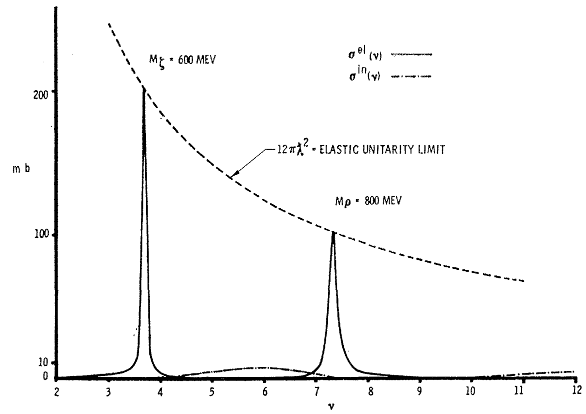

FIG. 4. The elastic and inelastic cross sections shown as full and dash-dot lines, respectively, for the cutoff calculation. The pure elastic limit is shown by the dashed line and the limit of the inelastic cross section is 1/4 of this.

$\omega+\pi$ intermediate state have little effect as can be seen in Fig. 2. It should be noted that $\mathrm{Re}(\nu f^{-1})$, $C_0+\nu N$, and νM all have zeros at or near the point where $\delta_R=\pi$; this is true in general if δ_R only passes once through π in the low-energy region.

A similar result is given by a calculation in which σ^{in} is *not* cut off. In particular, for the values of the parameters $M_\eta=475$ MeV, $G_\eta^2=1$ the two di-pion resonances occur at $M_\zeta=640$ MeV and $M_\rho=730$ MeV, and the widths are $\Gamma_\zeta=35$ MeV and $\Gamma_\rho=40$ MeV. In the latter result the contribution from the $\omega+\pi$ intermediate state is not included. The first peak in $[\nu/(\nu+1)]^{\frac{1}{2}}(R-1)$, in the case of no cutoff, has a maximum of 41 and the curve then falls rapidly almost to zero at the position of the second di-pion resonance, after which the curve rises rapidly again to about 6 and finally approaches a value corresponding to $R=2$ as $\nu\to\infty$. The theoretical maximum of σ^{el} is $\sigma_{\max}{}^{\mathrm{el}}=3\pi\lambda\!\!\!^{-2}(1+e^{-2\delta_I})^2$ and at the second resonance ($M_\rho=730$ MeV) this is 93% of the pure elastic maximum.

The position of the first resonance is very sensitive to the value of the inelastic threshold. The four-pion intermediate state threshold is, in fact, at $\nu=3$, but in the inelastic model used in the present work the threshold is that corresponding to $\eta+\pi$, and in virtue of this circumstance the η mass has been varied. All the results showing two resonances in the P wave are obtained for $M_\eta<500$ MeV. The widths of the ζ and ρ resonances are in satisfactory agreement with the experimental results, but some of our results show more than two resonances in the P wave, and it may be that the ρ consists of two narrow resonances close together.

An improved calculation of σ^{in} could be obtained by determining R from the R_{11}, R_{12}, R_{22} elements of the reaction matrix, where R_{11} is the amplitude for $\pi+\pi\leftrightarrow\pi+\pi$, R_{12} for $\pi+\pi\leftrightarrow\pi+\eta$, and R_{22} for $\pi+\eta\leftrightarrow\pi+\eta$. This calculation has the advantage of coupling σ^{in} to σ^{el}.

Provided that the inelastic cross section does not vary too rapidly we conclude from these results that the inelastic intermediate states can be treated as a perturbation on the previously published solutions for elastic pion-pion scattering,[2,3] since they have little effect on the nearby left cut in the inverse amplitude which dominates the problem. These results do not depend sensitively on the model used to represent the four-pion intermediate state.

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Soft Pion Emission Induced by Electromagnetic and Weak Interactions*

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We consider some extensions of the relation which has been obtained by Nambu and Lurié between an elastic process and the accompanying soft pion emission process under the assumption of approximate γ_5 invariance. In particular, a generalized formula is found which enables one to describe the electropion production $e+N\to N+e+\pi$ and the neutrino-pion production $\nu+N\to N+e+\pi$ at the threshold of $N\pi$ system in terms of the vector (Hofstadter) and the axial-vector form factors. Explicit forms of the cross sections are given.

1. INTRODUCTION

THE possibility of approximate chirality conservation in the pion-nucleon system was first observed in connection with weak interactions, where the lepton seems to be coupled to nearly conserved axial vector as well as vector currents. The physical aspects of the conserved axial vector currents, however, are very much different from those of the vector currents associated with the ordinary conservation laws. One may regard the former as a case of hidden symmetry which does not

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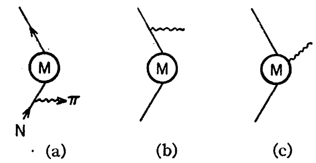
FIG. 1. Three diagrams contributing to M_{rad}.



manifest itself as a quantum number, but only implies that a certain expectation value remains constant in time.

