multiples of the reciprocal lattice vectors. Locking does occur for the linear spin wave configurations in erbium[46] and thulium.[42] For both cases, $p=7$. But the energetic cause is easy to understand in these two cases. The magnetic interaction responsible for the ordered state is the $s-f$ exchange interaction between a linear SDW in the conduction electrons and the localized f electrons. This interaction is negligible for atoms at nodes of the linear SDW; and there will always be a significant fraction of lattice sites at or very near such nodes for an incommensurate linear SDW. This loss of interaction energy is avoided by a commensurate SDW wave vector. The phase of the SDW can adjust so that no lattice site is near a node. Consequently, a locking mechanism is provided. This cannot occur for a spiral spin wave configuration, since a spiral SDW has no nodes. It is interesting that the locking in erbium stops when a spiral component appears, as might be anticipated, since the (perpendicular) spiral component eliminates the nodes, even though the linear component remains.

[46] J. W. Cable, E. O. Wollan, W. C. Koehler, and M. K. Wilkinson, J. Appl. Phys. **32**, 49S (1961).

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Radiative Corrections to Decay Processes Mediated by Vector Bosons*

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The lowest order electromagnetic corrections to β and μ decay have been calculated in an intermediate vector-boson theory. Assuming universal coupling, the μ lifetime agrees with the experimental value if the boson's mass is chosen to be that of a K meson. The results are relatively insensitive to the value chosen for the cutoff.

I. INTRODUCTION

The lowest order radiative corrections to decays through a $(V-A)$ Fermi interaction have been calculated by Kinoshita and Sirlin and by Berman.[1] In a universal coupling theory these corrections affect the lifetime of the muon in two ways: in the contributions to the μ decay itself and in the contributions to the O^{14} decay. The latter decay determines the coupling constant to be used in all weak interactions. For universal coupling, the most recent experimental data indicate a significant discrepancy between the predicted and observed μ lifetimes.[2]

It has been suggested that decay processes be described not by a Fermi interaction but by two Yukawa-type interactions mediated by a vector boson.[3] The intermediate vector-boson (IVB) description reduces to the Fermi theory for infinite boson mass. We calculate below the lowest order electromagnetic corrections to μ and β decay in an IVB theory. We then determine the μ lifetime for universal coupling and compare it with the experimental value.

* Supported in part by National Science Foundation.

[1] S. M. Berman, Phys. Rev. **112**, 267 (1958); T. Kinoshita and A. Sirlin, *ibid.* **113**, 1652 (1959).

[2] J. W. Butler and R. O. Bondelid, Phys. Rev. **121**, 1770 (1961).

[3] See, e.g., T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960).

II. INTERACTION LAGRANGIAN AND CUTOFF DEPENDENCE

Consider the following type of interaction Lagrangian as a basis for all weak processes[4]:

$$L_1=ig\bar{\psi}_X\gamma_\mu\frac{(1+i\gamma_5)}{2}\psi_Y\phi^{\mu*}+\text{H.c.},\tag{1}$$

where ψ_Y destroys a positively charged fermion Y, ϕ^μ creates a positively charged IVB, B^+, and the fermion X is neutral. g is a real, semiweak coupling constant. The electromagnetic field is introduced by replacing $\partial_\mu U^*$ in the free Lagrangian by $(\partial_\mu+ieA_\mu)U^*$. Here U^* is a charged field operator, and A_μ is the electromagnetic vector potential. Additional interactions corresponding to anomalous magnetic moments will not be considered here. The interaction Lagrangian is now of the form

$$\begin{aligned}L=L_1-ie\bar{\psi}\gamma_\mu\psi A^\mu-ie(A_\mu\phi_\nu{}^*\partial^\mu\phi^\nu-A_\mu\phi_\nu{}^*\partial^\nu\phi^\mu\\-\partial^\mu\phi_\nu{}^*A_\mu\phi^\nu+\partial_\nu\phi_\mu{}^*A^\mu\phi^\nu)\\-e^2(A_\mu A^\mu\phi_\nu{}^*\phi^\nu-A_\mu A^\nu\phi^{\mu*}\phi_\nu),\end{aligned}\tag{2}$$

where e is positive, and ψ destroys a negatively charged fermion.

[4] We follow the notation of J. M. Jauch and F. Rohrlich, *Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1955).

