

# Electron Capture and Nuclear Matrix Elements of $\text{Be}^7$ †

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We calculate the  $\text{Be}^7$  electron capture rate from continuum orbits as a function of electron temperature and concentration and consider the effect on the decay rate, for some typical stellar conditions, of the statistical distribution of electrons, the electron-nucleus Coulomb interaction, relativistic and nuclear size corrections, the imperfect overlap between initial and final atomic states, and electron screening in bound decay. We also analyze the experimental information on  $\text{Be}^7$  and  $\text{Li}^7$  to obtain accurate experimental Gamow-Teller matrix elements for both the ground- and excited-state transitions and compare these matrix elements with the predictions of some simple nuclear models. This comparison supports the conclusion of other authors that near the beginning of the  $1p$  nuclear shell  $L$ - $S$  coupling is better satisfied than  $j$ - $j$  coupling.

## I. INTRODUCTION

THE large value for the  $\text{He}^3(\alpha, \gamma)\text{Be}^7$  cross section found experimentally by Holmgren and Johnston<sup>1</sup> led to studies by Fowler<sup>2</sup> and Cameron<sup>3</sup> of the possibility of completing the proton-proton chain with reactions involving  $\text{Be}^7$ . The latter authors<sup>2,3</sup> showed that the proton-proton chain in the sun is completed more frequently through the  $\text{He}^3(\alpha, \gamma)\text{Be}^7$  reaction than through the  $\text{He}^3(\text{He}^3, 2p)\text{He}^4$  reaction, in regions of high temperature and high helium-to-hydrogen ratio. They emphasized that the rate of energy generation and the effective energy release by the proton-proton chain depend critically on the ratio of proton-capture lifetime to electron-capture lifetime in  $\text{Be}^7$ . Fowler<sup>2</sup> and Cameron<sup>3</sup> also showed that the possibility of detecting on earth neutrinos emitted by the sun or other stars depends sensitively on the ratio of the  $\text{Be}^7$  proton-capture lifetime to the electron-capture lifetime. Davis and his collaborators<sup>4</sup> are currently performing an experiment to determine if the solar neutrino flux is detectable by the inverse electron-capture process  $\text{Cl}^{37}(\nu, e^-)\text{Ar}^{37}$ .

The present work provides an accurate formula for the rate of capture of continuum electrons by  $\text{Be}^7$  as a function of electron density and temperature.<sup>5</sup> In deriving this formula, we consider the statistical distribution of the electrons, the electron-nucleus Coulomb interaction, relativistic and nuclear size corrections, the imperfect overlap between initial and final atomic states, and electron screening in bound decay. For temperatures of the order of  $2 \times 10^7 \text{K}$ , our results are in good agreement (14%) with a previous calculation by Cameron,<sup>3</sup> but at higher temperatures it is necessary to

consider more accurately the electron-nucleus Coulomb interaction.

Using Hartree-Fock atomic wave functions and accurately determined experimental parameters, we analyze existing information to obtain the experimental Gamow-Teller nuclear matrix elements that govern the  $\text{Be}^7$  decay. We estimate that the experimental value thus determined for the ground-state (excited-state) transition matrix element is accurate to 10% (15%). We calculate the Gamow-Teller matrix elements on the basis of some simple nuclear models and compare with the experimental values. The results support the conclusion by other authors<sup>6</sup> that near the beginning of the  $1p$  nuclear shell  $L$ - $S$  coupling is better satisfied than  $j$ - $j$  coupling.

