclear charge distribution incorporated into the calculations.

TABLE IV. Parameters for parabolic approximation to the real part of the optical potential giving best fit to the experimental results.

Ion	He^4	B^{11}	C^{12}	N^{14}	O^{16}	Ne^{20}
r_0 (F)	1.17	$\left\{\begin{matrix}1.23\\1.24\end{matrix}\right\}$	1.24	1.24	1.25	1.26
V_0 (MeV)	−66.6	−70	−70	−70	−70	−70
d (F)	0.574	$\left\{\begin{matrix}0.50\\0.48\end{matrix}\right\}$	0.48	0.48	0.48	0.44

IV. CONCLUSIONS

Although the parabolic approximation generally describes the experimental excitation functions much better than the square well, it is still difficult to match the data at the maximum and minimum energies with those 5 to 15 MeV above the minimum. This effect is noticeable in the O^{16} results and is quite marked

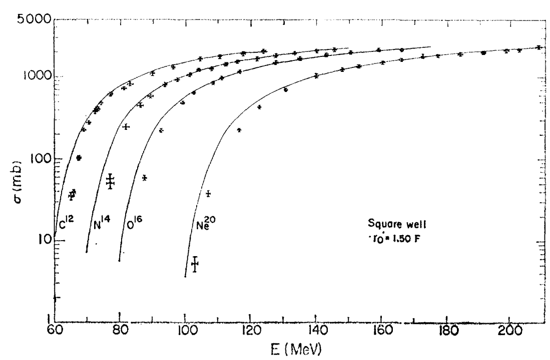

FIG. 2. Comparison of the experimental excitation functions for bombardment of U^{238} with C^{12}. N^{14}, O^{16}, and Ne^{20} with calculations based on a square-well nuclear potential and $r_0=1.50$ F (solid line).

[29] J. Wing, W. J. Ramler, A. L. Harkness, and J. R. Huizenga, Phys. Rev. **114**, 163 (1959).
[30] A. R. Britt and H. C. Quinton, Phys. Rev. **120**, 1768 (1960).
[31] J. R. Huizenga, R. Vandenbosch, and H. Warhanek, Phys. Rev. **124**, 1964 (1961).

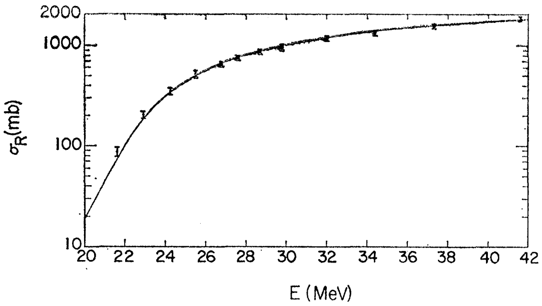

FIG. 3. Excitation function for the sum of the (α,fission), (α,xn), (α,pnx), and (α,$\alpha'n$) reactions from U^{238} compared with parabolic approximation to the real part of the optical potential for $r_0=1.17$ F, $V_0=-66.6$ MeV, and $d=0.574$ F.

with Ne^{20} (Fig. 4). This difficulty stems from too rapid a decrease in the calculated σ_R near the barrier. This effect has also been observed by Huizenga *et al.* in fitting total reaction cross sections for He^4 bombardment of U^{238} with an optical-model calculation.[31] Although we are in excellent agreement with their work, our He^4 data do not extend to sufficiently low energies to show this effect. Huizenga points out that the use of an energy-dependent real potential gives an energy dependence that fits the data more precisely.

Another possible explanation for this effect may reside in the fact that the calculation of the Coulomb potential in this model assumes the two interacting nuclei to be point charges. Use of a Coulomb potential accounting for nuclei with finite charge distributions such as those of Hill and Ford[32] should decrease the slope of σ_R near the barrier somewhat, by lowering the Coulomb barrier. The assumption that the interacting nuclei can be represented by point charges also may account for the increase in r_0 as a function of heavy-ion mass. That is, we have neglected the finite charge distribution of the heavy ion in the calculation, so that one might expect r_0 to increase regularly with increasing projectile Z. The possibility also exists that some perturbation of the cross-section calculation may arise because of the deformed shape of the uranium nucleus.

A third possibility is that we may be observing fission reactions resulting from Coulomb excitation near the barrier. Because Coulomb excitation is an electromagnetic interaction, it is not included in the calculation of σ_R. The large charges of the projectile and target enhance the probability for such reactions. We have calculated the cross section for $E1$ and $E2$ Coulomb excitation[33] to a level necessary for fission to proceed—about 6 MeV above the ground state. Using single-particle transition probabilities, we estimate the cross section for $Ne^{20}+U^{238}$ at 103 MeV to be 10 μb for $E1$ excitation and 50 μb for $E2$ excitation. The observed cross section at this energy is 5 mb, so that, unless the transition probabilities are two orders of magnitude too small, Coulomb-excited fission should be small here.

The parameters derived from fitting the experimental data for the various heavy ions should be useful in calculating total reaction cross sections and average angular momenta for heavy-ion bombardment of other heavy targets. However, it should be stressed that any quantitative interpretation of heavy-ion reactions must take into account the perturbations created by nuclear surface reactions.

Specifically, it should be pointed out that the cross section for compound-nucleus formation is substantially smaller than the total reaction cross section. The probability for surface reactions seems to be relatively independent of target mass.[6–8] The dependence on projectile energy is not clear cut, but products of transfer reactions have been observed in substantial amounts at energies near the Coulomb barrier.[34] Hence, it may be concluded that the surface reaction is a general feature of heavy-ion nuclear reactions.

