Seniority Mixing in $1f_{7/2}$ Nuclei*

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(Received June 6, 1962)

The (d,t) experiments on nuclei in which the $1f_{7/2}$ shell is filling with neutrons and protons show several strong $l=3$ peaks. This is interpreted as showing that the eigenfunctions describing nuclei in this region do not have well defined seniority. The Elliott generating procedure provides an alternative classification for the eigenfunctions and leads to states of mixed seniority. The strengths of the transitions calculated by use of these wave functions are generally in good agreement with experiment.

INTRODUCTION

RECENT studies of (d,t) reactions on even-even nuclei in which the $1f_{7/2}$ shell is filling with both protons and neutrons have exhibited more than one strong $l=3$ transition.[1,2] A simple explanation of the presence of two such peaks might be that the two peaks correspond, respectively, to $1f_{7/2}$ and $1f_{5/2}$ pickup. An argument of this type has been used to explain the gross structure in (d,p) results.[3] For (d,t) experiments, however, this explanation is not satisfactory for several reasons. Firstly, the intensity of the transition to the excited state is quite comparable to the intensity of the ground-state transition. For $Ti^{50}(d,t)Ti^{49}$ this ratio is approximately $\frac{1}{2}$. This would imply a very large admixture of $(1f_{5/2}{}^2)_0$ in the ground-state wave function of Ti^{50}, whereas the experimental results for heavier nuclei such as Ni^{58} show that a relatively small $(1f_{5/2}{}^2)_0$ admixture[4] is present even when the next shell is being filled. Secondly, the sum of the strengths for pickup is proportional to the number of nucleons present in the shell.[5] In order for the results on both odd-A and even-A targets to be even reasonably consistent with each other, it is necessary for the transitions to the excited states to be classified as corresponding to pickup of $1f_{7/2}$ neutrons. Thirdly, if large $1f_{5/2}$ admixtures are present in the wave functions, it is also reasonable to expect comparable $2p_{3/2}$ admixtures to be present. The experiments show weak $l=1$ transitions, corresponding to not more than a few percent of the wave function.

If the wave functions describing nuclei in which the $1f_{7/2}$ shell is filling with neutrons and protons have well-defined isotopic spin[6] and seniority, then the (d,t) pickup reaction on an even-even target should populate only two levels with $l=3$ in the odd-A daughter, namely the $J=\frac{7}{2}$ states with seniority one and isotopic spin $T\pm\frac{1}{2}$. The ratio of the strengths of the transitions to these two states is [7–9]

$$\frac{G_+}{G_-}=\frac{\text{strength of reaction to } T+\frac{1}{2} \text{ level}}{\text{strength of reaction to } T-\frac{1}{2} \text{ level}}=\frac{N-2T}{2T(N+2T+2)}, \quad (1)$$

where N is the number of nucleons filling the shell. The strength $G_\pm$ is defined in terms of the spectroscopic factor $\mathcal{S}$ as

$$G_\pm=(\tfrac{1}{2}\; T\pm\tfrac{1}{2}\; T_z-T_z'\; T_z'\,|\,T\; T_z)^2\, \mathcal{S}, \quad (2)$$

where the bracket is a Clebsch-Gordan coefficient, and T_z and T_z' are the initial and final values of the z component of isotopic spin. This ratio gives a value of 1/27 for the $Ti^{50}(d,t)Ti^{49}$ case. Thus, the excited group has much too large an intensity to be attributed to a transition to the $T_1=\frac{7}{2}$ state of Ti^{49}. Furthermore, following the arguments of French and Macfarlane,[8] one can estimate the excitation energy of the $T+\frac{1}{2}$ level to be

$$\begin{aligned}(\Delta E)^{A-1}&=E_c+(Q_{dt}{}^A-Q_{d\mathrm{He}^3}{}^A)+(Q_{\gamma n}{}^t-Q_{\gamma p}{}^{\mathrm{He}^3})\\&=[E_c+(Q_{dt}{}^A-Q_{d\mathrm{He}^3}{}^A)-0.7634 \text{ MeV}], \quad (3)\end{aligned}$$

where $(\Delta E)^{A-1}$ is the difference in energy between the $T+\frac{1}{2}$ and $T-\frac{1}{2}$ levels in the nucleus $(A-1)$, E_c is the Coulomb correction which can be calculated approximately from the work of Swamy and Green,[10] $Q_{dt}{}^A$ is the Q value for the (d,t) reaction on the nucleus A, $Q_{d\mathrm{He}^3}{}^A$ is the Q value for the (d,He^3) reaction on the nucleus A, $Q_{\gamma n}{}^t$ refers to the (γ,n) reaction on tritium and $Q_{\gamma p}{}^{\mathrm{He}^3}$ to the (γ,p) reaction on He^3. For example, in Ti^{49} one would estimate that the $T_1=\frac{7}{2}$ level lies at an excitation of about 9 MeV.