## II. ELECTRON CAPTURE FROM CONTINUUM ORBITS

The allowed continuum electron capture rate for a single nucleus of charge  $Z$ , nuclear energy release  $W_0$ , corresponding to one electron in a volume  $V$  is<sup>7,8</sup>

$$\lambda = \frac{G_V^2(W_0 + W)^2 \xi F(Z, W)}{2\pi V}, \quad (1)$$

where  $W$  is the total energy of the electron and  $\xi$  is the usual allowed combination of nuclear matrix elements,<sup>9</sup>

$$\xi \equiv \langle 1 \rangle^2 + (C_A^2/C_V^2) \langle \sigma \rangle^2. \quad (2)$$

The function  $F(Z, W)$  is the ratio of the electron density at the nucleus calculated with a Coulomb distorted wave to the density calculated with a plane wave;  $G_V$  is the usual beta-decay coupling constant.

Assuming that the electrons in a star constitute a Fermi gas in which interactions among the electrons can

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<sup>1</sup> H. D. Holmgren and R. L. Johnston, *Phys. Rev.* **113**, 1556 (1959).

<sup>2</sup> W. A. Fowler, *Astrophys. J.* **127**, 551 (1958).

<sup>3</sup> A. G. W. Cameron, Atomic Energy of Canada, Limited Report, CRL-41, 1958 (unpublished), 2nd ed. See also A. G. W. Cameron, *Ann. Rev. Nuclear Sci.* **8**, 299 (1958).

<sup>4</sup> R. Davis, Jr. (private communication).

<sup>5</sup> At sufficiently low stellar temperatures, not all  $\text{Be}^7$  nuclei will be completely ionized and electron capture from bound orbits must be considered. See reference seven for the combined capture rate formula and for some indication of the temperatures at which bound capture becomes important.

<sup>6</sup> A. M. Lane, *Proc. Phys. Soc. (London)* **A66**, 977 (1953); D. Kurath, *Phys. Rev.* **101**, 216 (1956). See also J. P. Elliott and A. M. Lane, in *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 241.

<sup>7</sup> J. N. Bahcall, *Phys. Rev.* **126**, 1143 (1962).

<sup>8</sup> We use units throughout this paper in which  $\hbar = m = c = 1$ .

<sup>9</sup> All symbols have their usual meaning. For definitions, see E. J. Konopinski, *Ann. Rev. Nuclear Sci.* **9**, 99 (1959). We use the definition of reduced matrix elements suggested by Konopinski,

$$\langle I'(M') | S_{Jm}^+ | I(M) \rangle \equiv \langle I'(M') J(m) | I(M) \rangle \langle S_J \rangle.$$

be neglected, the rate of capture of electrons from continuum orbits is<sup>7</sup>

$$\lambda = G_V^2 \xi K / 2\pi^3, \quad (3)$$

where the dimensionless quantity  $K$  is given by

$$K \equiv \int_0^\infty dp p^2 \frac{F(Z, W)(W_0 + W)^2}{1 + \exp(-\nu + W/kT)}. \quad (4)$$

If Boltzmann statistics are valid, and if  $2\pi\alpha Z \langle v^{-1} \rangle_{av} \gg 1$ ,  $\alpha^2 Z^2 \ll 1$ , and  $kT \ll mc^2$ , the statistical factor  $K$  can be evaluated approximately; the result is<sup>7,10</sup>

$$K_B \cong \pi\alpha Z n_e (2\pi)^{3/2} (kT)^{-1/2} q^2 (1 - (15/8)kT) \times [(1 + 2\mu_0 + 2\mu_0^2) + (1 + x_m/q)^2 (2x_m)^{5/4} e^{-f(x_m)} / (3\alpha Z k^2 T^2)^{1/2}], \quad (5a)$$

$$x_m \equiv (2^{-1}) [2\pi\alpha Z kT]^2, \quad (5b)$$

$$f(x) = (x/kT) + \pi\alpha Z (2/x)^{1/2}, \quad (5c)$$

and

$$\mu_0 = kT/q. \quad (5d)$$

The quantity  $q$  is the energy of the neutrino emitted in the decay process and is defined by the relation

$$q = W_0 + W \cong W_0 + 1. \quad (6)$$

Equations (5) differ from the corresponding Eqs. (17) in reference 7; everywhere Eqs. (17) in reference 7 contain  $W_0$ , Eqs. (5) contain  $q$ . Moreover, the error function that appears in reference 7 has been replaced by its asymptotic value for small temperatures and/or large nuclear charge.