Of particular importance is the effect of the calculated value of the average angular momentum transfer $\bar{l}$ in heavy-ion reactions. Thomas' values for $\bar{l}$ have been computed assuming complete absorption of the projectile for all l waves. The transfer of masses smaller than that of the projectile for the high-l-waves interactions gives a resultant $\bar{l}$ substantially lower than predicted. The effect of surface reactions on the calculated $\bar{l}$ for 166-MeV bombardment of U^{238} can be estimated from the existing data. We have found that a minimum of 25% of the fission in this reaction proceeds by a mechanism involving the transfer of fragments with A not greater than about 6.[35] Assuming that this 25% of the cross section is taken up by the partial cross sections for the highest l-wave impact parameters, recalculation of $\bar{l}$ lowers its value from 57.3 to 49.3 for the compound nucleus reactions. The transfer reactions will presumably result in much lower angular momentum transfer corresponding to particles of $A=6$ or less with

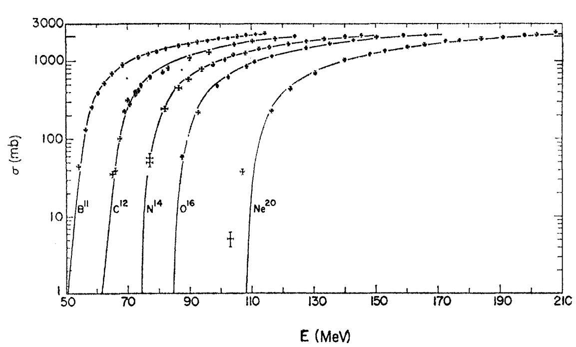

FIG. 4. Best fits for the parabolic approximation to the real part of the optical potential (solid line) to the excitation functions for fission of U^{238} with B^{11}, C^{12}, N^{14}, O^{16}, and Ne^{20}. (Parameters for calculation are given in Table IV.)

[32] K. W. Ford and D. L. Hill, Phys. Rev. **94**, 1617 (1954).
[33] K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Revs. Modern Phys. **28**, 432 (1956).
[34] J. M. Alexander and L. Winsberg, Phys. Rev. **121**, 529 (1961).
[35] T. Sikkeland and V. E. Viola, Jr. (unpublished data).

energies of 10.4 A MeV and impact parameters approximately equal to $r_0A^{1/3}$. Thus, $\bar{l}$ for the sum of the fissioning nuclei will be lowered even more. For fission of lighter nuclei where the transfer reactions do not lead to fission,[6] the value of $\bar{l}$ for the fissioning nuclei will be approximately that of the compound nuclei that are formed.

These considerations are important in any attempts to analyze fission fragment angular distributions with heavy ions. Because the angular momentum enters into the theoretical interpretation of these distributions as $\bar{l}^2$, the uncertainties in the average angular momentum created by the surface reactions affect the conclusions quite strongly. This problem also hinders the treatment of data from isomer ratios for metastable states formed from heavy-ion systems.

ACKNOWLEDGMENTS

The authors are indebted to Albert Ghiorso for his interest and many helpful discussions concerning this work. We are grateful to Homer E. Conzett, Richard M. Diamond, George J. Igo, T. Darrah Thomas, and Bruce D. Wilkins for their assistance in the interpretation of these results. Robert M. Latimer and Walter F. Stockton prepared the excellent silicon detectors, and Charles A. Corum designed the fission chamber. We also thank Roberta B. Garrett and Suzanne M. Hargis for their valuable aid in processing the data. The HILAC crew is acknowledged for excellent beams and other assistance during the experiments.

PHYSICAL REVIEW VOLUME 128, NUMBER 2 OCTOBER 15, 1962

Finite Nuclear Size Effects in β Decay*

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The finite nuclear size effects are of significant importance in the study of the second-order corrections to the allowed beta transitions, evaluation of nuclear matrix elements, and in all cases where the ξ approximation is not valid. Accurate electronic radial functions are computed by considering the finite nuclear size effects and the finite de Broglie wavelength effects. A summary of the computation procedure is given, and a comparison of the calculated beta-decay functions is made with the corresponding Coulomb functions.

1. INTRODUCTION

THE extensive work done in the last three years has led to the general acceptance of the vector and the axial vector interactions for the processes of nuclear beta decay. A considerable interest has developed in the following types of problems: (1) a study of second-order effects, (2) evaluation of nuclear matrix elements, (3) precision measurements of beta polarization, and (4) a detailed analysis of the $\beta-\gamma$ (circular polarization) correlation experiments. For all these investigations, one needs to know accurate electronic functions, which occur in the theoretical formulas. For example, empirical values of the nuclear matrix elements are obtained by fitting the relevant experiments with the theoretical formulas, and then these can be compared with those computed on the basis of a particular nuclear model.

In the computation of beta decay functions, there are two important effects to be considered: (1) the finite nuclear size effects[1] and (2) the finite de Broglie wavelength effect.[2] The corrections due to the finite nuclear size effects are those arising from a consideration of a charge distribution inside the nucleus. For this purpose, a nucleus is generally considered as a sphere of radius $1.2A^{1/3}$ F, and of a uniform charge distribution. This is in contrast to a point nucleus, i.e., only Coulomb field potential. As a usual practice, the electronic radial functions are evaluated at the nuclear surface. These electronic radial functions for a finite nucleus can be expressed (outside the nucleus) as a proper combination of the regular and the irregular solutions of the Dirac equation with a Coulomb potential. It turns out that some of the beta decay functions are very sensitive to

* The contribution of one of us (M.E.R.) was partially supported by the U. S. Atomic Energy Commission.

[1] M. E. Rose and D. K. Holmes, Phys. Rev. **83**, 190 (1951). Also see M. E. Rose and D. K. Holmes, Oak Ridge National Laboratory Report ORNL-1022 (unpublished).
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