* Work performed under the auspices of the U. S. Atomic Energy Commission.

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[9] Equation (1) holds for the pickup from an odd-even nucleus and also when seniority is not a good quantum number. In these cases, the strength to the single level must be replaced by the sum of the strengths to all the $T+\frac{1}{2}$ levels in the numerator and to all the $T-\frac{1}{2}$ levels in the denominator.
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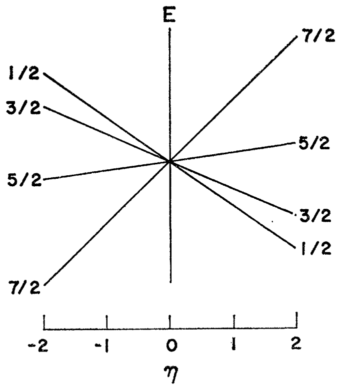

FIG. 1. Energy-level diagram for the $1f_{7/2}$ shell as a function of the deformation η.

The previous considerations force us to the conclusion that if one wishes to retain the usual model of pickup, then seniority cannot be a good quantum number when neutrons and protons are filling the $1f_{7/2}$ shell. The ground state of an odd-A nucleus then no longer has seniority one, but contains an admixture of seniority three and higher. Similarly, the excited states now contain a component which is seniority one. Therefore, although the model allows only a seniority change of one during pickup, it is now possible that the (d,t) reaction proceeds to excited states of an odd-A nucleus by virtue of the mixed seniority of these levels.

When neutrons and protons are filling the $1f_{7/2}$ level, the generating procedure proposed by Elliott[11] gives a classification scheme which differs from the seniority classification. In this scheme, part of the residual interaction between the nucleons in the shell is taken into account by allowing the well in which the particles move to have a Y_2 deformation. This means that the $1f_{7/2}$ level is split into four states (Fig. 1)—each one being doubly degenerate and characterized by the quantum number k, the projection of the angular momentum on the nuclear symmetry axis.[12] For a given sign of the deformation, these states are filled so that the energy is a minimum, two neutrons ($\pm k$) and two protons ($\pm k$) being allowed in each level. In this way one can form a normalized N-particle determinant χ_K of these eigenfunctions with well-defined isotopic spin and angular-momentum projection K. It is assumed that, in an actual shell-model calculation, the lowest eigenvector with spin I is well approximated by projecting out and normalizing that part of χ_K that has angular momentum I. The wave functions obtained by this prescription have mixed seniority. It has previously been shown that the use of these state vectors gives results in better accord with experiment[13] than can be realized with pure seniority eigenfunctions. It is the purpose of the present paper to further test the validity of the above generating procedure by examining whether or not the (d,t) reduced widths for transitions to the lowest state with a given angular momentum can be predicted by these eigenfunctions.

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THEORY

The antisymmetric wave function of N nucleons in the level j coupled to isotopic spin T and angular momentum I may be written in terms of the fractional-parentage coefficients as

$$\psi_{MT_z}{}^{ITs}(j^N)=\sum_{J_1T_1s_1}\sum_{mM_1}\sum_{\mu\nu}\langle j^{N-1}(J_1T_1s_1)]\!]j^N(ITs)\rangle \times(jJ_1mM_1|IM)(\tfrac{1}{2}T_1\mu\nu|TT_z) \times\chi_{m\mu}{}^{j\frac{1}{2}}\phi_{M_1\nu}{}^{J_1T_1s_1}(j^{N-1}), \quad (4)$$

where $\langle\cdots]\!]\cdots\rangle$ is the fractional-parentage coefficient and the index s denotes any additional quantum numbers needed to specify the state. The spectroscopic factor $\mathcal{S}$ for the pickup from the N-particle state $j^N(ITs)$ to the $(N-1)$-particle state $j^{N-1}(J_1T_1s_1)$ is proportional to the square of the fractional-parentage coefficient. Thus, the strength of the transition is

$$G_{T_1}=N(\tfrac{1}{2}T_1(T_z-T_z')T_z'|TT_z)^2 \times|\langle j^{N-1}(J_1T_1s_1|]\!]j^N(ITs)\rangle|^2. \quad (5)$$