Equations (5) were obtained by making use of the approximation<sup>7</sup>

$$F(Z, W) \cong 2\pi\eta [1 + \exp(-2\pi\eta)], \quad (7)$$

where

$$\eta \equiv \alpha Z v^{-1}. \quad (8)$$

The term in Eq. (5a) involving  $x_m$  arose from the term  $\exp(-2\pi\eta)$  in Eq. (7). Thus the term involving  $x_m$  is of the order of<sup>11</sup>

$$\exp(-2\pi\alpha Z v^{-1}) \cong \exp[-2\pi\alpha Z (2/\pi kT)^{1/2}]. \quad (9)$$

The central temperature in the sun is less than or of the order of  $1.5 \times 10^7$  °K,<sup>12</sup> and thus the correction term (9) is about 5% for Be<sup>7</sup> electron capture in the sun.<sup>13</sup> Hence we can omit for Be<sup>7</sup> the term involving  $x_m$  in

<sup>10</sup> Equations (5) are valid for low temperatures and/or large nuclear charge, since the formulas were obtained by using Eq. (7). The term involving  $x_m$  is not given very accurately by Eq. (5); if this term is appreciable, then numerical integration is probably necessary in order to obtain an accurate value for  $K$ .

<sup>11</sup> Formula (12c) is a general result for systems obeying Maxwell-Boltzmann statistics.

<sup>12</sup> R. L. Sears, *Mém. soc. roy. sci. Liege* **3**, 479 (1960).

<sup>13</sup> Equation (5a) yields a similar estimate for the correction term.

Eq. (5a), although this term must be included in order to obtain accurate results for the electron-capture lifetime of hydrogen or helium, Equation (5a) can be further simplified by observing that  $kT$  is less than or of the order of 1 keV for the sun while  $q$  is 863(385) keV for the ground-state (excited-state) decay of Be<sup>7</sup>. Thus we may neglect  $kT$  everywhere with respect to  $q$  in Eq. (5a). The dimensionless quantity  $K$  can then be written in the form

$$K_B \cong \pi\alpha Z n_e (2\pi)^{3/2} (kT)^{-1/2} q^2, \quad (10)$$

and the continuum electron-capture rate is given simply by

$$\lambda = (2/\pi kT)^{1/2} G_V^2 \alpha Z n_e q^2 \xi. \quad (11)$$

Expression (11) for the allowed electron-capture rate in a nondegenerate Fermi gas of electrons can be obtained heuristically in a simple way by substituting in Eq. (1)

$$n_e = 1/V, \quad (12a)$$

$$F(Z, W) \cong 2\pi\alpha Z \langle v^{-1} \rangle, \quad (12b)$$

and<sup>11</sup>

$$\langle v^{-1} \rangle = (2/\pi kT)^{1/2}. \quad (12c)$$

Formula (11) is then obtained immediately.

### III. BOUND ELECTRON CAPTURE

The allowed capture rate for an electron bound in an atomic orbit with quantum numbers<sup>14</sup>  $n, \kappa, \mu$  is

$$\lambda_{n, \kappa, \mu} = G_V^2 q^2 \xi |\psi_{n, \kappa}(0)|^2 / 2\pi, \quad (13)$$

where<sup>14,15</sup>

$$|\psi_{n, -1}(0)|^2 = (4\pi)^{-1} g_{n, -1}^2(0), \quad (13a)$$

$$|\psi_{n, +1}(0)|^2 = (4\pi)^{-1} f_{n, -1}^2(0), \quad (13b)$$

The quantity  $\Delta b_{n, \kappa}$  is the change in the atomic binding energy when an electron with quantum numbers  $n, \kappa$  is captured; this binding energy change is negligible for Be<sup>7</sup>. Only electrons with  $\kappa$  equal to plus or minus one undergo allowed decay; electrons with  $\kappa$  different from one possess orbital angular momentum and hence have their decay rates retarded by centrifugal repulsion. The form of the functions  $f$  and  $g$  is well known.<sup>14</sup>

