Magnetoplasma Resonance in Germanium

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Microwave magnetoplasma resonances have been studied in small samples of p- and n-type germanium as a function of carrier concentration, sample shape, and field configuration. For sufficiently small samples in a uniform microwave field, a simple drift-velocity treatment appears to explain all of the observed phenomena, even when more than one type of carrier and tensor masses are involved. In the case of p-type germanium, it is demonstrated that a majority carrier can act to short out the plasma effects of a minority carrier and that one can observe a "cyclotron resonance" line even when the plasma frequency is high compared to the frequency of the applied field. In n-type germanium, the tensor mass results in a large longitudinal magnetoplasma effect, which is absent in indium antimonide where the mass is isotropic.

INTRODUCTION

MAGNETOPLASMA resonances have been studied in germanium as a function of carrier concentration, sample shape, and orientation. The resonances were observed by placing small ellipsoids of germanium in a microwave cavity and measuring the absorption of microwave power as a function of applied dc magnetic field. Magnetoplasma resonance was first observed in a solid by Dresselhaus, Kip, and Kittel[1] with an approximately fixed concentration of a single type of carrier. Resonances due to the interaction of two different types of carriers have also been observed recently.[2]

In considering the interaction of electromagnetic waves with carriers in solids, one of two simple boundary value problems is generally assumed. The first is that of plane wave impinging on a semi-infinite slab of material. If the applied magnetic field is normal to the surface of the material and the band structure and crystal orientation are such that there is no component of the carrier motion normal to the surface, polarization charges do not build up on the surface and depolarization fields need not be considered.[3] The case where the magnetic field is in the plane of the surface could possibly involve polarization effects. This situation has not been solved in general, although the case where the mean free path of the carriers is large compared to the skin depth has been extensively treated; it is the situation in which one can observe Azbel'-Kaner cyclotron resonance in metals.[4]

The other simple boundary-value problem, and the one used in the present work, is that of a small ellipsoidal sample in a uniform microwave field. This is the usual situation for microwave cyclotron resonance work. There may now exist depolarization fields originating on the charges built up on the surfaces by the applied electric field. When the carrier concentration is large enough that the plasma frequency is comparable to the applied frequency, the question of the depolarization fields—often avoided in the semi-infinite surface treatment—becomes the dominant issue. Studies of the polarization effects in this simple geometry can give insight into the transport mechanisms which cancel the polarization effects in the extreme anomalous case (e.g., the transverse Azbel'-Kaner effect) and also into the problems polarization may introduce when the skin depth is comparable with the mean free path as it is for the semimetals and some heavily doped but high-mobility semiconductors. *Note added in proof.* An interesting treatment of this problem has been given recently by E. Burstein, P. J. Stiles, D. N. Langenberg, and R. F. Wallis [Phys. Rev. Letters **9**, 260 (1962)]. The treatment in the present paper follows the drift velocity treatment of Dresselhaus, Kip, and Kittel[1] and its extension by Kittel.[5]

THEORY

If a small ellipsoidal sample is placed in a microwave cavity in a region of uniform electric field, the field and the polarization within the sample will also be uniform. By a small sample, we mean one which has at least one dimension small compared to the penetration depth of the microwaves and all dimensions small compared to the wavelength inside the sample material.[6] We also require that the sample be far enough from the walls of the cavity that the presence of the sample does not affect the surface charge distribution on the walls.[7] We will always assume that the applied field is parallel to

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[1] G. Dresselhaus, A. F. Kip, and C. Kittel, Phys. Rev. **100**, 618 (1955).

[2] R. E. Michel and B. Rosenblum, Phys. Rev. Letters **7**, 234 (1961).

[3] See for example, J. K. Galt, W. A. Yager, F. R. Merritt, and B. B. Citlin, Phys. Rev. **114**, 1396 (1959) and B. Lax and G. Wright, Phys. Rev. Letters **4**, 16 (1960).

[4] See B. Lax and J. G. Mavroides, *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic Press Inc., New York, 1960), Vol. 11 and references therein.

[5] C. Kittel, Proceedings of the Conference on Radio and Microwave Spectroscopy, Duke University, 1957 (unpublished).

[6] The samples are, however, large compared to a Debye length; therefore the plasma modes discussed by P. Wolff [Phys. Rev. **112**, 66 (1958)] are not observed for the reasons he gives.

[7] In this connection, one can show that a plasma uniformly filling a condenser and driven by a constant voltage source exhibits no plasma resonance absorption. If it were driven by a constant current source, a resonance would exist. If the plasma occupies only a small fraction of the distance between the plates, the space between the plasma and the plates acts as a high impedance and therefore the plasma itself is effectively driven by a constant current source and does show a resonance at the usual plasma frequency.

a principal axis of the sample and thus the internal field will be parallel to the applied field.