This point was analyzed and elucidated in some detail in a previous paper.[1] It was realized that a change in nucleon isotopic chirality, which we will define to be minus twice the helicity times velocity times isospin, in any reaction results in the emission of a soft pion, so that one obtains a specific relation between the elastic and inelastic reaction amplitudes much resembling the one for ordinary bremsstrahlung. The formula was applied to the case of pion-nucleon elastic scattering and associated pion production, with a reasonable agreement with experimental results in a case where the elastic scattering is dominated by the 3–3 resonance. Since the theoretical formula is actually supposed to be valid only at energies large compared to the pion mass, the agreement may be a fortuitous one.

The purpose of the present paper is twofold. First, as a supplement to the previous paper, NL, we would like to make some more comments about the formula, in particular with regard to its relation to dispersion theory and Feynman diagrams. This enables one to prove the conjectured equality of the pion coupling appearing in our formula and the coupling defined in dispersion theory. Also, it will allow a natural generalization which incorporates meson emission with small but nonzero momentum.

The second and main purpose of the paper is to consider soft pion production induced by a small external perturbation such as a photon (real or virtual) or a neutrino, which does not commute with isotopic chirality. Namely, we have in mind processes of the type

$$e+N\rightarrow N+\pi+e,$$
$$\nu+N\rightarrow N+\pi+e.$$

The violation of chirality conservation due to the external perturbation necessitates a modification of the previous formula. This modification turns out to be very simple to the first order in the external perturbation. Moreover, it is of such a nature that both of the above mentioned processes are controlled by common form factors, namely, the vector (Hofstadter) form factors and the axial vector form factor, which appear intermingled with each other.

Thus, the group theoretical similarity between electromagnetic and weak processes becomes indeed a close one. In practice, this will also enable one to measure these form factors by using either process, and in turn check the validity of our underlying assumptions. Some calculations for this purpose will be presented in later sections.

2. DIAGRAMMATIC INTERPRETATION OF THE FORMULA

Recapitulating the derivation of the previous paper, let M be a reaction amplitude involving a single nucleon, and $M_{\mathrm{rad}}{}^{\alpha}$ the amplitude for an associated soft pion emission with zero momentum and isospin α. These two are related by

$$iM_{\mathrm{rad}}{}^{\alpha}=f[\chi_N{}^{\alpha,\mathrm{in}},M], \tag{2.1}$$

where $\chi_N{}^{\alpha}$ is the nucleon chirality operator

$$\chi_N{}^{\alpha,\mathrm{in}}=-\tau^{\alpha}\boldsymbol{\sigma}\cdot\mathbf{p}/E_p, \tag{2.1$'$}$$

and $f=g/2m$ is the pion-nucleon coupling constant. This result was obtained from the assumption $\chi^{\mathrm{in}}=\chi^{\mathrm{out}}$, where $\chi=\chi_N+\chi_\pi$, with $\chi_\pi{}^{\alpha,\mathrm{in}}=(1/f)\int\dot{\phi}^{\alpha,\mathrm{in}}d^3x$ being the pion contribution to χ. In case M is the invariant amplitude without the positive energy projection operator, the alternative form of Eq. (2.1) becomes

$$\begin{aligned}iM_{\mathrm{rad}}{}^{\alpha}=&f\{[(m/E)\gamma_4\gamma_5-\gamma_5]\tau^{\alpha}M\\&+M\tau^{\alpha}[(m/E)\gamma_5\gamma_4-\gamma_5]\}\\=&f[(m/E)\gamma_4\gamma_5\tau^{\alpha},M]-f\{\gamma_5\tau^{\alpha},M\}.\end{aligned} \tag{2.2}$$

A very simple interpretation of Eq. (2.2), which bears resemblance to a corresponding theorem for bremsstrahlung,[2] can be made in the following way. From the viewpoint of dispersion theory, the pion emission amplitude consists of the nucleon pole contribution corresponding to the diagrams (a) and (b) of Fig. 1, and the rest which is indicated by the diagram (c). The pole diagrams give

$$\begin{aligned}&g\tau^{\alpha}\gamma_5\frac{1}{i\gamma\cdot(p'+k)+m}M+M\frac{1}{i\gamma\cdot(p-k)+m}\gamma_5\tau^{\alpha}g\\&=g\tau^{\alpha}\gamma_5\frac{-i\gamma\cdot k}{2p'\cdot k+k^2}M+M\frac{-i\gamma\cdot k}{2p\cdot k-k^2}\gamma_5\tau^{\alpha}g.\end{aligned} \tag{2.3}$$

In the limit $\mathbf{k}=0$, and then $k_0=\mu\rightarrow 0$, this reduces to the commutator term in Eq. (2.2). The second (anticommutator) term must then correspond to the diagram (c). Note that M above is the mass shell amplitude corresponding to the real elastic scattering. In the Feynman-Dyson picture, on the other hand, diagrams (a) and (b) will contain off-the-mass shell corrections which must be lumped with contributions from the other irreducible diagrams.

It is interesting that the nonpole contribution in Eq. (2.2) is exactly in the form of an infinitesimal change in M generated by a γ_5 transformation. This is not surprising since, for example, in the Nishijima model[1]

[1] Y. Nambu and D. Lurié, Phys. Rev. **125**, 1429 (1962), hereafter referred to as NL. References to relevant literature are given there.

