Adair Spin Analysis with Parity Nonconservation

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A variation of the Adair spin analysis is considered for an unstable boson produced in association with a hyperon whose decay violates parity conservation.

THE breakdown of parity conservation in certain hyperon decay reactions, e.g., $\Lambda\to p+\pi^-$, provides a convenient tool for the measurement of hyperon polarization. We discuss the possibility of exploiting this in a variation of the familiar Adair method[1] for determining the spins of unstable particles or resonances which are produced in association with hyperons. In particular, consider a reaction in which a spinless particle collides with a nucleon to produce a hyperon and an unstable boson; suppose that the boson has definite spin l and decays into two spinless particles, e.g.,

$$\pi^-+p\to K^{*0}+\Lambda^0,\quad K^{*0}\to K^++\pi^-;$$
$$K^-+p\to\rho^0+\Lambda^0,\qquad \rho^0\to\pi^++\pi^-.$$

Let $\mathbf{q}$ be a unit vector along the decay axis in the boson rest frame; and let $\mathbf{p}$ be a unit vector along the direction of the pion produced in the hyperon decay, as referred to the rest frame of the hyperon. Finally, let $\mathbf{k}$ and $\mathbf{k}'$ be unit vectors along the incident and outgoing beams, in the over-all barycentric system. The general connection between the spin l of the unstable boson and the distribution in angle of $\mathbf{q}$ is contained in the statement that the angular distribution involves spherical harmonics of order no larger than $2l$ and is invariant under $\mathbf{q}\to-\mathbf{q}$. Any departure from isotropy in $\mathbf{q}$ implies nonvanishing spin, and a lower limit on the spin is set by the order of the highest spherical harmonic which appears in the angular distribution. The point is this: The angular distribution, in general, involves harmonics up to and including the order $2l$, but the coefficients in the general case depend on dynamical details and could, some of them, be nearly or completely vanishing.

One virtue of parity nonconservation in hyperon decay is simply that it gives rise to a variety of correlation terms which can in principle be determined separately, thus increasing the chances that some spherical harmonic of the limiting order $2l$ will show up. The general structure of the distribution in $\mathbf{q}$, $\mathbf{p}$, $\mathbf{k}'$ is given by

$$f_l(\mathbf{q},\mathbf{p},\mathbf{k}',\mathbf{k})=a_l+b_l\mathbf{p}\cdot(\mathbf{q}\times\mathbf{k})+c_l\mathbf{p}\cdot(\mathbf{q}\times\mathbf{k}')+d_l\mathbf{p}\cdot(\mathbf{k}\times\mathbf{k}'),$$

where the functions a_l, b_l, c_l, d_l depend on the scalar product variables $\mathbf{k}'\cdot\mathbf{k},\mathbf{q}\cdot\mathbf{k},\mathbf{q}\cdot\mathbf{k}'$. In general, f_l contains even powers of $\mathbf{q}$ through the $2l$th power. The last three terms above represent correlation effects associated with parity nonconservation in hyperon decay.

[1] Robert K. Adair, Phys. Rev. **100**, 1540 (1955).

On integrating over the azimuth of $\mathbf{k}'$ about $\mathbf{k}$ we obtain

$$f_l(\mathbf{q},\mathbf{p},\mathbf{k}'\cdot\mathbf{k},\mathbf{k})=A_l(\mathbf{k}'\cdot\mathbf{k},\mathbf{q}\cdot\mathbf{k})+B_l(\mathbf{k}'\cdot\mathbf{k},\mathbf{q}\cdot\mathbf{k})\mathbf{p}\cdot(\mathbf{q}\times\mathbf{k}),$$

where, in general, A_l contains even powers of $\mathbf{q}\cdot\mathbf{k}$ up through the $2l$th power and B_l contains odd powers of $\mathbf{q}\cdot\mathbf{k}$ up through to the $(2l-1)$th power. In the case $l=0$ the B_l term cannot show up at all and A_l must be a constant in $\mathbf{q}\cdot\mathbf{k}$. If $l>0$ the B_l term need not show up—this is a matter of luck. Suppose, however, that it does show up and that one wishes to test the conjecture that the boson resonance has spin $l=1$. The function A_l will now, in general, contain a term quadratic in $\mathbf{q}\cdot\mathbf{k}$ and a term independent of this variable, but the relative weights depend on dynamical details. For $l=1$ the shape of B_l is fully specified, however; namely, it must be linear in $\mathbf{q}\cdot\mathbf{k}$. These remarks hold for any $\mathbf{k}'\cdot\mathbf{k}$. Thus, to test for $l=1$ one can lump data together for all directions of $\mathbf{k}'$:

$$f_l(\mathbf{q},\mathbf{p},\mathbf{k})=\tilde{A}_l(\mathbf{q}\cdot\mathbf{k})+\tilde{B}_l(\mathbf{q}\cdot\mathbf{k})\mathbf{p}\cdot(\mathbf{q}\times\mathbf{k}),$$

and $\tilde{B}_l\sim(\mathbf{q}\cdot\mathbf{k})$ for $l=1$.

To sharpen matters further for general l, let us now consider, following Adair, the special configuration in which the scattered and incident beams lie along the same axis, $\mathbf{k}'=\pm\mathbf{k}$. With $\mathbf{q}\cdot\mathbf{k}=\cos\theta$, one finds here for the angular distribution in θ the expression

$$f_l\sim\{[P_l^0(\theta)]^2+|\rho|^2[P_l^1(\theta)]^2\}+2\alpha(\mathrm{Im}\rho)P_l^1(\theta)P_l^0(\theta)\mathbf{p}\cdot\mathbf{n},$$

where $\mathbf{n}$ is a unit vector in the direction of $\mathbf{q}\times\mathbf{k}$ and the P_l^m are associated Legendre functions. The parameter ρ is, up to a factor $[(l-1)!/(l+1)!]^{\frac{1}{2}}$, the ratio of spin-flip and non-spin-flip amplitudes; and α is the asymmetry parameter for hyperon decay. The parity-conserving term in the brackets involves an adjustable parameter $|\rho|^2$. The other term, however, has a *shape* which is fully specified for given l, though its very occurrence, if it is to be noticeable, involves a matter of luck, requiring that the spin-flip and non-spin-flip amplitudes be comparable in magnitude and out of phase to an appreciable extent.

The expressions for $P_l^1(\theta)P_l^0(\theta)$ are recorded below for low values of l.

$$l=1:\quad \tfrac{1}{2}\sin2\theta,$$
$$l=2:\quad \tfrac{3}{4}(3\cos^2\theta-1)\sin2\theta,$$
$$l=3:\quad \tfrac{3}{8}(5\cos^2\theta-3)(5\cos^2\theta-1)\sin2\theta.$$