Equation (13) neglects the difference between initial and final atomic states due to the change of nuclear charge by one unit. This effect has been studied for Be<sup>7</sup> by Benoist-Gueutal<sup>16</sup> who concludes that the Be<sup>7</sup> capture rate might be decreased by as much as 34% compared to the value given in Eq. (13), due to the imperfect overlap of initial and final atomic states. The estimate of Benoist-Gueutal only indicates the need for further study, since no accurate atomic wave functions were available for the excited states of Li<sup>7</sup>. The Li<sup>7</sup>

<sup>14</sup> We use the notation of M. E. Rose, *Relativistic Electron Theory* (John Wiley & Sons Inc., New York, 1961) for relativistic electron wave functions and quantum numbers.

<sup>15</sup> J. N. Bahcall, *Phys. Rev.* **124**, 495 (1961).

<sup>16</sup> P. Benoist-Gueutal, *Ann. Phys. (New York)* **8**, 593 (1953).

atom is usually left in the 1s2s<sup>2</sup> excited state following Be<sup>7</sup> electron capture.

The present writer has reinvestigated the effect on weak interaction decay rates of a different nuclear charge in initial and final atomic states. Our method differs from the one used by Benoist-Gueutal and has been briefly described in a previous publication.<sup>15</sup> Preliminary unpublished results by the present writer suggest that the imperfect overlap between individual atomic states does not inhibit the total Be<sup>7</sup> decay rate by nearly as much as the 34% upper bound estimated by Benoist-Gueutal.<sup>17</sup> The numerical results presented in this paper ignore the imperfect overlap between initial and final atomic states since the present writer's estimate for the smallness of this effect is not contradicted by the experimental comparisons described in Sec. V of this paper.

In terrestrial experiments, Be<sup>7</sup> decays by allowed electron capture to both the ground and the first excited states of Li<sup>7</sup>. We denote by  $q$  ( $q^*$ ),  $\xi$  ( $\xi^*$ ), and  $\lambda$  ( $\lambda^*$ ), the ground- (excited-) state neutrino energy, nuclear matrix elements, and transition probability respectively. The laboratory transition probability can then be written,

$$\lambda_{\text{lab}} = \lambda + \lambda^* = G_V^2 A (q^2 \xi + q^{*2} \xi^*), \quad (14)$$

where

$$A = (4\pi^2)^{-1} [g_{1,-1}^2(0) + g_{2,-1}^2(0)]. \quad (15)$$

#### IV. NUMERICAL RESULTS

In this section, we present some numerical results for the Be<sup>7</sup> electron capture rate from continuum orbits under typical stellar conditions. The stellar capture rate can be written as a sum of two terms, analogous to Eq. (14) for the laboratory decay rate, by making use of Eq. (10). Forming the ratio of  $\lambda_{\text{star}}$  and  $\lambda_{\text{lab}}$ , we obtain

$$\lambda_{\text{star}}/\lambda_{\text{lab}} = \tau_{\text{lab}}/\tau_{\text{star}} = A^{-1} (2/\pi kT)^{1/2} \alpha Z n_e. \quad (16)$$

Since the laboratory half-life,  $\tau_{\text{lab}}$ , is accurately known, the atomic factor  $A$  is the only quantity that must still be determined before Eq. (16) can be used. Brysk and Rose<sup>18</sup> have shown that nuclear size effects on the wave functions (13a) and (13b) are negligible for nuclei as small as Be<sup>7</sup>. Relativistic effects can easily be estimated by examining the form of  $g$  appropriate to a pure Coulomb field. The Dirac  $g_{1,-1}$ , correct to terms of order  $\alpha^2 Z^2$ , has the value<sup>19</sup> (at the nuclear radius  $R$ )

$$g_{1,-1}(R) \cong 2(\alpha Z)^{3/2} e^{-x} \{1 + \alpha^2 Z^2 [(5/4) - \gamma - \ln 2x]\}, \quad (17)$$

<sup>17</sup> It is hoped that a complete account of this work will be available in the near future.