[2] F. Low, Phys. Rev. **110**, 974 (1958).

the pion coupling was generated in this fashion. When M itself happens to be γ_5 invariant, the nonpole contribution will vanish, as was the case considered in Sec. 2 of NL.

The fact that the pole contribution is separated from the rest is due to the singular behavior of the nucleon propagator in the limit $k\rightarrow 0$. As was discussed in NL, this limit is not unique, depending on the direction from which k approaches zero. Our procedure was to let $\mathbf{k}$ go to zero first in the coordinate system considered. Physically, it means that we extrapolate to mesons produced at rest (with mass μ small but finite) in that particular frame of reference. The extrapolated value will change appreciably as soon as we go to a second coordinate system in which the meson at rest in the first coordinate system appears with a momentum $|\mathbf{k}|$ comparable to μ.

The nonpole contribution, on the other hand, consists of a dispersion integral over a continuous mass spectrum starting at $m+\mu$. Its limit as $\mathbf{k}\rightarrow 0$ and $\mu\rightarrow 0$ will not become singular unlike the pole term since the phase space for many pion emission processes vanishes at the threshold. There is no infrared catastrophe in the case of pion bremsstrahlung, but only a mild nonunique behavior of one-pion emission amplitude as $k\rightarrow 0$.

The above observation shows that the interpretation of the two terms of Eq. (2.2) is unambiguous. This, in turn, proves that the coupling constant f in Eq. (2.2) is the same as the dispersion theoretical one since both appear as a residue of the same pole contribution.

Finally, a slight but important generalization of Eq. (2.2) naturally suggests itself. Namely, instead of taking the limit $k\rightarrow 0$, we may keep k small but finite as far as the pole contribution is concerned, since it is most sensitive to k. This brings two advantages. First, the entire formula becomes manifestly Lorentz invariant in contrast to Eq. (2.2). Second, the formula agrees with the exact expression for arbitrary k as far as the pole term is concerned. Our modified formula will thus read

$$iM_{\mathrm{rad}}{}^{\alpha}(k)=-f\left[\tau^{\alpha}\gamma_5\frac{2mi\gamma\cdot k}{2p'\cdot k+k^2}M+M\frac{2mi\gamma\cdot k}{2p\cdot k-k^2}\gamma_5\tau^{\alpha}\right]-f(\tau^{\alpha}\gamma_5M+M\gamma_5\tau^{\alpha}). \quad (2.4)$$

In practical applications, this modification will be an essential improvement since it includes at least part of the p-wave pion contribution (and higher waves) which, combined with the s wave, is necessary to keep M_{rad} Lorentz invariant. Usually such a p-wave contribution will be present in the measurement, and can even be the dominant one unless we select pions with very small momenta.

The second term of Eq. (2.4) is specific to our assumption of chiral invariance, and becomes rigorous in the limit $k\rightarrow 0$. In dispersion dynamics, one may regard it as a new boundary condition to be imposed on M_{rad} as a function of k.

3. REACTIONS INDUCED BY CHIRALITY NONCONSERVING PERTURBATIONS

We now turn to soft pion production initiated by a small perturbation which does not commute with isotopic chirality. As we shall consider later, the electromagnetic interaction and the leptonic interaction of the nucleon belong to this category, since they behave like some particular components of isospin.

Let us denote this perturbing Hamiltonian by $\bar{H}'$. The chirality operator χ^{α} will change with time according to

$$i\dot{\chi}^{\alpha}=[\chi^{\alpha},\bar{H}'],$$

so that

$$\begin{aligned}\chi^{\alpha,\mathrm{out}}&=\chi^{\alpha,\mathrm{in}}-i\int_{-\infty}^{\infty}[\chi^{\alpha}(t),\bar{H}'(t)]dt\\&=S^{-1}\chi^{\alpha,\mathrm{in}}S,\\\chi^{\alpha,\mathrm{in}}S-S\chi^{\alpha,\mathrm{in}}&=-iS\int_{-\infty}^{\infty}[\chi^{\alpha}(t),\bar{H}'(t)]dt.\end{aligned} \quad (3.1)$$

Here S is the S matrix generated by the entire Hamiltonian. By taking an appropriate matrix element, the left-hand side leads to the same combination of M and M_{rad} as before, while the right-hand side gives a correction due to the symmetry breaking effect of $\bar{H}'$. To determine the latter, however, we have to know the complete dynamics of the system.

The problem becomes much simplified if we consider the effect of $\bar{H}'$ only to the first order. Suppose a reaction is generated by $\bar{H}'$ itself. Then the part of S we are interested in on the left-hand side will be of the order $\bar{H}'$, whereas the S on the right-hand side may be replaced by S_0 due to strong interactions alone. We thus obtain

$$(i/f)M_{\mathrm{rad}}{}^{\alpha}=[\chi_N{}^{\alpha,\mathrm{in}},M]+S_0[\chi^{\alpha},H'], \quad (3.2)$$

where we have removed the energy-momentum conservation factor from the amplitude, so that the time integral of $\bar{H}'$ is replaced by the Hamiltonian density H'.

