PHYSICAL REVIEW VOLUME 128, NUMBER 3 NOVEMBER 1, 1962

Impulse Approximation Calculation of High-Energy Deuteron-Deuteron Elastic Scattering†*

ARNOLD TUBIS AND BERNARD CHERN
Purdue University, Lafayette, Indiana

(Received May 25, 1962, revised manuscript received July 2, 1962)

The high-energy deuteron-deuteron elastic scattering cross section and vector polarization has been calculated on the basis of the impulse approximation with the neglect of off-energy-shell effects in the nucleon-nucleon scattering matrix and effects of multiple scattering. The results are in terms of the free nucleon-nucleon scattering amplitudes and an appropriate Fourier transform of the deuteron wave function. For simplicity the D-state component of the deuteron has been neglected throughout this calculation. Numerical results are given for 312 and 620 MeV. At the two energies considered, the results for the cross section are comparable to those for nucleon-deuteron scattering. The validity of the approximations used and possible improvements of the calculation are discussed.

I. INTRODUCTION

THE elastic scattering and polarization of high-energy deuterons by complex nuclei has been the subject of several recent experimental and theoretical investigations.[1-5] Baldwin and co-workers[1] measured the cross section and vector polarization for deuteron scattering from lithium, beryllium, carbon, and aluminum at several energies with a maximum of 157 MeV. Button and Mermod[2] measured the cross section and vector and tensor polarizations for deuteron scattering from carbon and beryllium at 420 MeV. The cross section and polarizations for deuteron scattering from spin-zero target nuclei have been calculated by Strapp,[3] Sakamoto and his collaborators,[4] and Kerman and Campbell.[5] All of these workers use the impulse approximation[6] which has proven fairly successful in the treatment of high energy nucleon-nucleus scattering and polarization.[7-11]

The Sakamoto group and Kerman and Campbell made direct use of the free nucleon-nucleon scattering amplitudes by applying the usual Chew approximation[6,7] of neglecting the internal motion of the nucleons in the deuteron and target during the virtual nucleon-nucleon scatterings. Their results for the cross section and polarizations at small angles were, for the most part, in qualitative agreement with the results of Button and Mermod. However, several of the tensor polarizations calculated by Sakamoto *et al.* were in rather violent disagreement with experiment even at angles as small as 8° in the center-of-mass system. It was pointed out that multiple-scattering corrections might account for these discrepancies. The deuteron tensor polarizations were found to depend sensitively on the D-state component of the deuteron wave function.[4] This had been previously noted in the case of nucleon-deuteron scattering.[8]

In this paper, we will treat high-energy elastic deuteron-deuteron scattering by means of the impulse approximation.[12] Deuteron-deuteron scattering represents the simplest example of deuteron-nucleus scattering. Because of the relatively diffuse deuteron structure, multiple scattering corrections to the impulse approximation should be less important than in the scattering of deuterons by heavier nuclei. Furthermore, the well-defined deuteron structure should permit one to add multiple scattering corrections[3,13] and off-energy-shell effects of the nucleon-nucleon scattering[14] much more readily than in the cases where heavier nuclei are involved. Thus, high-energy deuteron-deuteron scattering should provide the most definitive test of the impulse approximation and its correction terms in deuteron-nucleus reactions. Because of the presence of two deuterons, we would expect deuteron-deuteron scattering to be more sensitive to the deuteron structure than say nucleon-deuteron scattering.[15] We thus have

† This work was supported in part by the United States Air Force.

* A preliminary version of this work was presented at the Washington, D. C., meeting of the American Physical Society, April, 1962.

[1] J. Baldwin, O. Chamberlain, E. Segrè, R. Tripp, C. Weigand, and T. Ypsilantis, Phys. Rev. **103**, 1502 (1956).

[2] J. Button and T. Mermod, Phys. Rev. **118**, 1333 (1960).

[3] H. P. Stapp, Phys. Rev. **107**, 607 (1957).

[4] Y. Sakamoto, T. Takemiya, and H. Tanaka, Progr. Theoret. Phys. (Kyoto) **27**, 24 (1962).

[5] L. J. Campbell and A. K. Kerman, *Proceedings of the Rutherford Jubilee International Conference on Nuclear Physics, Manchester, 1961* (Heywood and Company, London, 1961), pp. 141–142.

