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Symmetry Model for the New Resonances

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A global symmetry is proposed which produces what seems to be a sensible classification of boson resonances and of N^*, Y^* resonances. A novel feature is the prediction that the cascade particle be spin 3/2.

1. INTRODUCTION

MANY global symmetry schemes have been proposed since the original proposals of Gell-Mann[1] and Schwinger.[2] None of these schemes have proved very successful in giving selection rules and relations between cross sections which are in agreement with experiments.[3] Nevertheless, there is some experimental indication that in the newly discovered resonances there exist sets of states, with fairly small mass differences, which contain more than one value of total isotopic spin and of strangeness. This provides a new incentive to consider possible symmetries which go beyond those implied by isotopic spin and strangeness conservation.

Recently Lee and Yang[4] have discussed some invariance models which predict that analogs of the 3, 3 pion-nucleon resonance will appear in the π-hyperon system. Gell-Mann[5] has discussed a group which unites pseudoscalar mesons (π,K,ψ_0) in an octet and $J=1^-$ resonances (ρ,ω,K') in another octet. Sakurai[6] has made several related proposals. A general discussion of models based upon simple Lie groups is given by Speiser and Tarski[7] and Behrends *et al.*[8]

In the present work a symmetry scheme is proposed which is somewhat different from any of those mentioned above, though related to that of reference 5. Global symmetry models being plentiful, some special justification should be given for proposing a new one. This we are unable to provide, except in terms of the results which will emerge. This work was admittedly done with one eye on very preliminary experimental data. However, many of the new resonances seem not only to be fitted, but to be predicted by the group to be considered. Before giving the details, we therefore describe briefly the outcome.

There will exist, in the direct product of the baryon representation with its conjugate, a meson representation containing an isotopic triplet with strangeness zero. Therefore, we may have a pion well separated in mass from other bosons. However, mesons with strangeness ± 1 will occur always in sets which also contain bosons of strangeness zero. Hence, we predict a set of states with $S=0$ and angular momentum 0^-, near in mass to the K meson. We predict another set, with angular momentum 1^-, near the K' resonance in mass.

The representation of minimum dimensionality which contains a pi-nucleon, $T=3/2$, isobar also includes a Y^* with $T=0$ and a Y^* with $T=1$. The cascade particle, Ξ, will belong to this set of $J=3/2$ states in our theory.

2. THE GROUP

The group which is of most interest to us is a subgroup of SU_6. Let us begin, however, with the full group SU_6. The possible significance of this full symmetry will be discussed in Sec. 5.

We choose a six-dimensional baryon representation, B, which consists of the unitary transformations on

$$\begin{bmatrix} p \\ n \\ \Lambda \\ \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{bmatrix}.$$

As explained above, the cascade particle will occur in another multiplet along with the π-N and π-hyperon resonances.

In the direct product, $B^*\times B$, we find representations of dimension one and of dimension 35. These correspond to possible sets of bosons which may have couplings to the baryon pairs. Clearly, the 35-boson representation is the one of interest to us.

The subgroup of SU_6 which we want to consider is $SU_3\otimes SU_2'$. We shall refer to this as the group from now on. The manner in which these operations act upon our sextet of baryons can most easily be defined by writing

$$B=\begin{bmatrix} N \\ Y \\ Z \end{bmatrix},$$

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[1] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).

[2] J. Schwinger, Ann. Phys. (New York) **2**, 407 (1957).

[3] See especially A. Pais, Phys. Rev. **110**, 574 (1958); **110**, 1480 (1958); **112**, 624 (1958); **122**, 317 (1961).

[4] T. D. Lee and C. N. Yang, Phys. Rev. **122**, 1954 (1961).

[5] M. Gell-Mann, Phys. Rev **125**, 1067 (1962).

[6] J. J. Sakurai, Phys. Rev. Letters **7**, 426, 355 (1961); and earlier work.

[7] D. R. Speiser and J. Tarski (to be published).

[8] R. E. Behrends, J. Dreitlein, C. Fronsdal, and B. W. Lee, Revs. Modern Phys. **34**, 1 (1962).

where

$$N=\begin{pmatrix} p \\ n \end{pmatrix}, \quad Y=\begin{bmatrix} \Sigma^+ \\ \frac{1}{\sqrt{2}}(\Sigma^0+\Lambda^0) \end{bmatrix}, \quad Z=\begin{bmatrix} \frac{1}{\sqrt{2}}(\Sigma^0-\Lambda_0) \\ Z^- \end{bmatrix}.$$