1. Single Type of Carrier

The equation of average motion of the particles in terms of the average displacement $\mathbf{r}$ from their equilibrium position is

$$m^*(\ddot{\mathbf{r}}+\rho\dot{\mathbf{r}})=e\mathbf{E}_{\text{eff}}+(e/c)\dot{\mathbf{r}}\times\mathbf{H}_0, \tag{1}$$

where m^* is the effective mass, which we initially assume is independent of energy and momentum, $\rho(=1/\tau)$ is the collision frequency, $\mathbf{E}_{\text{eff}}$ is the effective internal electric field seen by each carrier, and $\mathbf{H}_0$ is the applied magnetic field. We assume that these carriers are neutralized by ions uniformly distributed and rigidly fixed in the lattice.

In the effective-mass approximation, we can treat the carriers as essentially free particles with a mass m^*. We therefore do not have to consider any Lorentz or cavity contribution to the effective field on the electron. This has been shown by Darwin[8] for approximately free carriers and generalized by Nozières and Pines.[9] If an appreciable Lorentz correction were required, our experimental results would be different. We may therefore set

$$\mathbf{E}_{\text{eff}}=\mathbf{E}-\mathbf{L}\cdot\mathbf{P}, \tag{2}$$

where $\mathbf{E}$ is the applied electric field, $\mathbf{P}$ is the uniform polarization of the sample, and $\mathbf{L}$ is the depolarization tensor which is diagonal if the applied field is in a principal direction of the ellipsoid. We will for the present make the further specification that $\mathbf{H}_0$ is perpendicular to $\mathbf{E}$ and that the elements of $\mathbf{L}$ in the two principal directions perpendicular to $\mathbf{H}_0$ are equal. ($\mathbf{L}$ can now be considered a scalar.) The polarization $\mathbf{P}$ is the sum of the polarization of the crystal lattice and the polarization of the free carriers:

$$\mathbf{P}=\chi\mathbf{E}_{\text{eff}}+Ne\mathbf{r}, \tag{3}$$

where χ is the dielectric susceptibility of the lattice and N is the density of free carriers. $Ne\mathbf{r}$ is just the dipole moment per unit volume due to the free carriers. With Eqs. (2) and (3), Eq. (1) may be rewritten:

$$\begin{aligned} m^*(\ddot{\mathbf{r}}+\rho\dot{\mathbf{r}})+[L/(1+\chi L)]Ne^2\mathbf{r} \\ =e\mathbf{E}/(1+\chi L)+(e/c)\dot{\mathbf{r}}\times\mathbf{H}_0. \end{aligned} \tag{4}$$

This is just the equation for a driven, damped, harmonic oscillator in a magnetic field. Without the magnetic field, the driving force, or the damping, the system could freely oscillate at an angular frequency

$$\omega_p=[LNe^2/m^*(1+\chi L)]^{1/2},$$

the plasma frequency. If we assume $\mathbf{E}=\mathbf{E}_0e^{i\omega t}$ and that the applied field is circularly polarized, we can readily solve for the steady-state value of $\dot{\mathbf{r}}$. The current is given by $\mathbf{j}=Ne\dot{\mathbf{r}}$ and we can define a complex conductivity σ such that $\mathbf{j}=\sigma\mathbf{E}_0/(1+\chi L)$. The power absorption $\mathcal{P}$ is given by

$$\mathcal{P}=\text{Re}\left(\frac{\mathbf{E}_0}{1+\chi L}\cdot\mathbf{j}^*\right)=\frac{\mathbf{E}_0^2}{(1+\chi L)^2}\text{Re}(\sigma). \tag{5}$$

From the solution of Eq. (4) we obtain

$$\text{Re}(\sigma)=\frac{Ne^2\tau}{m^*}\left(\frac{1}{1+(\omega'-\omega_c)^2\tau^2}\right), \tag{6}$$

where $\omega'=\omega-\omega_p^2\omega^{-1}$ and $\omega_c=eH_0/m^*c$.

One can see by inspection that the resonance maximum occurs when $\omega'=\omega_c$. When $\omega_p=0$, $\omega'=\omega$ and we have the usual cyclotron resonance condition. As ω_p increases, the resonance line shifts towards lower magnetic field passing through zero field when $\omega_p=\omega$ and moves to higher negative values of magnetic field for $\omega_p>\omega$. The absolute width of the resonance line remains constant if τ remains constant. We note that when $\omega_p\gg\omega$, the magnetic field for resonance becomes independent of the mass of the carriers. One therefore sees that as long as $\omega_p\ll\omega$, one can observe ordinary cyclotron resonance, but at larger carrier concentrations there is no absorption resonance associated with $\omega_c=\omega$.

2. Two Types of Carriers

We will now consider the case when two different types of carriers exist in the sample. We will assume that the carriers have the same magnitude of charge but not necessarily the same sign. They may have different masses and perhaps different collision frequencies. The total polarization in this case is just

$$\mathbf{P}=\chi\mathbf{E}_{\text{eff}}+LN_1e_1\mathbf{r}_1+LN_2e_2\mathbf{r}_2, \tag{7}$$

where N_1, N_2; $\mathbf{r}_1$, $\mathbf{r}_2$ and e_1, e_2 are the density, average displacement and charge of the first and second type of carrier, respectively. A pair of coupled equations can be written for the motion of both types of carriers. Again we assume that any charge unbalance of these carriers is neutralized by a uniform charge fixed in the lattice.