[6] G. F. Chew, Phys. Rev. **74**, 809 (1948); **80**, 196 (1950).

[7] *Nuclear Forces and the Few-Nucleon Problem*, edited by T. C. Griffith and E. A. Power (Pergamon Press, New York, 1960).

[8] L. Castillejo and L. S. Singh, reference 7, p. 193.

[9] L. Castillejo and L. S. Singh, Nuovo cimento **11**, 131 (1959).

[10] H. A. Bethe, Ann. Phys. (New York) **3**, 190 (1958).

[11] H. Postma and R. Wilson, Phys. Rev. **121**, 1229 (1961).

[12] After the present work was completed, we received a preprint from Dr. Olle Brander of Chalmers Institute of Technology, Gotteborg, Sweden. He has developed a formalism for treating elastic deuteron-deuteron scattering in the impulse approximation which is based on the explicit construction of the 9×9 dimensional deuteron-deuteron scattering matrix in terms of the elementary nucleon-nucleon scattering amplitudes. No numerical results are included. The method which we discuss in the following sections does not involve the explicit construction of this 9×9 matrix and leads to a formally simpler and much more transparent treatment.

[13] R. J. Glauber, reference 7, p. 233.

[14] K. L. Kowalski and D. Feldman, J. Math. Phys. **2**, 499 (1961); Bull. Am. Phys. Soc. 4, 357 (1962); T. Fulton and P. Schwed, Phys. Rev. **115**, 973 (1959).

[15] As a simple example of this, we note from Eq. (11) that the second power of the usual sticking factor appears in the cross-section formula. It appears only to the first power in nucleon-deuteron elastic scattering.

the possibility of gaining important information on the deuteron structure from this study.

To our knowledge, the only deuteron-deuteron elastic-scattering experiments performed to data have been at energies below 20-MeV.[16] We hope that the present work will help to stimulate high-energy (300–1000 MeV) experimental work in this area.

In Sec. 2, we derive the impulse approximation form of the deuteron-deuteron elastic-scattering amplitude. For simplicity, we neglect the deuteron D-state component and the internal momenta in the nucleon-nucleon scattering matrices. Expressions for the cross section and polarization are given in Sec. 3. In Sec. 4, we present and discuss some numerical results for the cross section and vector polarization at 312 and 620 MeV. Finally, in Sec. 5, we discuss the accuracy and possible improvement of the present work.

The treatment in this paper is in contrast to a previous calculation by Runge[17] who used simple two-nucleon central forces in a Born approximation calculation. There have been several treatments of low-energy elastic deuteron-deuteron scattering,[18] where the distortion of the deuteron is important and must be taken into account. At the energies considered in this paper, distortion effects should be negligible.

2. SCATTERING AMPLITUDE

In this section we use the standard procedure[19,20] for treating rearrangement collisions of groups of identical particles in which the proper symmetrization of the wave function is ignored at first and then later included. The treatment here is equivalent to the usual impulse approximation treatment using the T matrix.[7,21]

The Schrödinger equation for the four-body system is

$$(T+V)\psi=E\psi, \tag{1}$$

where

$$T=T_1+T_2+T_3+T_4,$$
$$V=V_{12}+V_{13}+V_{14}+V_{23}+V_{24}+V_{34}.$$

The subscripts 1 and 3 refer to the protons and 2 and 4 refer to the neutrons. The initial two-deuteron system is taken to be 1,2 and 3,4. Making the usual transformation to the center-of-mass coordinates

$$4\mathbf{R}=\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3+\mathbf{r}_4,$$
$$\mathbf{r}_{12}=\mathbf{r}_1-\mathbf{r}_2,$$
$$\mathbf{r}_{34}=\mathbf{r}_3-\mathbf{r}_4,$$
$$2\mathbf{r}_\alpha=(\mathbf{r}_1+\mathbf{r}_2)-(\mathbf{r}_3+\mathbf{r}_4),$$

we obtain

$$T=T_{12}+T_{34}+T_R+T_{r_\alpha}$$
$$=-\frac{\hbar^2}{m}\nabla_{12}{}^2-\frac{\hbar^2}{m}\nabla_{34}{}^2-\frac{\hbar^2}{8m}\nabla_{\mathrm{R}}{}^2-\frac{\hbar^2}{2m}\nabla_{r_\alpha}{}^2,$$

where m is the nucleon mass.