Since χ^{α} is the generator of an infinitesimal γ_5 transformation, the second term on the right involves the infinitesimal change induced on H'. We may therefore state the result in the following way: When a reaction M is induced by $\bar{H}'$, the radiative process M_{rad} consists of two parts, one due to the change in nucleon chirality, and the other directly generated by an equivalent perturbation Hamiltonian $-i[\chi^{\alpha},\bar{H}']$. This latter, when taken between real Heisenberg states, must naturally include all the renormalizations and radiative corrections. The factor S_0 entering in Eq. (3.2) expresses the final-state interaction of the reaction products excluding the soft pion. It will drop out when the final state consists of a single particle.

4. ELECTROPION PRODUCTION

In this section, we apply our previous formula (3.2) to pion production by virtual photon. The electromag-

netic interaction of the nucleon and the pion is given by the Lagrangian density

$$L'=j_\mu A_\mu=ei\bar{\psi}\gamma_\mu[(1+\tau_3)/2]\psi A_\mu -ei\tfrac{1}{2}(\phi T_3\partial_\mu\phi-\partial_\mu\phi T_3\phi)A_\mu, \quad (4.1)$$

where T is the pion isospin matrix. The chirality operator χ^α is

$$\chi^\alpha=\int[\bar{\psi}\gamma_4\gamma_5\tau^\alpha\psi+(i/f)\partial_4\phi^\alpha]d^3x. \quad (4.2)$$

It is not necessary to use ∂_4-ieA_4 above, as the field may be turned on after $t=-\infty$. We may also use $-L'$ instead of H' without caution to the first order in e. So let us evaluate $[\chi^\alpha,L']$. With the standard commutation relations, we find

$$i[\chi^\alpha,L']=\epsilon_{3\alpha\beta}J_\mu{}^\beta A_\mu,$$
$$J_\mu{}^\beta=e[i\bar{\psi}\gamma_\mu\gamma_5\tau^\beta\psi-(1/f)\partial_\mu\phi^\beta]=e\chi_\mu{}^\beta. \quad (4.3)$$

This part of pion production then is generated by the isotopic axial vector (chirality) current rather than the vector current! In the above calculation, explicit forms (4.1) and (4.2) were used, but actually it is not necessary nor general. Only the group properties of vector and axial vector isotopic currents are essential. Note also that the isoscalar part of L' commutes with χ^α, and that J_μ is twice what should be more properly called isotopic axial vector current.

Now if the electropion production is described by M_{rad} ,the corresponding "elastic" process is obviously the elastic scattering of the nucleon by a virtual photon described by the Hofstadter form factors; namely,

$$\bar{u}_{p'}Mu_p=\langle p'|j_\mu A_\mu|p\rangle =e\bar{u}_{p'}F_\mu(p',p)A_\mu(q)u_p, \quad q=p'-p,$$
$$F_\mu(p',p)=i\gamma_\mu[\tfrac{1}{2}F_1{}^S(q^2)+(\tau_3/2)F_1{}^V(q^2)] +(i\sigma_{\mu\nu}q_\nu/2m)[\tfrac{1}{2}F_2{}^S(q^2)+(\tau_3/2)F_2{}^V(q^2)]. \quad (4.3)$$

Similarly, we may write

$$\langle p'|J_\mu{}^\alpha A_\mu|p\rangle=e\bar{u}_{p'}G_\mu{}^\alpha(p',p)A_\mu(q^2)u_p,$$
$$G_\mu{}^\alpha(p',p)=i\gamma_\mu\gamma_5\tau^\alpha G_1(q^2)+\gamma_5\tau^\alpha(q_\mu/q^2)G_2(q^2) =[i\gamma_\mu\gamma_5+2m\gamma_5(q_\mu/q^2)]\tau^\alpha G_1(q^2). \quad (4.4)$$

The last form follows, of course, from chirality conservation (neglecting pion mass and proton-neutron mass difference). Combining Eqs. (3.2), (4.3), and (4.4), we obtain

$$(i/f)M_{\text{rad}}{}^\alpha=e[\chi_N{}^{\alpha,\text{in}},F_\mu(p',p)A_\mu(q)] +ie\epsilon_{3\alpha\beta}G_\mu{}^\beta(p',p)A_\mu(q), \quad (4.6)$$

where, for the commutator term, one has to use Eq. (2.2) or (2.4). A few remarks are in order.

(1) It is instructive to interpret the formula in terms of diagrams. The commutator term corresponds to the diagrams in Fig. 1, where M is replaced by the electromagnetic vertex. The additional axial vector term, on the other hand, may be identified with the "photoelectric" term and an associated "direct production" term illustrated in Fig. 2. Again, the identification of the "photoelectric" term is unambiguous because it is the only term with a pion pole. Remember in this connection that the meson pole will have, in general, a denominator $(q-k)^2+\mu^2$, which in the limit $k=0$, $\mu=0$, becomes q^2. The form factor $G_1(q^2)$ must be interpreted as the combined effect of meson electromagnetic form factor, meson propagator correction, and the meson-nucleon vertex form factor. It is then clear that $G_1(0)=1$ since the residue of the pion pole in M_{rad} should equal fe. In other words, the axial vector form factor, like the vector form factor, should have no renormalization in order to be consistent with dispersion theory under strict γ_5 invariance. [In actual β decay, $G_1(0)\approx\mathbf{1.2}$.] This property has long been conjectured, but no direct proof seems to exist.