In the generating procedure, the lowest energy eigenvector of spin I is assumed to be given by

$$\psi_{MT_z}{}^{IT(K)}(x)=(2I+1)C_K{}^I\int dR\, D_{MK}{}^I(R)\chi_K(Rx), \quad (6)$$

where $C_K{}^I$, which is calculated from Eq. (6) of reference 13, is a constant which normalizes $\psi_M{}^{I(K)}$, $D_{MK}{}^I(R)$ is the rotation matrix, and the coordinate system (Rx) is oriented with respect to the laboratory system x. The N-particle determinant χ_K is constructed by filling the lowest Nilsson orbits (Fig. 1) for either positive or negative deformation. For example, for positive deformation

$$\chi_K(Rx)=|\nu_{1/2}(Rx)\nu_{-1/2}(Rx)\nu_{3/2}(Rx)\cdots \times\pi_{1/2}(Rx)\cdots|, \quad (7)$$

where ν stands for neutron and π for proton, and the vertical bars indicate that this is a normalized determinant. The single-particle wave functions ν_k and π_k have z component of angular momentum equal to k in the (Rx) coordinate system and are understood to have angular momentum $j=\frac{7}{2}$. The determinant of Eq. (7) can be decomposed in terms of minors and written as

$$\chi_K(Rx)=(1/\sqrt{N})[\nu_{1/2}(Rx)|\nu_{-1/2}(Rx)\nu_{3/2}(Rx)\cdots \times\pi_{1/2}(Rx)\cdots|-\nu_{-1/2}(Rx)|\nu_{1/2}(Rx)\nu_{3/2}(Rx)\cdots \times\pi_{1/2}(Rx)\cdots|+\cdots]. \quad (8)$$

Making use of the fact that

$$\phi_k{}^j(Rx)=\textstyle\sum_m D_{mk}{}^{*j}(R)\phi_m{}^j(x), \quad (9)$$

and of the relationship

$$(2I+1)\int dR\, D_{MK}{}^{I}(R)D_{mk}{}^{*j}(R)D_{M_1K_1}{}^{*J}(R)=(jJmM_1|IM)(jJkK_1|IK), \tag{10}$$

enables us to write Eq. (6) as

$$\begin{aligned}\psi_{MT_z}{}^{IT(K)}(x)=&C_K{}^I(1/\sqrt{N})\sum_{JMm}(jJmM_1|IM)(2J+1)\\&\times\Big((jJ\tfrac{1}{2}K-\tfrac{1}{2}|IK)\nu_m{}^j(x)\int dR\,D_{M_1K-\frac{1}{2}}{}^J(R)|\nu_{-1/2}(Rx)\nu_{3/2}(Rx)\cdots\pi_{1/2}(Rx)\cdots|\\&-(jJ-\tfrac{1}{2}K+\tfrac{1}{2}|IK)\nu_m{}^j(x)\int dR\,D_{MK+\frac{1}{2}}{}^J(R)|\nu_{1/2}(Rx)\nu_{3/2}(Rx)\cdots\pi_{1/2}(Rx)\cdots|+\cdots\Big)\\=&C_K{}^I(1/\sqrt{N})\sum_{JM_1m}\sum_{T_1s_1}(jJmM_1|IM)[A_{T_z-\frac{1}{2}}{}^{JT_1s_1}\nu_m{}^j(x)\phi_{M_1T_z-\frac{1}{2}}{}^{JT_1s_1}(x)\\&+B_{T_z+\frac{1}{2}}{}^{JT_1s_1}\pi_m{}^j(x)\phi_{M_1T_z+\frac{1}{2}}{}^{JT_1s_1}(x)],\end{aligned} \tag{11}$$

where we have used the convention that $t_z=+\frac{1}{2}$ for a neutron. The state vectors $\phi_{M_1T_z+\frac{1}{2}}{}^{JT_1s_1}(x)$ are a complete set of antisymmetric $(N-1)$-particle wave functions for the configuration j^{N-1} and the expansion coefficients A and B are the overlap integrals of the ϕ's with the coefficient of $\nu_m{}^j$ and $\pi_m{}^j$, respectively. For example,

$$\begin{aligned}A_{T_z-\frac{1}{2}}{}^{JT_1s_1}=\Big\langle\phi_{M_1T_z-\frac{1}{2}}{}^{JT_1s_1}(x)|\,(2J+1)\Big((jJ\tfrac{1}{2}K-\tfrac{1}{2}|IK)\\\times\int dR\,D_{M_1K-\frac{1}{2}}{}^J(R)|\nu_{-1/2}(Rx)\nu_{3/2}(Rx)\cdots\pi_{1/2}(Rx)\cdots|-\cdots\Big)\Big\rangle.\end{aligned} \tag{12}$$