Since the deuteron D-state component will not be considered in this paper, the properly normalized wave function describing the initial two-deuteron state in the center-of-mass system is

$$\psi_i=\varphi_D(r_{12})\varphi_D(r_{34})e^{i\mathbf{k}_i\cdot\mathbf{r}_\alpha}|\chi_i\rangle. \tag{2}$$

Here, φ_D represents the spatial part of the deuteron wave function, normalized to unity, and

$$|\chi_i\rangle=|\chi_i(1234)\rangle$$

the spin state of the two deuterons. The magnitude of $\mathbf{k}_i$, which represents the relative wave number of the two initial deuterons, is given by $\mathbf{k}_i{}^2=(2m/\hbar^2)(E-2E_D)$ where E is now the energy in the center-of-mass system and E_D the deuteron binding energy.

One expands ψ in a complete orthonormal set of product wave functions, $\varphi_D{}^i(1,2)\varphi_D{}^j(2,4)$, of the two di-nucleon systems and substitutes this into Eq. (1). One now makes use of the fact that

$$\begin{aligned}(T_{12}+V_{12})\varphi_D{}^i&=E^i\varphi_D{}^i,\\(T_{34}+V_{34})\varphi_D{}^j&=E^j\varphi_D{}^j,\end{aligned} \tag{3}$$

and multiplies Eq. (1) on the left by the complex conjugate of $\varphi_D(r_{12})\varphi_D(r_{34})|\chi_f\rangle$. Upon integrating over the coordinates and summing over the spins of 1,2 and 3,4, one obtains a differential equation in $\mathbf{r}_\alpha$ for the deuteron expansion coefficient in the original series expansion of ψ. Solving this by the Green's function technique[14] and utilizing the outgoing-wave boundary condition, one gets

$$\psi_{r_\alpha\to\infty}\to\psi_i+\frac{e^{ikr_\alpha}}{r_\alpha}f(\mathbf{k}_f|\mathbf{k}_i)\varphi_D(r_{12})\varphi_D(r_{34})|\chi_i\rangle, \tag{4}$$

where

$$\mathbf{k}_f=k\mathbf{r}_\alpha/r_\alpha,\quad k=|\mathbf{k}_i|=|\mathbf{k}_f|,$$

[16] A list of references pertaining to low-energy experiments is given by G. Ernst and S. Flugge, Z. Physik **162**, 448 (1961).
[17] R. J. Runge, Phys. Rev. **85**, 1052 (1952).
[18] J. L. McHale, Jr., Ph.D. thesis, Indiana University, 1951 (unpublished); P. G. Burke and W. Laskar, C. R. Acad. Sci. Paris **246**, 3044 and 3158 (1958); G. Ernst and S. Flugge, reference 16; W. Laskar, C. Tate, and P. G. Burke, reference 7, p. 559.
[19] L. I. Schiff, *Quantum Mechanics* (McGraw Hill Book Company, New York, 1955), 2nd. ed.; N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, New York, 1949), 2nd ed.
[20] J. R. Rook and L. J. B. Goldfarb, Nuclear Phys. **27**, 79 (1961).
[21] A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (New York) **8**, 551 (1959).

and

$$\langle\chi_f|f(\mathbf{k}_f|\mathbf{k}_i)|\chi_i\rangle$$

$$=\left\langle\chi_f\middle|\left\{-\frac{m}{2\pi\hbar^2}\int d\mathbf{r}_{12}d\mathbf{r}_{34}d\mathbf{r}_\alpha\, e^{-i\mathbf{k}_f\cdot\mathbf{r}_\alpha}\varphi_D{}^*(r_{12})\varphi_D{}^*(r_{34})[V_{13}+V_{14}+V_{23}+V_{24}]\varphi_D(r_{12})\varphi_D(r_{34})e^{i\mathbf{k}_i\cdot\mathbf{r}_\alpha}\right\}\middle|\chi_i\right\rangle. \quad (5)$$

In addition, we have used the Born approximation in replacing ψ by ψ_i in the scattering amplitude, Eq. (5).