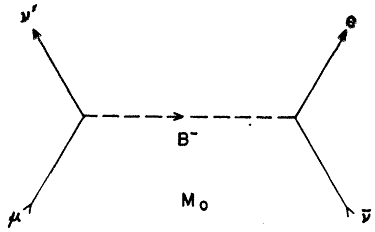

FIG. 1. Feynman diagram for μ decay labeled by the corresponding matrix element.

In momentum space the free IVB field propagator is

$$\Delta_{\mu\nu}(k^2)=\frac{g_{\mu\nu}+k_\mu k_\nu/m^2}{k^2+m^2-i\epsilon}, \tag{3}$$

where m is the IVB mass.

The lowest order radiative corrections to the Fermi theory were calculated for an unrenormalized weak coupling constant, and the results are cutoff dependent. The IVB calculations will be carried out for an unrenormalized coupling g, and the results will again be cutoff dependent.

The Fermi interaction is unrenormalizable.[5] This difficulty persists if the electromagnetic field is introduced. The interactions of IVB's with fermion pairs and the electromagnetic field are also unrenormalizable.[6] In the lowest order radiative corrections to the IVB theory the unrenormalizability of the interactions will introduce a quadratic divergence into the IVB self-energy part.

III. μ DECAY

The decay of a negative muon in an IVB theory proceeds through the following interactions: First the μ decays into a neutrino and a virtual IVB. This process is described by the interaction Lagrangian

$$L_{1\mu}=ig\bar{\psi}_{\nu'}\gamma_\omega\frac{(1+i\gamma_5)}{2}\psi_\mu\phi^\omega. \tag{4}$$

Then the virtual IVB decays into an electron and an antineutrino. The second process is described by the interaction Lagrangian[7]

$$L_{2\mu}=ig\bar{\psi}_e\gamma_\omega\frac{(1+i\gamma_5)}{2}\psi_\nu\phi^{\omega*}. \tag{5}$$

The Feynman diagram describing μ decay is shown in Fig. 1. As $m\rightarrow\infty$ the IVB propagator reduces to $g_{\mu\nu}/m^2$, and since g^2/m^2 is proportional to the Fermi coupling constant G, the Fermi interaction is obtained.

The lowest order electromagnetic corrections to μ decay are described by the diagrams shown in Fig. 2. As in the Fermi theory, these diagrams can be divided into two groups: One group involves the emission and reabsorption of a virtual photon, and the other the emission of a real photon which is not observed. The interference terms in the expression $|\sum_{i=0}^{7}M_i|^2$ and the terms of the expression $|\sum_{i=8}^{10}M_i|^2$ constitute the second-order radiative corrections to μ decay. The inner-bremsstrahlung contribution includes an infrared divergence which is removed by giving the photon a small mass, ϵ_1.[8] This divergence cancels a similar expression contributed by the matrix elements involving virtual photons.

Diagrams 1 through 4 contain self-energy parts. A subtraction is made from these parts in order to renormalize the masses of the charged particles involved in the decay process. This subtraction eliminates M_4. M_3 is particularly interesting since it is quadratically divergent, whereas the other matrix elements we consider are either cutoff independent or logarithmically divergent. The matrix elements obtained when one uses the Fermi interaction are only logarithmically divergent. Thus, results obtained with the IVB theory are more sensitive to the value of the cutoff chosen than those obtained with the Fermi theory. However, the IVB self-energy part in M_3 has the same effect upon all decay rates so long as momentum transfers are ignored. Therefore, in a universal coupling theory the net contribution of this self-energy part to a given decay rate is zero. This is because the direct contribution just cancels that from the redefinition of the coupling constant (see V).

Evaluating the Feynman integrals, squaring matrix elements, and summing over final states, one obtains the electron spectrum for μ decay. With the assumption that the cutoff and m are much larger than the mass of the muon and all momenta appearing in μ decay, the

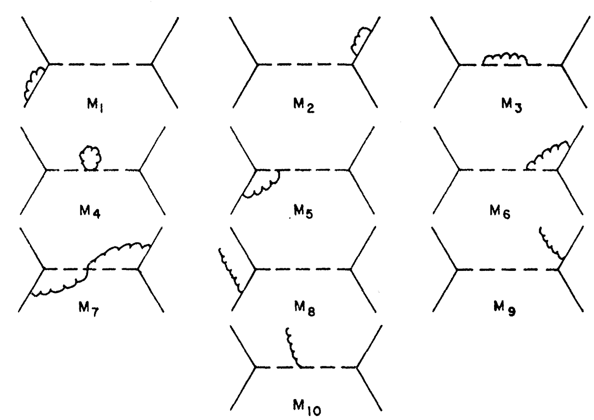

FIG. 2. Feynman diagrams for lowest order radiative corrections to μ decay labeled by the corresponding matrix elements.