Since the determinant from which we generate ψ has well-defined isotopic spin, it follows that the T_z dependence of A and B must factor out as a Clebsch-Gordan coefficient, i.e.,

$$A_{T_z-\frac{1}{2}}{}^{JT_1s_1}=(\tfrac{1}{2}T_1\tfrac{1}{2}T_z-\tfrac{1}{2}|TT_z)A(JT_1s),\quad B_{T_z+\frac{1}{2}}{}^{JT_1s_1}=(\tfrac{1}{2}T_1-\tfrac{1}{2}T_z+\tfrac{1}{2}|TT_z)A(JT_1s), \tag{13}$$

so that finally the generated wave function may be written as

$$\psi_{MT_z}{}^{IT(K)}(x)=C_K{}^I(1/\sqrt{N})\sum_{JM_1m}\sum_{T_1\mu\nu}\sum_{s_1}(jJmM_1|IM)(\tfrac{1}{2}T_1\mu\nu|TT_z)A(JT_1s_1)\chi_{m\mu}{}^{j\frac{1}{2}}(x)\phi_{M_1\nu}{}^{JT_1s_1}(x). \tag{14}$$

On comparing Eq. (4) with Eq. (14) one sees that the fractional-parentage coefficient is just

$$\langle j^{N-1}(JT_1s_1)]\!]j^N(ITs)\rangle=C_K{}^I(1/\sqrt{N})A(JT_1s_1). \tag{15}$$

In particular, we are interested in calculating the strength of the transition to the state of lowest energy for given J in the $(N-1)$-particle nucleus. According to the generating procedure, this wave function is given by an expression similar to Eq. (6)—except that in this case $\chi_K(Rx)$ would refer to an $(N-1)$-particle determinant obtained by filling the lowest Nilsson orbits. The overlap integral $A(JT_1s_1)$ of Eq. (12) is calculated in a straightforward manner from the equations given in reference 13.

RESULTS

The theoretical and experimental results are given in Table I. The calculations have been performed by generating from both signs of the deformation as well as for the pure seniority wave functions. The experimental results are obtained from (d,t) reactions initiated by 21.5-MeV deuterons.[1,2,4]

In order to compare results, it was necessary to convert the experimental information into a suitable form (such as number of particles) which measures the strength of the transition. For the Ti isotopes, Yntema[1] normalized his results by summing the peak differential cross sections for all $l=3$ transitions in the $Ti^{50}(d,t)Ti^{49}$ reaction and assumed that this number corresponds to 7.2 particles. The strength of an individual reaction is then given by the peak cross section relative to this normalization. This procedure neglects the dependence of the cross section on the Q value of the reaction. The precise form of this energy dependence is not clear. Use of the Butler formalism,[14] however, provides a way of incorporating an energy dependence into the analysis of the data. In this way we obtained results which are more consistent than those of Yntema. The experimental data for the Ti isotopes (Table I) are obtained

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TABLE I. Strengths of transitions. The strengths of transitions to the lowest state of spin J in the final nucleus are calculated by use of wave functions generated from positive deformation, negative deformation, and seniority. The last column lists the experimental value for these transitions.

Target	Level in the final nucleus Energy	Spin	Strength + Deformation	− Deformation	Seniority	Experiment
Ti^{46}	0	$7/2^-$	3.27	3.00	3.33	2.15
Ti^{47}	0.88	2^+	0.78	0.43	···	0.57
	2.07	4^+	0.88	1.27	···	0.86
	3.30	6^+(?)	0.08	0.06	···	0.13
Ti^{48}	0.16	$7/2^-$	3.74	3.67	5.60	2.45
Ti^{49}	0	0^+	0.25	0.25	0.25	0.23
	0.98	2^+	0.88	0.45	1.19	0.68
	2.33	4^+	0.50	0.82	2.14	0.54
	3.30	6^+(?)	0.05	1.37	3.09	(2.30)
Ti^{50}	0	$7/2^-$	4.90	4.90	7.71	4.05
Cr^{52}	0	$7/2^-$	3.31	4.23	7.20	3.58
Fe^{54}	0	$7/2^-$	3.75	5.52	6.00	4.43

in the above manner on the assumption that the normalized results for the $Ti^{50}(d,t)Ti^{49}$ reaction correspond to 7.7 particles.[15] Although the summations of strengths to the lower T components of Ti^{45}, Ti^{46}, Ti^{48}, and Ti^{49} are in roughly the expected ratios, that for Ti^{47} is low. This may be due to transitions above the range of the experiment.