We now take the symmetries of the system into account. The wave function for the four-nucleon system must be antisymmetric under the exchange of two neutrons or protons. Hence, the proper symmetrization operator for ψ is

$$A=1+P_{13}P_{24}-P_{13}-P_{24}, \quad (6)$$

where P_{ij} interchanges the spin and space coordinates of particles i and j. Thus,

$$A\psi_{r_\alpha\to\infty}\to(1+P_{13}P_{24})\psi_i+\frac{e^{ikr_\alpha}}{r_\alpha}f(\mathbf{k}_f|\mathbf{k}_i)\varphi_D(r_{12})\varphi_D(r_{34})|\chi_i\rangle, \quad (7)$$

where now

$$\langle\chi_f|f(\mathbf{k}_f|\mathbf{k}_i)|\chi_i\rangle=\left\langle\chi_f\middle|\left\{-\frac{m}{2\pi\hbar^2}\int d\tau e^{-i\mathbf{k}_f\cdot\mathbf{r}_\alpha}\varphi_D{}^*(r_{12})\varphi_D{}^*(r_{34})\right.\right.$$

$$\left.\left.\times[V_{13}+V_{14}+V_{23}+V_{24}](1+P_{13}P_{24}-P_{13}-P_{24})\varphi_D(r_{12})\varphi_D(r_{34})e^{i\mathbf{k}_i\cdot\mathbf{r}_\alpha}\right\}\middle|\chi_i\right\rangle. \quad (8)$$

The impulse approximation as used here now consists of replacing the nucleon-nucleon scattering amplitudes obtained in the Born approximation by the experimentally determined free nucleon-nucleon scattering amplitudes and ignoring the contribution of the internal motions of the deuterons. The latter step is the usual Chew procedure[6] of neglecting off-energy shell effects in the nucleon-nucleon scattering. Performing the exchange operations and integrations in Eq. (8) leads to the following expressions:

$$V_{13}\text{ term}=\langle\chi_f|f_{1,3}{}^{pp}(\tfrac{1}{2}\mathbf{k}_f|\tfrac{1}{2}\mathbf{k}_i)\{F(\mathbf{k}_i-\mathbf{k}_f)-F(\mathbf{k}_i+\mathbf{k}_f)P_{24}{}^\sigma\}|\chi_i\rangle. \quad (9a)$$

$P_{ij}{}^\sigma$ is the spin exchange operator for particles i and j, and f_{ij} is the free nucleon-nucleon scattering matrix for nucleons i and j.

$$F(\Delta\mathbf{k})=2\left|\int d\mathbf{r}\;\varphi_D{}^*(r)\varphi_D(r)\exp(\tfrac{1}{2}i\Delta\mathbf{k}\cdot\mathbf{r})\right|^2=2S(\Delta\mathbf{k}/2).$$

Note that $F(\Delta\mathbf{k})$ is two times the usual sticking factor.[6] Similarly,

$$V_{14}\text{ term}=\langle\chi_f|[F(\mathbf{k}_i-\mathbf{k}_f)f_{1,4}{}^{pn}(\tfrac{1}{2}\mathbf{k}_f|\tfrac{1}{2}\mathbf{k}_i)+F(\mathbf{k}_i+\mathbf{k}_f)f_{1,4}{}^{pn}(\tfrac{1}{2}\mathbf{k}_f|-\tfrac{1}{2}\mathbf{k}_i)P_{13}{}^\sigma P_{24}{}^\sigma]|\chi_i\rangle$$

$$+\frac{m}{2\pi\hbar^2}(2\pi)^3\int d\mathbf{q}\left\langle\chi_f\middle|\left[\left(E_D-\frac{\hbar^2}{4m}(\mathbf{k}_i+\mathbf{k}_f+2\mathbf{q})^2\right)\varphi(\tfrac{1}{2}\mathbf{k}_i+\tfrac{1}{2}\mathbf{k}_f+\mathbf{q})\varphi(\tfrac{1}{2}\mathbf{k}_i-\tfrac{1}{2}\mathbf{k}_f+\mathbf{q})\varphi^*(\mathbf{q})\varphi^*(\mathbf{q}+\mathbf{k}_i)P_{13}{}^\sigma\right.\right.$$