(2) Equation (4.6) is compatible with gauge invariance and the Kroll-Ruderman theorem for photoproduction at threshold. Gauge invariance is obvious because both terms in Eq. (4.6) satisfy the continuity equation. As for the Kroll-Ruderman theorem, we observe that in the case of a real incident photon, the reaction M cannot go as a real process except in the limit $q=0$, which in turn can only lead to M_{rad} for emission of a pion with $k=0$. In the nucleon rest system, which is now equal to the center-of-mass system, the term with χ_N vanishes since $v_N=0$, whereas the axial vector term with $G(0)=1$ agrees with the content of the Kroll-Ruderman theorem. In this sense Eq. (4.6) may be regarded as a generalized Kroll-Ruderman theorem for a virtual photon.

(3) In the more realistic case of finite k (and μ), we may follow the prescription of Sec. 2, and construct M_{rad} as a sum of pole terms and a specific correction characteristic of the γ_5 invariance. There are now two nucleon and one pion pole terms, plus three nonpole additions. However, this is not yet satisfactory, as gauge invariance is violated. In order to restore gauge invariance, another correction has to be added. We propose thus the following form:

$$\begin{aligned}(1/ef)M_{\text{rad}}{}^\alpha=&i\tau^\alpha\gamma_5\frac{2mi\gamma\cdot k}{2p'\cdot k+k^2}F_\mu(p'+k,\,p)A_\mu(q)\\&+F_\mu(p',\,p-k)A_\mu(q)\frac{2mi\gamma\cdot k}{2p\cdot k+k^2}i\gamma_5\tau^\alpha\\&+\frac{1}{2i}[\tau^\alpha,\tau^3]\frac{2m\gamma_5(2k_\mu-q_\mu)A_\mu(q)}{q^2-2k\cdot q}G(q^2)\\&+i\tau^\alpha\gamma_5F_\mu(p',p)A_\mu(q)\\&+F_\mu(p',p)A_\mu(q)i\gamma_5\tau^\alpha\\&-(1/2i)[\tau^\alpha,\tau^3]i\gamma_\mu\gamma_5A_\mu(q)G(q^2)\\&-(1/2i)[\tau^\alpha,\tau^3]i\gamma\cdot k\gamma_5q_\mu A_\mu(q)\\&\times\{F_1{}^V(q^2)-G(q^2)\}/q^2, \quad (4.7)\end{aligned}$$

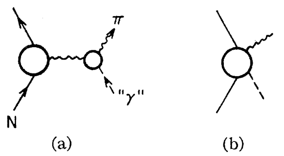


FIG. 2. Diagrams responsible for the axial vector term in electropion production.

where $G(q^2)=G_1(q^2)/G_1(0)$. The last term is the new addition designed to satisfy gauge invariance.[3] Since this is purely longitudinal, however, it does not make any real contribution to the physical amplitude. The assignment of the normalized form factor G to the pion pole term is rather arbitrary unlike the nucleon pole case, but it seems to be the simplest assumption insofar as we do not have any theory about the deviation from strict γ_5 invariance. [Note that this correction term is free of an unphysical singularity at $q^2=0$.]

5. CALCULATION OF CROSS SECTION

We will proceed to make some calculations on the electropion production cross section based on the previous results. Since the amplitude (4.7) is pretty complicated, however, we consider only the easiest problem. This is the total cross section for producing a pion at threshold of the final-nucleon pion system, for a given momentum transfer, q, large compared to pion mass. In this case, the produced pion will be at rest relative to the outgoing nucleon, so that we consider our formula (4.6) in the rest system of the outgoing nucleon. But then the final chirality of the nucleon becomes zero as it is proportional to velocity. Moreover, the pion pole term may be dropped since, after putting $k=0$, it is purely longitudinal ($\sim q_\mu A_\mu$) as shown in Eq. (4.4). We get therefore the simplified form

$$(i/fe)M_{\mathrm{rad}}{}^{\alpha}=\{-F_\mu(p',p)\chi_N{}^\alpha(p) -\tfrac{1}{2}[\tau^\alpha,\tau^3]i\gamma_\mu\gamma_5 G(q^2)\}A_\mu(q), \quad (5.1)$$
$$A_\mu(q)=j_\mu{}^{(e)}/q^2,$$

$j_\mu{}^{(e)}$ being the electron current. The chirality $\chi_N{}^\alpha$ above refers to that of the incident nucleon, which must be measured in the rest frame of the outgoing nucleon. In the helicity representation, $\chi_N{}^\alpha$ then turns out to be

$$\chi_N{}^\alpha(p)=-\tau^\alpha hv=-\tau^\alpha h|q|(m^2+q^2/4)^{1/2} \times(m^2+q^2/2)^{-1}. \quad (5.2)$$