When the reactions with targets of Ti^{50}, Cr^{52}, and Fe^{54} are compared, we find the total strengths to be roughly in the ratio 3:2:1, whereas we would anticipate near equality. It has been found that the total strength for the $V^{51}(d,t)V^{50}$ reaction is close to that for a Cr^{52} target.[2] The analysis of (d,t) reactions in the iron-nickel region has produced consistent results using the V^{51} reaction as a normalization for the $1f_{7/2}$ strength.[4] It is, therefore, disturbing that the titanium and Fe^{54} results are not reasonably consistent with this normalization. We have, therefore, normalized all the titanium results with $Ti^{50}(d,t)Ti^{49}$ as a base and separately normalized both $Cr^{52}(d,t)Cr^{51}$ and $Fe^{54}(d,t)Fe^{53}$. Although this procedure is certainly open to question, it enables us to make comparisons at this time.

$Ti^{46}(d,t)Ti^{45}$

The seniority calculation predicts a strength of 3.33 particles, which is close to the strength of 3.27 particles calculated for positive deformation. The experimental value is 2.14 particles for the reaction to the ground state and there is another group at approximately 1.75 MeV with a strength of 1.35 particles. The result for negative deformation is 3.00 particles, which is still far from the observed value. The $T=\frac{3}{2}$ state should lie about 5 MeV above the ground state, beyond the range of excitation studied in the experiment. This nuclide provides by far the worst disagreement between theory and experiment.

[15] The total strength to the $T=\frac{5}{2}$ levels in Ti^{49} should be 7.7 particles.

$Ti^{47}(d,t)Ti^{46}$

The transition to the ground state of Ti^{46} is forbidden for a pure $1f_{7/2}$ configuration and is not seen experimentally. This confirms our contention that the $1f_{5/2}$ component is negligible. For positive deformation, the calculated strengths of the transitions to the 2^+ and 4^+ states are in good agreement with the experimental values, particularly for the transition to the 4^+ level. Although the location of the 6^+ level is not certain experimentally, a reasonable location is about 3.3 MeV, where a weak peak is observed. The calculated strength of this transition is in good agreement with the observed peak, when the experimental error is considered. From the over-all consistency of our results, this may be taken as preliminary evidence for locating the 6^+ state. As is generally the case for pickup from odd-neutron targets, most of the reaction strength goes to levels having Q values similar to those of neighboring even-even nuclei. A pure seniority calculation is not simple because of the $\frac{5}{2}^-$ spin of Ti^{47}.

$Ti^{48}(d,t)Ti^{47}$

The experimental value for the transition to the lowest $\frac{7}{2}^-$ state is 2.45 particles. The calculated values of 3.74 for positive deformation and 3.67 for negative deformation are not in particularly good agreement, but certainly are a vast improvement over the pure seniority value. There is a question in the evaluation of the experimental number for this nucleus because the over-all normalization is low. If we normalize, assuming all the $T=\frac{3}{2}$ strength has been observed, then the ground-state transition has a strength of 3.2 particles, which is substantially closer to the theoretical value. The $T=\frac{5}{2}$ strength should appear at about 7.5 MeV excitation.

$Ti^{49}(d,t)Ti^{48}$

The calculated strength of the ground-state transition is 0.25 particle for all of the methods and is in excellent agreement with the experimental value of 0.23. The observed strength of the transition to the 2^+ state lies between the values for positive and negative deformation, while the strength of the 4^+ transition is very close to the value for positive deformation. The comparison for the 6^+ transition is hampered by the presence of a second 4^+ state close to the first 6^+. Since it was not possible to separate these two states experimentally, the apparent poor agreement between theory and experiment should not be considered as a true discrepancy. Both the calculated and experimental results indicate that the bulk of the transition strength is for reactions proceeding to levels other than the lowest 0^+, 2^+, 4^+, and 6^+ states. The pure seniority calculation requires that the transitions to these states exhaust the sum rule.