$$\left.\left.+\left(E_D-\frac{\hbar^2}{4m}(\mathbf{k}_i-\mathbf{k}_f+2\mathbf{q})^2\right)\varphi(\tfrac{1}{2}\mathbf{k}_f-\tfrac{1}{2}\mathbf{k}_i+\mathbf{q})\varphi(-\tfrac{1}{2}\mathbf{k}_i-\tfrac{1}{2}\mathbf{k}_f+\mathbf{q})\varphi^*(\mathbf{q})\varphi^*(\mathbf{q}-\mathbf{k}_i)P_{24}{}^\sigma\right]\middle|\chi_i\right\rangle, \quad (9b)$$

where

$$\varphi(\mathbf{q})=\frac{1}{(2\pi)^{\frac{3}{2}}}\int d\mathbf{r}\;e^{-i\mathbf{q}\cdot\mathbf{r}}\varphi_0(r),$$

$$V_{23}\text{ term}=\langle\chi_f|[f_{2,3}{}^{np}(\tfrac{1}{2}\mathbf{k}_f|\tfrac{1}{2}\mathbf{k}_i)F(\mathbf{k}_i-\mathbf{k}_f)+f_{2,3}{}^{np}(\tfrac{1}{2}\mathbf{k}_f|\tfrac{1}{2}\mathbf{k}_i)F(\mathbf{k}_i+\mathbf{k}_f)P_{13}{}^\sigma P_{24}{}^\sigma]|\chi_i\rangle$$

$$+\frac{m}{2\pi\hbar^2}(2\pi)^3\int d\mathbf{q}\left\langle\chi_f\middle|\left\{\left(E_D-\frac{\hbar^2}{4m}(\mathbf{k}_i-\mathbf{k}_f+2\mathbf{q})^2\right)\varphi(\tfrac{1}{2}\mathbf{k}_i+\tfrac{1}{2}\mathbf{k}_f+\mathbf{q})\varphi(\tfrac{1}{2}\mathbf{k}_i-\tfrac{1}{2}\mathbf{k}_f+\mathbf{q})\varphi^*(\mathbf{q})\varphi^*(\mathbf{q}+\mathbf{k}_i)P_{13}{}^\sigma\right.\right.$$

$$\left.\left.+\left(E_D-\frac{\hbar^2}{4m}(-\mathbf{k}_i-\mathbf{k}_f+2\mathbf{q})^2\right)\varphi(\tfrac{1}{2}\mathbf{k}-\tfrac{1}{2}\mathbf{k}_i+\mathbf{q})\varphi(-\tfrac{1}{2}\mathbf{k}_i-\tfrac{1}{2}\mathbf{k}_f+\mathbf{q})\varphi^*(\mathbf{q})\varphi^*(\mathbf{q}-\mathbf{k}_i)P_{24}{}^\sigma\right\}\middle|\chi_i\right\rangle, \quad (9c)$$

$$V_{24}\text{ term}=\langle\chi_f|f_{2,4}{}^{nn}(\tfrac{1}{2}\mathbf{k}_f|\tfrac{1}{2}\mathbf{k}_i)[F(\mathbf{k}_i-\mathbf{k}_f)-F(\mathbf{k}_i+\mathbf{k}_f)P_{13}{}^\sigma]|\chi_i\rangle. \quad (9d)$$

Neglecting those terms in (9b) and (9c) which do not contribute appreciably at high energies, we obtain