[5] See, e.g., the discussion of renormalization in H. Umezawa, *Quantum Field Theory* (North-Holland Publishing Company, Amsterdam, 1956), Chap. XV.

[6] Attempts to renormalize IVB interactions have not been successful. See, e.g., S. Glashow, Nuclear Phys. **10**, 107 (1959), and A. Komar and A. Salam, Nuclear Phys. **21**, 624 (1960).

[7] We consider ν and ν' to be distinct since $\mu\rightarrow e+\gamma$ has not been observed.

[8] For a treatment of this divergence, see the references in footnote 1.

spectrum for the decay of a muon at rest is

$$P(x)dx=\frac{g^4M_\mu^5}{3\times 2^8\pi^3m^4}(3-2x)x^2dx\left[1+\left\{\frac{M_\mu^2x(2-x)}{m^2(3-2x)}+\frac{3M_\mu^4x^2(5-2x)}{10m^4(3-2x)}\right\}-\frac{\alpha}{4\pi}(a+b)\right]. \tag{6}$$

Here M_μ is the muon mass, $x=2E/M_\mu$, where E is the total electron energy, a is the contribution of the matrix elements involving virtual photons, and b is the contribution of the inner bremsstrahlung matrix elements. The terms in curly brackets are nonradiative corrections to the Fermi theory and have been taken into account in the literature.[9] The quantities a and b are defined below. The term a is given by

$$\begin{aligned}a=&-4L(x)+4\ln^2(x)-4\ln(x)\ln(1-x)+\tfrac{2}{3}\pi^2+4\ln\left(\frac{M_e}{M_\mu}\right)\ln\frac{M_e}{M_\mu x^2}+6\ln\frac{M_e^2}{M_\mu^2}+\frac{12x^2-23x+12}{(3-2x)(x-1)}\ln x\\&+8-\frac{x}{3-2x}+2\ln\frac{M_\mu}{M_e}+4\left(1+\ln\frac{M_e}{M_\mu x}\right)\ln\frac{\epsilon_1^2}{M_e^2}+\left\{4x\frac{M_\mu^2}{m^2}\ln\frac{M_\mu^2}{m^2}+\left(\frac{3}{m^2/\Lambda^2-1}-3\frac{\Lambda^2}{m^2}-\frac{\Lambda^4}{m^4}+\frac{\Lambda^6}{3m^6}\right)\ln\frac{\Lambda^2}{m^2}\right.\\&\left.+\frac{M_\mu^2(12+5x-3x^2)}{3m^2(3-2x)}\ln\frac{m^2}{M_\mu^2}+\frac{M_\mu^2x(2-x)}{m^2(3-2x)}\ln\frac{m^2}{M_e^2}+4\left(4+\frac{\Lambda^2}{3m^2}-\frac{\Lambda^4}{3m^4}\right)F(\Lambda,m)-\frac{\Lambda^2}{2m^2}-\frac{2\Lambda^4}{3m^4}\right\}, \end{aligned}\tag{7}$$

where

$$F(\Lambda,m)=\left(\frac{\Lambda^2}{m^2}-\frac{\Lambda^4}{4m^4}\right)^{1/2}\left(\tan^{-1}\frac{(\Lambda^2/m^2-\Lambda^4/4m^4)^{1/2}}{1-\Lambda^2/2m^2}-\tan^{-1}\frac{(\Lambda^2/m^2-\Lambda^4/4m^4)^{1/2}}{-\Lambda^2/2m^2}\right),$$