<sup>18</sup> H. Brysk and M. E. Rose, *Revs. Modern Phys.* **30**, 1169 (1958).

<sup>19</sup> D. Layzer and J. Bahcall, *Ann. Phys. (New York)* **17**, 177 (1962).

where

$$x = \alpha Z R,$$

and  $\gamma$  is the Euler-Mascheroni constant. The dependence of  $g_{1,-1}(R)$  on the nuclear radius is weak and relativistic corrections, represented by the terms proportional to  $\alpha^2 Z^2$ , are much less than one percent. Similar results are valid for  $g_{2,-1}(R)$ .

Electronic screening causes the beryllium atomic wave function to differ appreciably at the origin from the value computed using a pure Coulomb field. Hartree and Hartree<sup>20</sup> computed accurate beryllium radial wave functions using the method of the self-consistent field with exchange; they found (atomic units):

$$g_{1,-1}(0) = 14.67, \quad (18a)$$

$$g_{2,-1}(0) = 2.67. \quad (18b)$$

The value of  $g_{1,-1}(0)$  given above is probably accurate to better than 1%. It differs by much less than 1% from the  $g_{1,-1}(0)$  computed by Hartree and Hartree<sup>21</sup> using a self-consistent field without exchange and also agrees to better than 1% with their<sup>21</sup>  $g_{1,-1}(0)$  computed, ignoring exchange, for Be<sup>++</sup>. The values of  $g_{1,-1}(0)$  computed by Hartree and Hartree for neutral Be, including exchange, and for Be<sup>++</sup>, ignoring exchange, agree to within a few tenths of a percent with the value determined by Pekeris<sup>22</sup> for Be<sup>++</sup> using an elaborate numerical technique.

The electron density calculated from Eq. (18a) is 16% less than the pure Coulomb value,

$$[g_{1,-1}^2(0)]_{\text{coulomb}} \equiv 4Z^3 = 256. \quad (19)$$

The fact that  $g_{1,-1}(0)$  is essentially the same for Be and for Be<sup>++</sup> shows that the 16% decrease in the self-consistent field decay rate compared to the pure Coulomb decay rate is due to the mutual screening of the two 1s electrons. The ratio of  $L$  to  $K$  capture determined solely from Eqs. (18) is  $3.3 \times 10^{-2}$ ; this is much less than the pure Coulomb ratio of  $12.5 \times 10^{-2}$ , although it has frequently been stated that for light atoms the  $L$  to  $K$  capture ratio is given accurately by pure Coulomb wave functions.

The value<sup>8</sup> of  $A$  computed from Eqs. (18) is  $2.19 \times 10^{-6}$ ; this estimate of  $A$  is probably accurate to one percent. The half-life  $\tau_{\text{lab}}$  is  $(4.61 \pm 0.01) \times 10^6$  sec.<sup>23</sup> Substituting these numbers in Eq. (16), we find:

$$\tau_{\text{star}}^{-1} \equiv \lambda_{\text{star}}/\ln 2 = 1.02 n_e T_6^{-1/2} \times 10^{-32} \text{ sec}^{-1}, \quad (20)$$

<sup>20</sup> D. R. Hartree and W. Hartree, *Proc. Roy. Soc. (London)* **A150**, 9 (1935).

<sup>21</sup> D. R. Hartree and W. Hartree, *Proc. Roy. Soc. (London)* **A149**, 210 (1935).

<sup>22</sup> C. L. Pekeris, *Phys. Rev.* **112**, 1649 (1958).