$$\langle\chi_f|f(\mathbf{k}_f|\mathbf{k}_i)|\chi_i\rangle=\langle\chi_f|\{F(\mathbf{k}_i-\mathbf{k}_f)[f_{1,3}{}^{pp}(\tfrac{1}{2}\mathbf{k}_f|\tfrac{1}{2}\mathbf{k}_i)+f_{1,4}{}^{pn}(\tfrac{1}{2}\mathbf{k}_f|\tfrac{1}{2}\mathbf{k}_i)+f_{2,4}{}^{nn}(\tfrac{1}{2}\mathbf{k}_f|\tfrac{1}{2}\mathbf{k}_i)+f_{1,3}{}^{np}(\tfrac{1}{2}\mathbf{k}_f|\tfrac{1}{2}\mathbf{k}_i)]$$
$$-F(\mathbf{k}_i+\mathbf{k}_f)[f_{13}{}^{pp}(\tfrac{1}{2}\mathbf{k}_f|\tfrac{1}{2}\mathbf{k}_i)-f_{1,4}{}^{pn}(\tfrac{1}{2}\mathbf{k}_f|-\tfrac{1}{2}\mathbf{k}_i)P_{13}{}^{\sigma}P_{24}{}^{\sigma}+f_{2,4}{}^{nn}(\tfrac{1}{2}\mathbf{k}_f|\tfrac{1}{2}\mathbf{k}_i)P_{13}{}^{\sigma}-f_{2,3}{}^{np}(\tfrac{1}{2}\mathbf{k}_f|-\tfrac{1}{2}\mathbf{k}_i)P_{13}{}^{\sigma}P_{24}{}^{\sigma}]\}|\chi_i\rangle. \quad (10)$$

The first term in Eq. (10) is large in the forward scattering direction and very small in the backward direction. The second term, which corresponds to the rearrangement or exchange process, peaks in the backward direction and is very small in the forward direction. This second term is usually neglected in T-matrix formulations of the impulse approximation.[7]

3. CROSS SECTION AND POLARIZATION

For the case of unpolarized deuterons in the incident beam and target material, one can write for the cross section[22]

$$\frac{d\sigma}{d\Omega}=\tfrac{1}{9}\sum_{i,f}\langle\chi_i|f^\dagger|\chi_f\rangle\langle\chi_f|f|\chi_i\rangle, \quad (11)$$

where the sum over the spin states is restricted to those that can be compounded from the spin-one states of deuterons 1,2 and 3,4 and $f^\dagger$ denotes the Hermitian conjugate of f. We can reformulate the expression for the cross section in terms of traces in the 16×16 dimensional spin space of the 4 nucleons by use of the triplet spin projection operators. Thus,

$$d\sigma/d\Omega=\tfrac{1}{9}\,\mathrm{tr}(\mathcal{P}^Tf^\dagger\mathcal{P}^Tf), \quad (12)$$

where

$$\mathcal{P}^T=\mathcal{P}_{1,2}{}^T\mathcal{P}_{3,4}{}^T=\tfrac{1}{4}(3+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\tfrac{1}{4}(3+\boldsymbol{\sigma}_3\cdot\boldsymbol{\sigma}_4).$$

Similarly, we can write the expectation value of any observable in the 9×9 spin space of two spin-one particles in terms of traces in 16×16 space. Thus,

$$\langle O\rangle_f=\mathrm{tr}(\mathcal{P}^Tf\mathcal{P}^Tf^\dagger\mathcal{P}^TO)/\mathrm{tr}(\mathcal{P}^Tf\mathcal{P}^Tf^\dagger). \quad (13)$$

Hence, the vector polarization of the deuteron is

$$\left\langle\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\cdot\hat{n}\right\rangle_f=\mathrm{tr}\left(\mathcal{P}^Tf\mathcal{P}^Tf^\dagger\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\cdot\hat{n}\right)\Big/$$
$$\times\,\mathrm{tr}(\mathcal{P}^Tf\mathcal{P}^Tf^\dagger), \quad (14)$$

where $\hat{n}=\mathbf{k}_i\times\mathbf{k}_f/|\mathbf{k}_i\times\mathbf{k}_f|$ is a unit vector perpendicular to the scattering plane and I_0 is the unpolarized cross section. Use has also been made of the explicit spin dependent form of the nucleon-nucleon scattering matrix[21] in eliminating one of the triplet projection operators. Equations (12) and (14) are of the same form as that given by Castillejo and Singh[7] for the case of nucleon-deuteron scattering. The general spin-one polarizations, $\langle T_{JM}\rangle_f$, are obtained by setting $O=T_{JM}$ in Eq. (13). The T_{JM} as first given by Lakin[23] for a system of spin one are

$$T_{00}=1,$$
$$T_{11}=-\tfrac{1}{2}\sqrt{3}(S_x+iS_y),$$
$$T_{10}=(\tfrac{3}{2})^{\frac{1}{2}}S_z,$$
$$T_{22}=\tfrac{1}{2}\sqrt{3}(S_x+iS_y)^2,$$
$$T_{21}=-\tfrac{1}{2}\sqrt{3}[(S_x+iS_y)S_z+S_z(S_x+iS_y)],$$
$$T_{20}=(1/\sqrt{2})(3S_z{}^2-2),$$
$$T_{J,-M}=(-1)^MT_{JM}{}^\dagger.$$