The transformations allowed will be firstly those of unitary, unimodular, 3×3 matrices acting bodily on B, where N, Y, Z are considered as single entities. These transformations are to be compounded with transformations of the form

$$\begin{bmatrix} N \\ Y \\ Z \end{bmatrix} \rightarrow \begin{bmatrix} UN \\ UY \\ UZ \end{bmatrix}, \tag{1}$$

where U is a unitary, unimodular 2×2 matrix. The baryon representation B is still irreducible for this subgroup of SU_6. This representation we now call $R(3\otimes 2')$ in an obvious notation. It is easy to see that the true isotopic spin rotation is contained. Under this group the representation 35 of SU_6 breaks up in the following way:

$$35=3+8+24. \tag{2}$$

Each of these representations may be easily understood in terms of a product of a representation of SU_3 by one of SU_2'. The representations of SU_3 are discussed in references 5, 7, 8, and by Wess.[9] The representations of SU_2' are those of the three-dimensional rotation group. Under SU_2' [Eq. (1)] the baryons transform as spin 1/2. Hence, in the direct product $B\times B^*$ we have either singlets or triplets as far as SU_2' is concerned. Thus, we see that

$$\begin{aligned} &\text{A.} \quad 3=1\otimes 3', \\ &\text{B.} \quad 8=8\otimes 1', \\ &\text{C.} \quad 24=8\otimes 3', \end{aligned}$$

where the first number on the right side of an equation refers to the dimensionality of a representation of SU_3 and the second to the dimensionality of a representation of SU_2'. The content of these representations is listed in Table I.

TABLE I. Representations of $SU_3\otimes SU_2$.

	Dimension	Quantum numbers		G for pseudoscaler	G for vector
A.	3	$T=1$	$S=0$	-1	1
B.	8	$T=1/2$	$S=\pm 1$		
		$T=0$	$S=0$	1	-1
		$T=1$	$S=0$	-1	1
C.	24	$T=0, 1, 2$	$S=0$	1	-1
		$T=1$	$S=0$	-1	1
		$T=1/2$	$S=\pm 1$		
		$T=3/2$	$S=\pm 1$		

[9] J. Wess, Nuovo cimento **10**, 15 (1960).

TABLE II. Predicted bosons.

		$J=1^-$	Mass/m_π		$J=0^-$	Mass/m_π
A.	3	ζ^a	4.2	A.	π	1
B.	8	K'	6.4	B.	K	3.4
		ρ^b	5.6		η^d	4.0 ($G=1, T=0, S=0$)
		ω^c	5.6		π'	? ($G=-1, T=1, S=0$)
C.	24 at higher mass			C.	24 at higher mass	

[a] R. Barlout and J. Heughebaert, A. Leveque, J. Meyer, and R. Omnes *et al.*, Phys. Rev. Letters **8**, 32 (1961).
[b] A. R. Erwin, R. March, W. D. Walker, and E. West, Phys. Rev. Letters **6**, 628 (1961).
[c] B. Maglić *et al.*, Phys. Rev. Letters **7**, 178 (1961).
[d] A. Pevsner *et al.*, Phys. Rev. Letters **7**, 421 (1961).

In the last two columns we have listed the number $G=C\exp(i\pi T_2)$ for the bosons with strangeness zero in the case of pseudoscalar and vector space time properties.

Let us now assume that we have one set of 35 $J=1^-$ states and also a set of 35 $J=0^-$ states, each broken into 3+8+24 as indicated in Table I. We very tentatively assign in Table II the states to the elementary particles and resonances.

Aside from the 48 states at higher mass we have here only one presently unclaimed particle, π'. We would emphasize that this tabulation according to the splitting of the 35 mesons predicted from SU_6 is only suggestive. There is room in our smaller group, $SU_3\otimes SU_2'$, for isolated boson states of arbitrary isotopic spin and strangeness zero; these correspond to identity representations of SU_3 combined with some representation of SU_2'.

3. $J=3/2$, P-WAVE RESONANCES

The minimum dimensionality of a representation of $SU_3\otimes SU_2'$ which contains a $T=3/2$, $S=0$ state, with baryon number one, is 12. The multiplet may be filled out in either of two ways. The first and most interesting way is to use the representation $6\otimes 2'$. The six-dimensional representation of SU_3 is discussed in references 5 and 9. This is to be compounded with the spin 1/2 representation of SU_2'. The content of this twelve-dimensional representation is given in Table III All these are observed except the last. We shall return to this last $N_{1/2}^*$ in the next section.