<sup>23</sup> F. Aijzenberg-Selove and T. Lauritsen, *Nuclear Phys.* **11**, 1 (1959).

TABLE I. Experimental parameters.

Quantity	Value <sup>a</sup>	Quantity	Value <sup>a</sup>
$q^2$	2.85	$G_V^2$	$(8.82 \pm 0.12) \times 10^{-24}$
$(q^*)^2$	0.568	$C_A^2/C_V^2$	$1.41 \pm 0.05$
$\lambda^*/\lambda$	$0.130 \pm 0.013$	$A$	$2.19 \times 10^{-6}$
$\tau_{1/2}$	$3.58 \times 10^{+27}$		

<sup>a</sup> We use units in which  $\hbar = m = c = 1$ . Except where shown explicitly, experimental errors are assumed negligible.

where  $\tau_{\text{star}}$  is the half-life for capture of an electron by a  $\text{Be}^7$  nucleus,  $n_e$  is the number of electrons per  $\text{cm}^3$ , and  $T_6$  is the temperature in units of  $10^6$  °K.

Let

$$n_e \equiv \rho / (\mu_e m_p), \quad (21)$$

where  $\rho$  is the density of the stellar matter in  $\text{g}/\text{cm}^3$  and  $m_p$  is the mass of the proton. Then,

$$\tau_{\text{star}}^{-1} = 6.12 \times 10^{-9} \rho / \mu_e T_6^{+1/2} \text{ sec}^{-1}. \quad (22)$$

Equations (20) and (22) are accurate to 1 or 2% if we ignore higher order correction terms in the expansion<sup>24</sup> of  $F(Z, W)$  and the imperfect overlap of initial and final atomic states.

Equation (22) is in good agreement with a heuristic estimate given by Cameron.<sup>3</sup> His result differs by only 14% from the value given by Eq. (22), the difference being due almost entirely to the neglect of electron screening in his calculation of  $A$ . For temperatures of the order of the central temperature in the sun, i.e.,  $T_6$  about 15, Eq. (29) yields a lifetime three times shorter than the order-of-magnitude estimate derived by Bethe and used by Fowler<sup>2</sup> in his estimates of the solar neutrino flux. As a first approximation, Bethe assumed that  $F(Z, W)$  was equal to one; it is about three for solar temperatures.

## V. NUCLEAR MATRIX ELEMENTS

We can extract experimental values for the nuclear matrix elements involved in  $\text{Be}^7$  electron capture by making use of the parameters listed in Table I. The values for  $q$ ,  $q^*$ ,  $\lambda^*/\lambda$ , and  $\tau_{1/2}$  were taken from the compilation of Ajzenberg-Selove and Lauritsen.<sup>23</sup> The value of  $G_V^2$  has recently been determined very accurately<sup>25</sup>; our knowledge of  $C_A/C_V$  is much less precise.<sup>26</sup> The value of  $A$  adopted here has been discussed in Sec. III. Unless explicitly shown in Table I, it is assumed that experimental uncertainties are unimportant in our considerations.

<sup>24</sup> For temperatures of the order of  $2 \times 10^7$  °K, these terms will decrease the predicted lifetime by about 5%, as we have seen in Sec. II. At lower temperatures, the correction terms in the expansion of  $F(Z, W)$  are completely negligible. For low densities and high temperatures, such as exist in our sun, electron screening in continuum orbits will be much less than in bound atomic orbits.

<sup>25</sup> R. K. Bardin, C. A. Barnes, W. A. Fowler, and P. A. Seeger, *Phys. Rev.* **127**, 583 (1962); J. W. Butler and R. O. Bondelid, *ibid.* **121**, 1770 (1961).

<sup>26</sup> A. Sosnovskii, P. Spivak, Yu. Prokoviev, I. Kutikov, and Yu. Dobrynin, *Soviet Phys.—JETP* **35**, 739 (1959).