In our case $\mathbf{S}=(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)/2$. It should be noted that what we have called the vector polarization is in Lakin's terminology the expectation value of the y component of the deuteron spin, i.e., $i\langle T_{11}\rangle=\tfrac{1}{2}\sqrt{3}\langle[(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)/2]\cdot\hat{n}\rangle$. The y direction is chosen perpendicular to the first plane of scattering.

Using the form of the free nucleon-nucleon scattering matrix as given by Kerman, McManus, and Thaler[21] and restricting ourselves to small-angle scattering, we have calculated the cross section and vector polarization in terms of the scattering coefficients A, B, C, E, and F given in reference 21. The explicit analytical forms appear in the Appendix. As was mentioned earlier, we have not calculated the tensor polarization because of our neglect of the D-state component of the deuteron which seemingly plays an important role there.

4. NUMERICAL RESULTS

Using the tables given in Kerman, McManus, and Thaler,[21] which are based on the Gammel-Thaler phase shifts,[24] we have plotted in Figs. 1 and 2 the cross section and vector polarization for deuteron energies of 312 and 620 MeV in the lab system.

It is interesting to note that the vector polarization and cross section are comparable to those obtained for nucleon-deuteron scattering in the impulse approximation.[8] The diminishing effect of the higher power of the striking factor on the deuteron-deuteron cross section, at the angles considered in the graph, is compensated for by the greater number of nucleon-nucleon scattering amplitudes involved. We have limited the present calculation to center of mass angles smaller than 30° since multiple scattering and off-energy-shell effects play an increasingly larger role at larger angles.

[22] The justification for neglecting symmetrization except in the scattering amplitude in Eqs. (11) through (14) follows from the general theory of scattering involving identical particles. See for example H. P. Stapp [Ph.D. thesis, University of California Radiation Laboratory Report UCRL-3098, 1955 (unpublished)].

[23] W. Lakin, Phys. Rev. **98**, 139 (1959).

[24] J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291, 1337 (1957).

We have also made a preliminary investigation of the effect of the point Coulomb interaction on the above results. (See Appendix.) One finds that for the cross section the effect of the Coulomb force is negligible at c.m. angles greater than 8° for the case of 312 MeV and 6° in the 620-MeV case.

5. FINAL COMMENTS

We believe that the analysis of elastic deuteron-deuteron scattering presented in this paper has the same validity as comparable analyses of nucleon-deuteron scattering.[7–9,11] An examination of the role of multiple scattering effects[3,13] and off-energy shell effects[14] in nucleon-deuteron scattering suggests that multiple scattering should be the most important correction to our analysis. The D-state component of the deuteron, as was mentioned earlier, probably plays an important role in the calculation of the tensor polarizations. We are at present adding deuteron D-state and multiple (double) scattering corrections to our analysis.

ACKNOWLEDGMENT

The authors wish to thank Dr. Olle Brander of Chalmers Institute of Technology, Göteburg, Sweden, for informing them of his work on deuteron-deuteron elastic scattering before publication.

APPENDIX

The form of the two-body nucleon-nucleon scattering matrices as given in reference 21 are

$$f_{ij}=A+B\boldsymbol{\sigma}_i\cdot\hat{n}\boldsymbol{\sigma}_j\cdot\hat{n}+C[\boldsymbol{\sigma}_i\cdot\hat{n}+\boldsymbol{\sigma}_j\cdot\hat{n}]+E\boldsymbol{\sigma}_i\cdot\hat{q}\boldsymbol{\sigma}_j\cdot\hat{q}+F\boldsymbol{\sigma}_i\cdot\hat{p}\boldsymbol{\sigma}_j\cdot\hat{p},$$

where $[\hat{q},\hat{n},\hat{p}]$ are vectors forming a right-handed coordinate system:

$$\hat{q}=\frac{\mathbf{k}_f-\mathbf{k}_i}{|\mathbf{k}_f-\mathbf{k}_i|},\quad \hat{n}=\frac{\mathbf{k}_i\times\mathbf{k}_f}{|\mathbf{k}_i\times\mathbf{k}_f|},\quad \hat{p}=\frac{\mathbf{k}_i+\mathbf{k}_f}{|\mathbf{k}_i+\mathbf{k}_f|},$$