$L(x)$ is the Spence function,[10] $-\int_0^x t^{-1}\ln(1-t)dt$, M_e is the electron mass, and Λ is the ultraviolet cutoff. The terms in curly brackets are the contributions due to the IVB and vanish if its mass, m, is infinite. For $\Lambda>2m$, $F(\Lambda,m)$ becomes

$$-\frac{1}{2}\left(\frac{\Lambda^4}{4m^4}-\frac{\Lambda^2}{m^2}\right)^{1/2}\ln\left(\frac{1-\Lambda^2/2m^2+(\Lambda^4/4m^4-\Lambda^2/m^2)^{1/2}}{1-\Lambda^2/2m^2-(\Lambda^4/4m^4-\Lambda^2/m^2)^{1/2}}\,\frac{-\Lambda^2/2m^2-(\Lambda^4/4m^4-\Lambda^2/m^2)^{1/2}}{-\Lambda^2/2m^2+(\Lambda^4/4m^4-\Lambda^2/m^2)^{1/2}}\right).$$

The quantity b is defined as

$$\begin{aligned}b=&\frac{4(8x^2-13x+5)}{3x(3-2x)}+\tfrac{2}{3}\pi^2-4\left(1+\ln\frac{M_e}{M_\mu x}\right)\ln\frac{\epsilon_1^2}{M_e^2}+\frac{2(5+12x-69x^2+46x^3)}{3x^2(3-2x)}\ln\frac{M_e}{M_\mu x}-8\ln x\\&+\frac{4(1+x)}{x}\ln(1-x)-4L(x)-4\ln^2\left(\frac{M_e}{M_\mu x}\right)-8\ln(x)\ln\frac{M_e}{M_\mu x}+8\ln(1-x)\ln\frac{M_e}{M_\mu x}.\end{aligned}\tag{8}$$

b is independent of m in the approximation that $m^2\gg M_\mu^2$.

Integrating $P(x)dx$ between $x=0$ and $x=1$, one obtains the μ decay rate

$$T_\mu^{-1}=\frac{g^4M_\mu^5}{3\times 2^9\pi^3m^4}\left\{1+\left[\frac{3M_\mu^2}{5m^2}+\frac{2M_\mu^4}{5m^4}\right]-\frac{\alpha}{2\pi}c\right\}, \tag{9}$$

where

$$\begin{aligned}c=3.3+&\left\{0.15\frac{M_\mu^2}{m^2}\ln\frac{m^2}{M_\mu^2}+\frac{M_\mu^2}{3m^2}\ln\frac{m^2}{M_e^2}\right.\\&+\frac{1}{2}\left(\frac{3}{m^2/\Lambda^2-1}-\frac{3\Lambda^2}{m^2}-\frac{\Lambda^4}{m^4}+\frac{\Lambda^6}{3m^6}\right)\ln\frac{\Lambda^2}{m^2}\\&\left.+2\left(4+\frac{\Lambda^2}{3m^2}-\frac{\Lambda^4}{3m^4}\right)F(\Lambda,m)-\frac{\Lambda^2}{4m^2}-\frac{\Lambda^4}{3m^4}\right\}.\end{aligned}\tag{10}$$

In order to simplify our results somewhat we have assumed that $M_\mu^2/m^2\ll 1$. This is a reasonable assumption, since one anticipates only small corrections to the Fermi theory. In addition, the fact that no IVB's have been observed suggests that their mass is greater than that of the K mesons. For such heavy bosons the term $2M_\mu^4/5m^4$ in Eq. (9) and the terms $0.15(M_\mu^2/m^2)\ln(m^2/M_\mu^2)+(M_\mu^2/3m^2)\ln(m^2/M_e^2)$ in Eq. (10) are negligible. The cutoff is undetermined; a value of one nucleon mass is often chosen as appropriate for weak interactions. In the final results below we consider a range of m from less than $M_{K\text{ meson}}$ to infinity and of Λ from less than M_{proton} to $10M_{\text{proton}}$. The correction to T_μ^{-1} for $m\to\infty$ agrees with the results of Berman and Kinoshita and Sirlin and is independent of Λ.