The experimental nuclear matrix elements can be computed from the following version of formula (13),

$$\begin{aligned} \xi &= \langle 1 \rangle^2 + (C_A^2/C_V^2) \langle \sigma \rangle^2 \\ &= \ln 2 [G_V^2 q^2 A \tau_{1/2} (1 + \lambda^*/\lambda)]^{-1}, \end{aligned} \quad (23)$$

and an analogous expression for  $\xi^*$ . For mirror nuclei, it is an excellent approximation to assume that  $\langle 1 \rangle = 1$  and thus Eq. (23) can be used to obtain an experimental value for the reduced Gamow-Teller matrix element  $\langle \sigma \rangle$ ; the Fermi matrix element  $\langle 1 \rangle^*$  vanishes for the excited state decay. The experimental matrix elements are shown in the second column of Table II; the largest uncertainty arises from the estimated experimental error in the determination of  $\lambda^*/\lambda$ . The estimated value of excited state Gamow-Teller matrix element  $\langle \sigma \rangle^*$  depends strongly upon the branching ratio  $\lambda^*/\lambda$ ; thus  $\langle \sigma \rangle^*$  is not as well known as  $\langle \sigma \rangle$ .

Table II also lists several theoretical predictions of the Gamow-Teller matrix elements. The third column of Table II contains the single-particle  $j$ - $j$  coupling values. A number of authors<sup>27</sup> have emphasized that the correct  $j$ - $j$  wave functions to use for light nuclei are eigenfunctions of the total angular momentum  $J$  and the total isotopic spin  $T$ . Thus column four lists values computed with eigenfunctions of the total isotopic spin; the entries in this column are explained in more detail below. The supermultiplet theory of Wigner<sup>28</sup> predicts the same matrix elements as the pure single-particle values; the supermultiplet predictions are shown in column five.

The ground state of  $\text{Be}^7$  has a spin of  $3/2$  and a total isotopic spin of  $1/2$ ; the ground-state configuration is assumed to be (in the notation of Mayer and Jensen<sup>29</sup>):  $(\pi p_{3/2})^2 (\nu p_{3/2})$ . The ground state of  $\text{Li}^7$  has the same total spin and isotopic spin and an analogous configuration. Mayer and Jensen have constructed the isotopic spin wave function in  $j$ - $j$  coupling for the  $\text{Be}^7$  and  $\text{Li}^7$  ground states.

The first excited state of  $\text{Li}^7$  has a total spin and isotopic spin both equal to one-half; the configuration of the excited state may be either  $(\nu p_{3/2})^2 (\pi p_{3/2})$  or  $(p_{3/2})^2 (p_{1/2})$ , according to the shell model.<sup>29</sup> The  $\text{Li}^{7*}$  configuration involving  $(p_{1/2})$  can be described by two independent wave functions which correspond to the two  $p_{3/2}$  nucleons being coupled to an intermediate angular momentum of zero or one. The antisymmetrized eigenfunctions of total spin and isotopic spin can be found by the method of Mayer and Jensen<sup>29</sup> or by direct construc-

<sup>27</sup> See, for example, M. Mizushima and M. Umezawa, *Phys. Rev.* **85**, 37 (1952) and B. H. Flowers, *Phil. Mag.* **43**, 1330 (1952). The first entry in column four of Table II has also been obtained by A. Winther and O. Høfoed-Hansen, *Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd.* **27**, no. 14 (1953). The latter authors pointed out the necessity of using isotopic spin eigenfunctions for the calculation of Gamow-Teller matrix elements.

<sup>28</sup> E. P. Wigner, *Phys. Rev.* **56**, 519 (1939).

<sup>29</sup> M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley & Sons, Inc., New York, 1953).

TABLE II. Comparison of experimental and theoretical nuclear matrix elements.