In order to calculate the square of M_{rad}, it is now convenient to go to the system where the initial and final momenta of the nucleon are equal and opposite with magnitude $|q|/2$ (the "Breit" system, see Fig. 3). The helicity used in Eq. (5.2) remains unchanged under this change of reference frame. As was shown by Yennie *et al.*,[4] the electromagnetic form factor F_μ contains two incoherent matrix elements in this coordinate system. They are the time component F_0 which flips helicity, and the transverse spatial component $F^\pm(\perp q)$ which conserves helicity. Omitting the isotopic spin, these matrix elements are given by

$$\langle p',h'|F_0|p,h\rangle=\delta_{h,-h'}F_c(q^2)(1+q^2/4m^2)^{-1/2},$$
$$\langle p',h'|F_\pm|p,h\rangle=[(h\pm1)/2]\delta_{h,h'}F_m(q^2) \times(1+q^2/4m^2)^{-1/2}|q|/2m,$$
$$F_c=F_1-(q^2/4m^2)F_2,$$
$$F_m=F_1+F_2,$$
$$F_\pm=(F_x\pm iF_y)/2, \quad (z\|q) \quad (5.3)$$

where the new charge and magnetic form factors F_c and F_m have been introduced.[5] In our problem, there is also the axial vector form factor G_μ, which turns out to have a nonzero matrix element only for the transverse component:

$$\langle p',h'|G_\pm|p,h\rangle=-\tfrac{1}{2}(1\pm h)\delta_{h,h'}G(q^2). \quad (5.3')$$

In a similar fashion, the helicity matrix elements of the electron current $j_\mu{}^{(e)}$ in the same reference frame are found to be

$$\langle l',h'|j_0|l,h\rangle=-e\delta_{h,h'}\cos(\theta_B/2),$$
$$\langle l',h'|j_\pm|l,h\rangle=e\delta_{h,h'}[h\mp\sin(\theta_B/2)]/2, \quad (5.4)$$

in the relativistic limit. Here θ_B is the electron scattering angle in this frame, being related to the laboratory angle θ by

$$\tan(\theta_B/2)=(1+q^2/4m^2)^{1/2}\tan(\theta/2). \quad (5.4')$$

From Eqs. (5.1)–(5.4), the cross section for soft pion production per unit energy range of the final electron at a fixed laboratory angle is obtained without much difficulty (see reference 4). The result is

$$\frac{d\sigma}{d\epsilon' d\Omega}=\alpha^2\frac{f^2}{4\pi^2}\frac{[2\mu(\epsilon_m'-\epsilon')]^{1/2}}{[1+\mu/m]^2}\frac{\cos^2(\theta/2)}{4\epsilon^2\sin^4(\theta/2)} \times\left[1+\frac{2\epsilon}{m}\sin^2(\theta/2)\right]^{1/2}S,$$

$p\rightarrow p+\pi^0$:

$$S=F_{pc}{}^2m^2q^2/(m^2+q^2/2)^2+[1+2\tan^2(\theta_B/2)] \times[F_{pm}q^2/(2m^2+q^2)]^2,$$

$n\rightarrow n+\pi^0$:

$$S=F_{nc}{}^2m^2q^2/(m^2+q^2/2)^2+[1+2\tan^2(\theta_B/2)] \times[F_{nm}q^2/(2m^2+q^2)]^2,$$

$p\rightarrow n+\pi^+$:

$$S=2F_{nc}{}^2m^2q^2/(m^2+q^2/2)^2+2[1+2\tan^2(\theta_B/2)] \times[G+F_{nm}q^2/(2m^2+q^2)]^2,$$

$n\rightarrow p+\pi^-$:

$$S=2F_{pc}{}^2m^2q^2/(m^2+q^2/2)^2+2[1+2\tan^2(\theta_B/2)] \times[G-F_{pm}q^2/(2m^2+q^2)]^2, \quad (5.5)$$

[3] A similar term is proposed in S. Fubini, Y. Nambu, and V. Wataghin, Phys. Rev. **111**, 329 (1958).

[4] D. R. Yennie, M. M. Lévy, and D. G. Ravenhall, Revs. Modern Phys. **29**, 144 (1957), Appendix.

[5] F. J. Ernst, R. G. Sachs, and K. C. Wali, Phys. Rev. **119**, 1105 (1960). See also W. R. Theis, Phys. Rev. Letters **8**, 45 (1962); L. N. Hand, D. G. Miller, and R. Wilson, *ibid.* **8**, 110 (1962).

F_p and F_n refer to the proton and neutron form factors, respectively. The invariant momentum transfer q^2, the incident electron energy ϵ, the maximum energy of inelastic electron ϵ_m', and the scattering angle θ are related by the following relations:

$$q^2=4\epsilon\epsilon_m' \sin^2(\theta/2)=2m(\epsilon-\epsilon_m'-\mu),$$
$$\epsilon_m'=(\epsilon-\mu-\mu^2/2m)/[1+(2\epsilon/m)\sin^2(\theta/2)]. \quad (5.5')$$

In Eqs. (5.5) and (5.5′), pion and electron masses are neglected compared with m, ϵ, and q except for purely kinematical corrections, so that they should be valid only under these conditions.