IV. β DECAY

The decay of a neutron in an IVB theory proceeds as follows: First the neutron decays into a proton and a

[9] S. A. Bludman, Phys. Rev. **124**, 947 (1961).
[10] See, e.g., K. Mitchell, Phil. Mag. **40**, 351 (1949).

virtual IVB. This process is described by the interaction Lagrangian

$$L_{\text{in}}=ig\bar{\psi}_P\gamma_\mu\frac{1+i\gamma_5}{2}\psi_N\phi^\mu. \tag{11}$$

This is followed by the decay of the virtual IVB as described in Sec. III. The Feynman diagrams we have to consider are analogous to those shown in Fig. 1 and Fig. 2. The effects of strong interactions are not considered here.[11] We again renormalize masses. Λ^2 and m^2 are taken to be much greater than M_e^2 and the squared momenta involved in β decay. This assumption is much better than the corresponding one made in Sec. III, and there are no significant nonradiative corrections to the β-decay electron spectrum.[12]

Proceeding as before, we obtain the electron spectrum

$$P(p)dp=\frac{g^4}{4\pi^3m^4}(E_0-E)^2p^2dp\left[1+\frac{\alpha}{2\pi}(d+e)\right]. \tag{12}$$

Here p is the electron momentum, E_0 is the maximum total electron energy, d is the contribution of the matrix elements involving virtual photons, and e is the contribution of the inner bremsstrahlung matrix elements.

The terms d and e are defined as follows:

$$\begin{aligned}d=&-\frac{1}{2\beta}\ln^2\left(\frac{1+\beta}{1-\beta}\right)-\frac{2}{\beta}L\left(\frac{2\beta}{1+\beta}\right)+\left(-2+\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}\right)\ln\frac{\epsilon_1^2}{M_e^2}+\beta\ln\frac{1+\beta}{1-\beta}-\tfrac{7}{2}-\tfrac{1}{2}\ln\left(\frac{\Lambda^2}{M^2}+1\right)-\frac{1}{2(\Lambda^2/M^2+1)}\\&-\ln\frac{\Lambda}{M_e}+2\ln\frac{M^2}{M_e^2}+\frac{\Lambda^2/M^2}{1-\Lambda^2/m^2}\left(\left[\left(\frac{M^2}{\Lambda^2}+\frac{3}{2}\right)\ln\frac{\Lambda^2}{M^2}-6\frac{M^2}{\Lambda^2}F(\Lambda,M)\right]-\{[\Lambda\to m]\}\right)\\&+\left\{\left(3+\frac{\Lambda^2}{m^2}-\frac{\Lambda^4}{3m^4}\right)\frac{\Lambda^2}{2m^2}\ln\frac{\Lambda^2}{m^2}-2\left(4+\frac{\Lambda^2}{3m^2}-\frac{\Lambda^4}{3m^4}\right)F(\Lambda,m)+\frac{\Lambda^2}{4m^2}+\frac{\Lambda^4}{3m^4}\right.\\&\left.+\frac{1}{1-m^2/\Lambda^2}\left(\left[\left(1+\frac{\Lambda^2}{2M^2}+\frac{\Lambda^4}{12M^4}\right)\ln\frac{\Lambda^2}{M^2}-\frac{1}{3}\left(8+\frac{\Lambda^2}{M^2}\right)F(\Lambda,M)-\frac{\Lambda^2}{6M^2}\right]-[\Lambda\to m]\right)\right\},\end{aligned} \tag{13}$$

and

$$\begin{aligned}e=&8-\frac{4(E_0-E)}{3E}+\ln\left(\frac{\epsilon_1^2}{M_e^2}\right)\left(2-\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta}\right)+\frac{2}{\beta}L(-\beta)-\frac{2}{\beta}L(\beta)+\frac{1}{\beta}L\left(\frac{1+\beta}{2}\right)-\frac{1}{\beta}L\left(\frac{1-\beta}{2}\right)\\&+\frac{1}{2\beta}\left(\frac{(E_0-E)^2}{3E^2}+\frac{8(E_0-E)}{3E}-8\right)\tanh^{-1}\beta+\frac{1}{\beta}\tanh^{-1}\beta\ln(1-\beta^2)\\&+4\ln\left(\frac{M_e}{E_0-E}\right)\left(1-\frac{1}{\beta}\tanh^{-1}\beta\right)+2\left(-2+\frac{1}{\beta}\tanh^{-1}\beta\right)\ln 2.\end{aligned} \tag{14}$$

In the above expressions M is the proton's mass, and $\beta=p/E$. As before, the terms in curly brackets vanish as $m\to\infty$, the inner bremsstrahlung contribution is independent of m for the assumptions made above, and the inverse tangent expressions are to be replaced by logarithms for the appropriate ranges of parameters. For infinite m and large Λ the spectrum reduces to the expression obtained by Kinoshita and Sirlin and is cutoff dependent.