The other twelve-dimensional representation is $3\otimes 4'$ and contains,

$$\begin{aligned} &N^*, \quad T=3/2, \quad S=0; \\ &Y_1^*, \quad T=1 \quad\;\; S=-1; \\ &Y_2^*, \quad T=2, \quad S=-1. \end{aligned}$$

This set corresponds to the global symmetry prediction of Lee and Yang.[4]

We tentatively accept the set of Table III which includes the cascade particle that was left out of the baryon sextet.

TABLE III. $J=3/2$ resonances.

Name	Quantum numbers		Mass/m_π
$N_{3/2}^*$	$T=3/2$	$S=0$	9.0
Y_1^*	$T=1$	$S=-1$	10.0[a]
Y_0^*	$T=0$	$S=-1$	11.0[b]
Ξ	$T=1/2$	$S=-2$	9.4
$N_{1/2}^*$	$T=1/2$	$S=0$	??

[a] M. H. Alston and M. Ferro-Luzzi, Revs. Modern Phys. **33**, 416 (1961).
[b] M. Ferro-Luzzi, R. D. Tripp, and M. B. Watson, Phys. Rev. Letters **8**, 28 (1962).

4. SELECTION RULES

We do not expect reliable selection rules from our symmetry, which is surely strongly broken. Our program is merely to classify the mass multiplets. However, one comment should be made; until $SU_3 \otimes SU_2'$ is broken, all those resonances belonging to the eight-dimensional representations are forbidden to decay into pions. This is because from the π representation $1\otimes 3'$ we may never construct the resonance representation $8\otimes 1'$. Also, N^*, Y^* are stable against pion decay in the presence of the symmetry since $6\otimes 2'$ is not contained in the product of $3\otimes 2'$ with $1\otimes 3'$. However, if the mass of $N_{T=\frac{1}{2}}^*$ is sufficiently high it has an allowed decay into $N+\eta$. Hence, we may expect that the decay of $N_{T=\frac{1}{2}}^*$ will be primarily into $N+3\pi$ and that its effect on elastic pion-nucleon scattering may therefore be small.

5. THE LARGER GROUP

In the context of a theory in which the mesons are bound baryon, antibaryon pairs we want to make some comments on the significance of our two stages of symmetry SU_6 and $SU_3\otimes SU_2$. It is worth remarking, to begin with, that 1^- and 0^- are the quantum numbers of baryon, antibaryon pairs in triplet and singlet S states, so that if the spin and parity assignments of Table II are correct, a bound-state picture is an attractive one.

Suppose then that the strongest force in the baryon-antibaryon system, which binds the mesons in the first place, has the full symmetry SU_6. Then we predict, aside from a possible singlet, one group of 35 degenerate bosons. Next let us assume a somewhat weaker interaction with only the symmetry $SU_3\otimes SU_2$. This will cause gross splittings between the three sets of multiplicity 3, 8, and 24. If, following an obvious superstition, we order these sets in such a way that mass increases with multiplicity, we obtain qualitatively the situation shown in Table II. We predict, therefore, that the remaining 48 pseudoscalar and vector bosons will indeed be found at higher energy.

In the final stage of breakdown of symmetry (still in the domain of strong interactions) all degeneracies are removed except those implied by isotopic spin conservation and by charge conjugation. In this stage the resonances comprising the octets are allowed to decay into pions.

6. OTHER POSSIBILITIES

Obviously if we give up the demand that there be only one set of 0^- bosons and one set of 1^- bosons and we give up, in addition, the requirement that the boson representations be contained in $B^*\times B$ we have much more freedom to fit future experimental developments. In particular, our symmetry $SU_3\otimes SU_2$ will accommodate any $S=0$ boson. The consequences of the theory will then be merely the prediction of $T=0$, $S=0$ and $T=1$, $S=0$ counterparts of K and of K', plus the classification of hyperon and nucleon resonances as in Table III.

We mention here only one specific variant. This has to do with the unknown parities of strange particles. We must, of course, assume equal Λ and Σ parity in order to have our group in the first place. Since the K is in a different representation from the pion, however, we are free to make it even parity (with respect to the $N\Lambda$ pair). In this case the $T=0$ particle associated with K will have a two-pion decay, the $T=1$ particle a radiative decay or a forbidden two-pion decay. These possibilities are not ruled out by experiment as yet. If the K is of even parity then our resonance from Table III, Y_0^*, will have a $D_{3/2}$ decay into $\bar{K}+N$.

7. CONCLUSIONS

Our symmetry has one obvious superiority over many other proposed schemes; it allows the prediction of interesting multiplets without demanding that particles with known large mass splittings belong to the same multiplet. For example, π and K are in different multiplets but we still can unite ρ, ω, and K' in a single one. Likewise the grouping of the cascade particle with the resonances is favorable from the standpoint of minimizing the mass differences within a single representation.

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