Equation (5.5) allows one to determine the form factors by measuring the soft pion inelastic cross section at a fixed angle and varying incident energy. In particular, the electric form factors predominate near the forward direction, while the magnetic and axial vector factors are the sole contributions at 180°. These features are common with the elastic cross section given by the Rosenbluth formula. In comparison with Eq. (5.5), the latter reads[4,5]

$$\frac{d\sigma}{d\Omega}=\alpha^2\frac{\cos^2(\theta/2)}{4\epsilon^2\sin^4(\theta/2)}\frac{1}{1+(2\epsilon/m)\sin^2(\theta/2)}\frac{1}{1+q^2/4m^2}\times\{F_c^2+[1+2\tan^2(\theta_B/2)](q^2/4m^2)F_m^2\}. \quad (5.6)$$

The electropion production was utilized by Panofsky and Allton,[6] and by Ohlsen[7] in order to determine the neutron magnetic form factor F_{n2}. They measured the cross section for pion production at the 3–3 resonance for varying q^2. Our formula requires a similar experiment to be done near threshold, which would be more difficult because of the smallness of the cross section. On the other hand, it will enable one to determine the axial-vector form factor if the vector form factors are known. The axial-vector form factor, according to our theory, reflects the various effects associated with a virtual pion, and therefore did not appear in the earlier theories.

6. NEUTRINO-PION PRODUCTION[8]

With the standard assumption of the bare V-A lepton-nucleon interaction, H' and M responsible for the elastic process are given by

$$H'=-ig/\sqrt{2}\sum_\pm \bar{\psi}\gamma_\mu(1+\gamma_5)\tau^\pm\psi j_\mu{}^{(L)\mp},$$
$$\bar{u}_{p'}Mu_p=g/\sqrt{2}\sum_\pm \bar{u}_{p'}[F_\mu{}^V+G_\mu]\tau^\pm u_p j_\mu{}^{(L)\mp}, \quad (6.1)$$

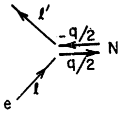


FIG. 3. Nucleon and electron momenta in the "Breit" system.

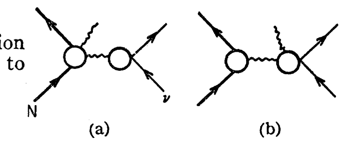


FIG. 4. Two kinds of pion pole diagrams contributing to neutrino-pion production.

where $j^{(L)\pm}$ is the lepton current, and $g=10^{-5}m^{-2}$ the β-decay coupling constant. If the weak interactions are mediated by an intermediate boson, there will arise a characteristic form factor multiplying $j^{(L)}$. Since the necessary modification is obvious, we will in the following ignore this possibility. The commutator $[\chi^\alpha,H']$ in Eq. (3.2) becomes

$$[\chi^\alpha,H']=-(ig/\sqrt{2})\sum_\pm \bar{\psi}\gamma_\mu(1+\gamma_5)\times[\tau^\alpha,\tau^\pm]\psi j_\mu{}^{(L)\mp},$$
$$\langle p'|[\chi^\alpha,H']|p\rangle=-(g/\sqrt{2})\sum_\pm \bar{u}_{p'}[F_\mu{}^V+G_\mu]\times[\tau_\alpha,\tau^\pm]u_p j_\mu{}^{(L)\mp}. \quad (6.2)$$

The radiation amplitude thus takes the form

$$(i/f)M_{\mathrm{rad}}{}^\alpha=(g/\sqrt{2})\sum_\pm\{[\chi_N{}^{\alpha,\mathrm{in}},(F_\mu{}^V+G_\mu)\tau^\pm]-(F_\mu{}^V+G_\mu)[\tau^\alpha,\tau^\pm]\}j_\mu{}^{(L)\mp}. \quad (6.3)$$

The interpretation of these terms is similar to the electroproduction case, except that we have here also an intrinsic axial vector interaction. The G_μ contains a pion pole, but in the first term the produced pion is emitted from the nucleon-pion vertex, whereas in the second term it is emitted from the pion-lepton vertex like the "photoelectric" term in electropion production (Fig. 4). According to this interpretation, it is again possible to construct a generalized expression corresponding to Eq. (4.7) for finite k, but we will not attempt to write down the explicit formula.