Following Kinoshita and Sirlin and letting $\beta\to 1$, one obtains the neutron decay rate by integrating the spectrum with respect to x from $x=0$ to $x=1$, where $x=E/E_0$. (One finds, as in the Fermi theory, that the decay rate is relatively insensitive to the value of β.) The result is

$$T_N^{-1}=\frac{g^4E_0^5}{120\pi^3m^4}\left(1+\frac{\alpha}{2\pi}f\right), \tag{15}$$

where

$$\begin{aligned}f=&3\ln\frac{M}{2E_0}-5.85-\ln\frac{\Lambda}{M}-\tfrac{1}{2}\ln\left(\frac{\Lambda^2}{M^2}+1\right)-\frac{1}{2(\Lambda^2/M^2+1)}+\frac{\Lambda^2/M^2}{1-\Lambda^2/m^2}\left(\left[\left(\frac{M^2}{\Lambda^2}+\frac{3}{2}\right)\ln\frac{\Lambda^2}{M^2}-6\frac{M^2}{\Lambda^2}F(\Lambda,M)\right]-\{[\Lambda\to m]\}\right)\\&+\left\{\frac{\Lambda^2}{2m^2}\left(3+\frac{\Lambda^2}{m^2}-\frac{\Lambda^4}{3m^4}\right)\ln\frac{\Lambda^2}{m^2}-2\left(4+\frac{\Lambda^2}{3m^2}-\frac{\Lambda^4}{3m^4}\right)F(\Lambda,m)+\frac{\Lambda^2}{4m^2}+\frac{\Lambda^4}{3m^4}\right.\\&\left.+\frac{1}{1-m^2/\Lambda^2}\left(\left[\left(1+\frac{\Lambda^2}{2M^2}+\frac{\Lambda^4}{12M^4}\right)\ln\frac{\Lambda^2}{M^2}-\frac{1}{3}\left(8+\frac{\Lambda^2}{M^2}\right)F(\Lambda,M)-\frac{\Lambda^2}{6M^2}\right]-[\Lambda\to m]\right)\right\}.\end{aligned} \tag{16}$$

[11] See, e.g., R. J. Blin-Stoyle and J. LeTourneux, Phys. Rev. **123**, 627 (1961), and B. V. Geshkenbein and V. S. Popov, J. Exptl. Theoret. Phys. (U.S.S.R.) **14**, 145 (1962).

[12] J. D. Childress, Phys. Rev. **123**, 1729 (1961).

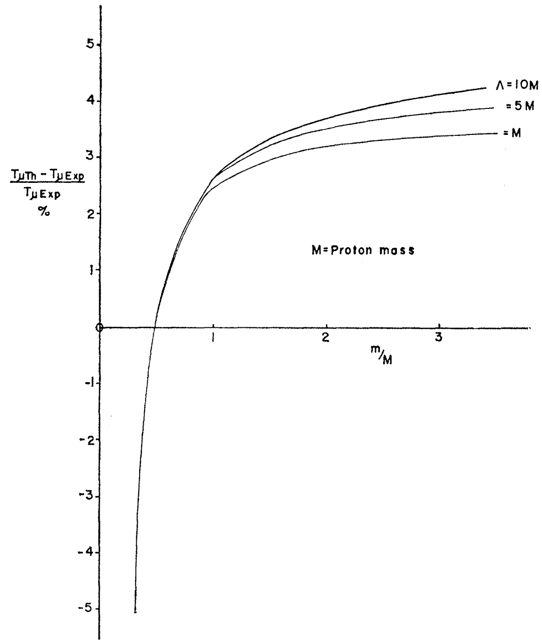

FIG. 3. Discrepancy between experimental and theoretical μ lifetimes as a function of intermediate vector boson mass with cutoff as a parameter.

In order to obtain the decay rate of O^{14}, we use the value of E_0 for that process: 2.3 MeV. [In that case the radiative corrections shown above are in addition to the Coulomb correction $F(Z,E)$.]