Matrix element	Experimental	Single-particle $j-j$	Isotopic spin $j-j$ <sup>a</sup>	Supermultiplet theory
$\langle\sigma\rangle^2$	$1.48\pm 0.15$	1.67	0.90	1.67
$\langle\sigma\rangle^{*2}$	$1.42\pm 0.22$	1.33	$(p_{3/2})^3: 0.44$ $J_{in}=0(p_{1/2}): 0.19$ $J_{in}=1(p_{1/2}): 0.12$	1.33

<sup>a</sup> Entries in this column are explained in the text.

tion. The excited state Gamow-Teller matrix element was computed using each of the three possible  $j-j$  isotopic spin wave functions; the results are listed below.

$$(\nu p_{3/2})^2(\pi p_{3/2}): \quad \langle\sigma\rangle^{*2}=4/9; \quad (24a)$$

$$(p_{3/2})^2_{J_{in}=0}(p_{1/2}): \quad \langle\sigma\rangle^{*2}=5/27; \quad (24b)$$

$$(p_{3/2})^2_{J_{in}=1}(p_{1/2}): \quad \langle\sigma\rangle^{*2}=12/100. \quad (24c)$$

The configuration label indicates which of the three Li<sup>7\*</sup>  $j-j$  eigenfunctions was used; the quantity  $J_{in}$  gives the value of the intermediate angular momentum to which the two  $p_{3/2}$  nucleons are coupled in the  $(p_{3/2})^2(p_{1/2})$  configuration. A similar notation has been used in Table II.

The  $p_{1/2}$  configuration yields in both cases a smaller value of  $\langle\sigma\rangle^{*2}$  than the  $(p_{3/2})^3$  configuration. This is because the orthogonality of the  $p_{3/2}$  and  $p_{1/2}$  single-particle wave functions permits only parts of the Be<sup>7</sup> and Li<sup>7\*</sup> wave functions to contribute to the matrix element if the Li<sup>7\*</sup> state contains, unlike the Be<sup>7</sup> state, a  $p_{1/2}$  single-particle component.

The trend of the numbers in Table II is in agreement with results of extensive intermediate coupling calculations<sup>6</sup> of other nuclear parameters for light nuclei. The intermediate coupling calculations show that  $L-S$  coupling is better satisfied than  $j-j$  coupling for nuclei near the beginning of the  $1p$  nuclear shell.<sup>30</sup> This fact

<sup>30</sup> C. W. Kim has confirmed this result by analyzing the  $ft$  values of light even- $A$  nuclei (private communication).

is reflected in Table II by the excellent agreement between the supermultiplet predictions and the experimental values; the isotopic spin  $j-j$  eigenfunctions yield matrix elements that are too small. It would be interesting to see how well the intermediate coupling wave functions that have been used to compute other nuclear parameters for mass number seven reproduce the experimental Gamow-Teller matrix elements.

If the imperfect overlap between initial and final atomic states caused the atomic factor  $A$  to be decreased significantly from the value adopted in Sec. III, the theoretical matrix elements listed in Table II would appear to be too small when compared with the experimental values. However, the agreement between theory and experiment is similar to the agreement found for nuclear magnetic moments,<sup>27,29</sup> and for magnetic moments there is, of course, no atomic overlap correction. We conclude that there is no evidence for a large decrease in the total electron capture probability due to an imperfect overlap between initial and final atomic states. An intermediate coupling calculation with the same wave functions that were used to calculate the Be<sup>7</sup> and Li<sup>7</sup> magnetic moments would allow one to place quantitative limits on the decrease in  $A$ , and thus the total electron capture probability, due to the imperfect atomic overlap.

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*Note added in proof.* R. W. Kavanagh [Nuclear Phys. **15**, 411 (1960)] measured the Be<sup>7</sup>( $p,\gamma$ )B<sup>8</sup> cross section and W. A. Fowler [Mem. Soc. Roy. Liege **3**, 207 (1960), Ser. 5] used Kavanagh's results to show that proton capture by Be<sup>7</sup> is rare in the sun.