Turning to the easier problem, the inelastic cross section at threshold for a given (and large) momentum transfer q can be calculated by the method in Sec. 5. The formula corresponding to Eq. (5.1) becomes

$$(i/f)M_{\mathrm{rad}}{}^\alpha=-(g/\sqrt{2})\sum_\pm\{\tau^\pm(F_\mu{}^V+G_\mu)\chi_N{}^{\alpha,\mathrm{in}}+[\tau^\alpha,\tau^\pm](F_\mu{}^V+G_\mu)\}j_\mu{}^{(L)\mp}. \quad (6.4)$$

The matrix elements of F_μ, G_μ, and $j_\mu{}^{(L)}$ in the Breit system are given in Eqs. (5.3)–(5.4′), but in the present case the two-component nature of the neutrino has to be taken into account. We quote the results for the various reactions[9]:

(a) $\nu+p\rightarrow p+e^-+\pi^+$,

(b) $\nu+n\rightarrow p+e^-+\pi^0$,

(c) $\nu+n\rightarrow n+e^-+\pi^+$,

(d) $\bar{\nu}+n\rightarrow n+e^++\pi^-$,

(e) $\bar{\nu}+p\rightarrow n+e^++\pi^0$,

(f) $\bar{\nu}+p\rightarrow p+e^++\pi^-$,

[6] W. K. Panofsky and E. A. Allton, Phys. Rev. **110**, 1155 (1958).
[7] G. G. Ohlsen, Phys. Rev. **120**, 584 (1960).
[8] On this general subject see, for example, Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) **23**, 1117 (1960); T. D. Lee and C. N. Yang, Phys. Rev. Letters **4**, 307 (1960); S. M. Berman, Proceedings of the International Conference on Theoretical Aspects of Very High-Energy Physics [CERN Report No. 61-22, (unpublished), p. 7.]

[9] For a dispersion-theoretic treatment of the problem, see Ya. N. Azimov, Zhur. Eksp. i. Teoret. Fiz. **41**, 1879 (1961); N. Dombey, Phys. Rev. **127**, 653 (1962); P. Dennery, *ibid.* **127**, 664 (1962).

$$\frac{d^2\sigma}{d\epsilon' d\Omega}=\left(\frac{g}{4\pi}\right)^2\frac{2f^2}{\pi^2}\frac{[2\mu(\epsilon_m'-\epsilon')]^{1/2}}{[1+\mu/m]^2}[1+(2\epsilon/m)\sin^2(\theta/2)]^{1/2}\frac{\epsilon_m'^2\cos^2(\theta/2)}{1+q^2/4m^2}S,$$

$$S_a=2(F_c{}^V)^2(1+v^2)+\sum_{h=\pm1}[F_m{}^V(qh/2m)-\lambda G_1]^2(1+hv)^2[1-h\sin(\theta_B/2)]^2/\cos^2(\theta_B/2),$$

$$S_b=(F_c{}^V)^2(4+v^2)+\tfrac{1}{2}\sum_{h=\pm1}[F_m{}^V(qh/2m)-\lambda G_1]^2(2+hv)^2[1-h\sin(\theta_B/2)]^2/\cos^2(\theta_B/2),$$

$$S_c=2(F_c{}^V)^2+\sum_{h=\pm1}[F_m{}^V(qh/2m)-\lambda G_1]^2[1-h\sin(\theta_B/2)]^2/\cos^2(\theta_B/2),$$

$$S_d=2(F_c{}^V)^2(1+v^2)+\sum_{h=\pm1}[F_m{}^V(qh/2m)-\lambda G_1]^2(1+hv)^2[1+h\sin(\theta_B/2)]^2/\cos^2(\theta_B/2),$$

$$S_e=(F_c{}^V)^2(4+v^2)+\tfrac{1}{2}\sum_{h=\pm1}[F_m{}^V(qh/2m)-\lambda G_1]^2(2+hv)^2[1+h\sin(\theta_B/2)]^2/\cos^2(\theta_B/2),$$

$$S_f=2(F_c{}^V)^2+\sum_{h=\pm1}[F_m{}^V(qh/2m)-\lambda G_1]^2[1+h\sin(\theta_B/2)]^2/\cos^2(\theta_B/2), \tag{6.5}$$

with the abbreviations

$$v=(q/m)(1+q^2/4m^2)^{1/2}(1+q^2/2m^2)^{-1}, \tag{6.5'}$$
$$\lambda=(1+q^2/4m^2)^{1/2}.$$

Other quantities are defined in Sec. 5. As before, these formulas are supposed to be valid when pion and electron masses may be neglected compared with ϵ, q, and m. They should also apply to processes involving the muon under similar conditions.

For the sake of comparison, the elastic cross sections[8] are given below in our notation.

(b′) $\nu+n\to p+e^-$,
(e′) $\bar{\nu}+p\to n+e^+$,

$$\frac{d\sigma}{d\Omega}=\left(\frac{g}{4\pi}\right)^2[1+(2\epsilon/m)\sin^2(\theta/2)]^{-3}\frac{8\epsilon^2\cos^2(\theta/2)}{1+q^2/4m^2}S,$$

$$S_{b'}=(F_c{}^V)^2+\tfrac{1}{2}\sum_{h=\pm1}[F_m{}^V(qh/2m)-\lambda G_1]^2\times[1-h\sin(\theta_B/2)]^2/\cos^2(\theta_B/2),$$

$$S_{e'}=(F_c{}^V)^2+\tfrac{1}{2}\sum_{h=\pm1}[F_m{}^V(qh/2m)-\lambda G_1]^2\times[1+h\sin(\theta_B/2)]^2/\cos^2(\theta_B/2). \tag{6.6}$$