A, B, C, E, and F depend on the energy, momentum transfer, and isotopic spin. In the present case, we evaluate them for a nucleon-nucleon center-of-mass energy of one fourth the deuteron-deuteron laboratory energy and a momentum transfer of $\frac{1}{2}|\mathbf{k}_i-\mathbf{k}_f|$.

For the cross section (ignoring the Coulomb interaction), one obtains

$$\begin{aligned}I_0=4S^2(\mathbf{q}/2)I=\{&4|A^{pp}+A^{pn}|^2\\&+(16/3)|C^{pp}+C^{pn}|^2\\&+(16/9)|B^{pp}+B^{pn}|^2\\&+(16/9)|E^{pp}+E^{pn}|^2\\&+(16/9)|F^{pp}+F^{pn}|^2\}4S^2(\mathbf{q}/2)\end{aligned}$$

Neglecting the Coulomb interaction, the vector polarization is

$$\left\langle\frac{\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2}{2}\cdot\hat{n}\right\rangle_f=\frac{16}{3I}\operatorname{Re}[(A^{pp}+A^{pn})^*(C^{pp}+C^{pn})]+\frac{32}{9I}\operatorname{Re}[(B^{pp}+B^{pn})^*(C^{pp}+C^{pn})].$$

A crude estimate of the Coulomb effect on the cross section and vector polarization can be obtained by incorporating the point-charge Coulomb scattering amplitudes in the proton-proton scattering amplitudes. The comments in Sec. 4 are based on this approach. This is to be contrasted with the more realistic treatment of the Coulomb interaction as given by Bethe.[10,11]

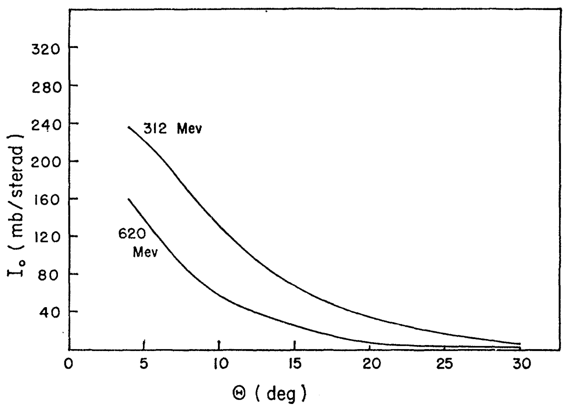

FIG. 1. The unpolarized differential cross section for elastic deuteron-deuteron scattering as a function of the polar angle θ in the center-of-mass system. The nucleon-nucleon amplitudes in Eq. (10) were taken from the work of Kerman, McManus, and Thaler (reference 21) while the factor $F(\mathbf{q})$ was evaluated using the Moravcsik [M. J. Moravcsik, Nuclear Phys. **7**, 113 (1958)] analytic form of the Gartenhaus deuteron wave function.

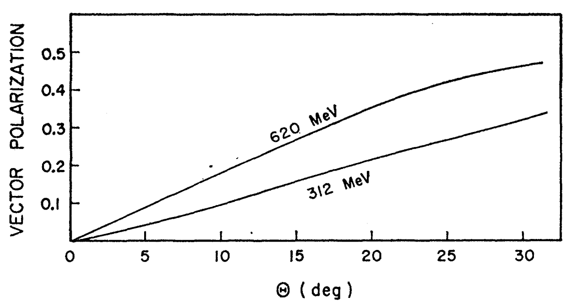

FIG. 2. The vector polarization of the deuteron as defined in Eq. (14) as a function of the polar angle θ in the center-of-mass system.

Note added in proof. We wish to thank Dr. O. Brander for calling our attention to an error in the equation for the cross-section as originally given in the Appendix. Dr. Brander's preprint referred to in the beginning of the article has now appeared in print in Nuclear Phys. **36**, 82 (1962).