V. μ LIFETIME AND UNIVERSAL COUPLING

The decay rates computed above contain a factor g^4/m^4 which is as yet undefined. The experimental value of the O^{14} decay rate is used to determine this coupling. Using Eq. (15), one has

$$\frac{g^4}{m^4}=\frac{120\pi^3 T_{O^{14}}^{-1}}{E_0^5[1+\alpha(2\pi)^{-1}f]}\approx\frac{120\pi^3 T_{O^{14}}^{-1}}{E_0^5}\left(1-\frac{\alpha}{2\pi}f\right). \quad (17)$$

Thus, the final expression for the decay rate of a muon in a universal coupling IVB theory is

$$T_\mu^{-1}\approx\frac{5M_\mu^5}{2^6E_0^5}T_{O^{14}}^{-1}\left(1+\left\{\frac{3M_\mu^2}{5m^2}\right\}-\frac{\alpha}{2\pi}(c+f)\right). \quad (18)$$

We note that the expression $(c+f)$ in Eq. (18) considered as a function of Λ and m has interesting properties: We set

$$(c+f)=h(\Lambda,m). \quad (19)$$

If we let $m\rightarrow\infty$, we have for $\Lambda\gg M$

$$h(\Lambda,\infty)\approx 0.5+3\ln(M/2E_0)+6\ln(\Lambda/M). \quad (20)$$

TABLE I. $(T_{\mu\ \mathrm{th}}-T_{\mu\ \mathrm{exp}})/T_{\mu\ \mathrm{exp}}$, in %, for various values of intermediate vector boson mass and ultraviolet cutoff.

m/M \ Λ/M	1	5	10
∞	3.5	4.4	4.9
2	3.2	3.5	3.7
1	2.4	2.6	2.6
$\frac{1}{2}$	0.0	0.1	0.1
$\frac{1}{3}$	−4.4	−4.5	−4.5

If we now let $\Lambda\rightarrow\infty$, we obtain

$$h(\infty,\infty)\rightarrow\lim_{x\rightarrow\infty}6\ln x. \quad (21)$$

If we first let $\Lambda\rightarrow\infty$, we have for $m\gg M$

$$h(\infty,m)\approx 0.5+3\ln(M/2E_0)+6\ln(m/M). \quad (22)$$

Letting $m\rightarrow\infty$, we obtain Eq. (21) again.

Finally, if we set

$$\Lambda^2/m^2=\lim_{\epsilon\rightarrow 0}(1+\epsilon) \quad (23)$$

and $\Lambda\gg M$, we have

$$h(\Lambda,\Lambda)\approx 6\ln(\Lambda/M)+3\ln(M/2E_0)-2.5. \quad (24)$$

Letting $\Lambda\rightarrow\infty$, we obtain Eq. (21) again.

Thus, $h(\Lambda,m)$ diverges only if both Λ and $m\rightarrow\infty$, and the divergent expression $\mathrm{Lim}_{\Lambda,m\rightarrow\infty}h(\Lambda,m)$ is independent of the manner in which the limits are taken.

Table I gives the discrepancy between the predicted and measured values of T_μ for a number of values of Λ and m. For $m=M/2$ and $\Lambda=M/3$ this discrepancy is −0.1%. Included in the results is a 1.4% discrepancy between experiment and the Fermi theory without radiative corrections. Table II shows the source of the corrections to the μ lifetime for selected values of Λ and m. Figure 3 presents the results graphically.

TABLE II. Corrections to $T_{O^{14}}$ and T_μ for selected values of cutoff and boson mass. $\Delta T_x=(T_{x\ \mathrm{corrections\ included}}-T_{x\ \mathrm{without\ corrections}})/T_{x\ \mathrm{without\ corrections}}$. The final column includes a 1.4% discrepancy between experiment and the Fermi theory without radiative corrections.