$Ti^{50}(d,t)Ti^{49}$

The calculated strength of 4.90 particles for both positive and negative deformation is reasonably close to the observed value of 4.05 particles. In this example the target has a closed neutron shell so that the seniority of the ground state must be zero. The pickup reaction, therefore, directly measures the fraction of the wave function of the ground state of Ti^{49} corresponding to seniority one (~52%).

$Cr^{52}(d,t)Cr^{51}$

For positive deformation, the calculated strength of the reaction to the ground state is 3.31 particles. This is in excellent agreement with the measured value of 3.58 particles. The value of 4.23 particles for negative deformation is in much better accord with the data than is the seniority value of 7.2. The $T=\frac{5}{2}$ state lies above the range of energy covered in the experiment.

$Fe^{54}(d,t)Fe^{53}$

Comparison of the data and calculation is difficult because of the normalization problem discussed earlier. The calculated values for the ground-state transition strength are 3.75 for positive deformation, 5.52 for negative deformation, and 6.0 for seniority. The $T=\frac{3}{2}$ state should be at about 4.5 MeV excitation, well within the range of excitation covered by the experiment, and the strength of the transition to this state should be 2.0 particles. It is expected that four transitions should be seen, three corresponding to $T=\frac{1}{2}$ and one to $T=\frac{3}{2}$. The experimental data indeed exhibit four strong peaks, one of which is at approximately 4.5 MeV excitation, but other groups lie nearby. If the normalization used is the same as for $V^{51}(d,t)V^{50}$ and $Cr^{52}(d,t)Cr^{51}$, the sum of the strengths of the transitions to Fe^{53} corresponds to only 3.6 particles as opposed to the required value of 8 particles. The ground-state transition, by far the strongest of all, has a strength of 2 particles. If the total reaction strength is renormalized to 8 particles, the ground-state transition has a strength of 4.42 particles, which is between the calculated values for positive and negative deformation, but still far from the seniority value. With this normalization, one of the excited states (~3.0 MeV) has a strength of 1.75 particles, a value close to the predicted strength for the $T=\frac{3}{2}$ state. For this nucleus, in particular, the range of Q values involved is somewhat different from those of all other nuclei studied, so that the extraction of transition strengths may be less reliable. We, therefore, feel that comparison with the (d,t) data is, at present, not a good test of the calculation. A better check may possibly be the $Fe^{54}(p,d)Fe^{53}$ reaction which has been studied with 22-MeV protons.[16] The result, obtained by an analysis based on the distorted-wave Born approximation, is a strength of 3.3 particles for the ground-state transition. This result is in excellent agreement with the calculation. On the other hand, the analysis[17] of $Fe^{54}(He^3,\alpha)Fe^{53}$, also by use of distorted waves, yields a value of 2 particles.

DISCUSSION

It is seen that, for even-even targets, generating from either positive or negative deformation predicts ground-state transition strengths which are generally in good agreement with the experimental values—the positive deformation yielding somewhat better results. For the odd-neutron targets, the strengths predicted for the 0^+, 2^+, 4^+, and 6^+ states for positive deformation are in remarkably good agreement with the experimental results. In all cases, the predictions are much better than those of a seniority calculation. We note that our predicted strengths for transitions to 2^+ states are about 30% higher than observed, while the 4^+ strengths are very close to the experimental values. A similar situation holds for the $Ti^{49}(d,p)Ti^{50}$ reaction.[18]

The present calculation attempts to simulate the effect of a residual interaction through the mechanism of a quadrupole deformation. It is obvious that this can only be an approximation to the truth. The qualitatively excellent agreement obtained is, therefore, somewhat better than one might expect. It would be interesting to do an actual shell-model calculation with a conventional residual two-body interaction to see how closely the wave functions agree with those obtained by the generating procedure.

A further experimental test of the mixed-seniority hypothesis is provided by a study of the (d,He^3) reaction, particularly on nuclei in which the neutron shell is full. In this latter case, only one strong $l=3$ peak should be observed. Such experiments are now in progress at this laboratory.[19]

ACKNOWLEDGMENTS

We should like to thank Dr. M. H. Macfarlane for several interesting discussions. We appreciate the cooperation of Dr. T. H. Braid and Dr. J. L. Yntema in providing us with data prior to publication.

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