Λ/M	m/M	Corrections included	$\Delta T_{O^{14}}$ (%)	ΔT_μ (%)	$\frac{T_{\mu\ \mathrm{th}}-T_{\mu\ \mathrm{exp}}}{T_{\mu\ \mathrm{exp}}}$ (%)
1	∞	radiative	−1.7	0.4	3.5
1	$\frac{1}{2}$	nonradiative	0	−3.0	−1.6
1	$\frac{1}{2}$	radiative	−2.8	−1.2	3.0
1	$\frac{1}{2}$	total	−2.8	−4.2	0
5	∞	radiative	−2.6	0.4	4.4
5	$\frac{1}{2}$	nonradiative	0	−3.0	−1.6
5	$\frac{1}{2}$	radiative	13.4	15.1	3.1
5	$\frac{1}{2}$	total	13.4	12.1	0.1

VI. CONCLUSIONS

We see that the theoretically predicted μ lifetime agrees with the experimental data for $m\approx M_K$ and that our numerical results are insensitive to the value chosen for the cutoff. Indeed, Eqs. (18), (19), and (22) imply that $T_\mu/T_{O^{14}}$ is cutoff independent if the IVB mass is finite. In the Fermi theory this ratio is logarithmically divergent. The above results were obtained by neglecting the momentum transfer in the diagrams involving boson self-energy parts. An examination of the boson propagators in these diagrams indicates that the leading momentum-transfer term contributing to the μ lifetime for universal coupling is $O(M_\mu M_e/m^2)$. That is, the quadratic divergence in the IVB self-energy parts will yield a significant contribution only when $(\alpha/2\pi)(\Lambda^2/m^2)(M_\mu M_e/m^2)\gtrsim 0.1\%$. For $m\approx M_K$ this implies $\Lambda\gtrsim 35M$. Therefore, the results shown in Fig. 3 are valid for a range of Λ from less than M to approximately $35M$.

Any possible significance of an IVB with a mass equal to that of a K meson must await the description of decay processes by a theory which leads to unique (and finite) results.

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Universal Neutrino Degeneracy

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Modern cosmological theories imply that the universe is filled with a shallow degenerate Fermi sea of neutrinos. In the steady state and oscillating models (and perhaps also the "big bang" theories) it can be shown rigorously that the proportion of filled neutrino levels (plus the proportion of filled antineutrino levels) is precisely one up to a finite Fermi energy E_F. The proof takes into account both absorption and the repressive effects of already filled levels on neutrino emission. Experiment shows that $E_F\lesssim 200$ eV for antineutrinos and $E_F\lesssim 1000$ eV for neutrinos. The degenerate neutrinos could be observed (if $E_F>10$ eV) by looking for apparent violations of energy conservation in β^- decay. In the steady state and evolutionary cosmologies E_F is much too low to ever be observed, but in the oscillating cosmologies $E_F\simeq 5R_c$ MeV, where R_c is the minimum radius of the universe in units of its present radius; thus experiment already shows that the universe will contract by a factor over 10^3, if at all. Astronomical evidence plus Einstein's field equation (without cosmological constant) require in an oscillating cosmology that $E_F<2\times 10^{-3}$ eV (so $R_c<10^{-9}$) and suggest that higher energy neutrinos may represent the bulk of the energy of the universe. A model universe incorporating this idea is constructed.

I. INTRODUCTION

We have previously pointed out[1] that neutrinos may be subject to an Olbers paradox even if photons are not. (An Olbers paradox is an infinite value for some total cosmic flux.) Neutrinos carry a quantum number, so that their number density must stay finite, however red shifted they may become. It was shown that the popular modern cosmologies do not lead to a neutrino Olbers paradox, but for very different reasons: In the steady-state cosmology the speed of neutrinos vanishes beyond a certain distance (if we use a time-independent metric). In the evolutionary cosmologies neutrino emission has only been going on for a finite time. In the oscillating cosmologies absorption of neutrinos must become important during the contracted phase.

But neutrinos differ also from photons in that they obey Fermi statistics. The question arises whether any cosmological theories give rise to a degenerate neutrino population.[2] The answer is definitely yes, but again characteristic differences among these theories appear. In any cosmology (such as the steady state or oscillating theories) in which neutrino emission has been going on for an infinite time, it will be shown rigorously that precisely one-half of all neutrinos and antineutrino energy levels are full at very low energy. (The neutrino levels may be full and the antineutrino levels empty, for example.) The same is likely to be true in a "big-bang" evolutionary theory. The calculation takes into account both absorption and the repressive effect of already filled levels.

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[1] S. Weinberg, Nuovo cimento **25**, 15 (1962). The preprint of this work contained some mistaken remarks about degeneracy which should be ignored.

[2] This speculation was first raised by K. M. Watson (private communication).