$$\begin{aligned} m_1^*(\ddot{\mathbf{r}}_1+\rho_1\dot{\mathbf{r}}_1)+\frac{LN_1e_1^2\mathbf{r}_1}{1+\chi L}+\frac{LN_2e_1e_2\mathbf{r}_2}{1+\chi L} \\ =\frac{e_1\mathbf{E}}{1+\chi L}+\frac{e_1}{c}\dot{\mathbf{r}}_1\times\mathbf{H}_0, \\ m_2^*(\ddot{\mathbf{r}}_2+\rho_2\dot{\mathbf{r}}_2)+\frac{LN_2e_2^2\mathbf{r}_2}{1+\chi L}+\frac{LN_1e_1e_2\mathbf{r}_1}{1+\chi L} \\ =\frac{e_2\mathbf{E}}{1+\chi L}+\frac{e_2}{c}\dot{\mathbf{r}}_2\times\mathbf{H}_0. \end{aligned} \tag{8}$$

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[8] C. G. Darwin, Proc. Roy. Soc. (London) **A146**, 13 (1934).
[9] P. Nozières and D. Pines, Phys. Rev. **109**, 762 (1958).

Kittel has studied solutions of these coupled equations in some limiting cases[5] for a semi-infinite plane geometry where $\mathbf{E}$ is perpendicular to $\mathbf{H}_0$ and both are in the plane of the surface. (This is the geometry for the transverse Azbel'-Kaner effect.) In the approximation he considers, while there are no actual resonances in the surface impedance, there are resonances in the components of the conductivity, which in these limiting cases are at the cyclotron field of the minority carriers. Thus, the majority carriers can act to short out the plasma effects of the minority.

In the present geometry of a small spheroidal sample, it is not difficult to solve for the resonant magnetic fields when $\rho_1=\rho_2=0$ for arbitrary carrier concentrations. One can actually observe these resonances since here the power absorption is determined by the real part of the conductivity. If we set $\mathbf{E}$, ρ_1, and ρ_2 equal to zero, Eqs. (8) can readily be solved in the following manner: When written in component form with $\mathbf{H}_0$ in the z direction the y-component equations are multiplied by (i) and added to the corresponding x-component equations. We define the variables $v_{1,2}=(\dot{x}_{1,2}+i\dot{y}_{1,2})$ and assume that $v_{1,2}=v_{01,02}e^{i\omega t}$. Equations (8) then become

$$\begin{aligned}\left(\omega^2 m_1^*-\frac{L}{1+\chi L}Ne_1^2-\mathbf{H}_0\frac{e_1}{c}\omega\right)v_{01}&\\ -\left(\frac{L}{1+\chi L}N_2e_1e_2\right)v_{02}&=0,\\ -\left(\frac{L}{1+\chi L}N_1e_1e_2\right)v_{01}&\\ +\left(\omega^2 m_2^*-\frac{L}{1+\chi L}Ne_2^2-\mathbf{H}_0\frac{e_2}{c}\omega\right)v_{02}&=0.\end{aligned} \tag{9}$$

To find the values of $\mathbf{H}_0$ for which solutions exist for these equations, we set the determinant of the coefficients of v_{01} and v_{02} equal to zero and solve the resulting secular equation. We obtain

$$\begin{aligned}\frac{2e_1H_0}{m_1^*c}=\left(1-\frac{\omega_{p1}^2}{\omega^2}\right)+\mu f\left(1-\frac{\omega_{p2}^2}{\omega^2}\right)&\\ \pm\left\{\left[\left(1-\frac{\omega_{p1}^2}{\omega^2}\right)+\mu f\left(1-\frac{\omega_{p2}^2}{\omega^2}\right)\right]^2\right.&\\ \left.+4f\mu\left[\frac{\omega_{p1}^2}{\omega^2}+\frac{\omega_{p2}^2}{\omega^2}-1\right]\right\}^{1/2},&\end{aligned} \tag{10}$$

where $\omega_{pi}^2=LN_ie^2/(1+\chi L)m_i^*$, $\mu=m_2^*/m_1^*$, and $f=+1$ if the charges have the same sign, and $f=-1$ if the charges have opposite sign. In spite of its appearance, Eq. (10) is symmetric in the two carriers. One sees that, in general, there are two magnetic fields at which one can observe a resonance.[10] When $\omega_{p1}^2/\omega^2\sim\omega_{p2}^2/\omega^2\sim 0$, Eq. (10) gives the usual two cyclotron resonance lines. If $\omega_{p2}^2/\omega^2\sim 0$ and $\omega_{p1}^2/\omega^2>0$ there is some coupling of the motion, but Eq. (10) reduces to give $e_2H_0/m_2^*c=\omega$ and $e_1H_{01}/m_1^*c=\omega-\omega_{p1}^2\omega^{-1}$, i.e., just the same resonant field for each carrier as if the other were not present. (If the inhomogeneous equations are solved it is found that the intensities of these lines are affected by the other carrier even though the positions are not.)

The most interesting case is when ω_{p1}^2/ω^2, $\omega_{p2}^2/\omega^2\gg 1$ which may be considered the case of a quasi-metal. The plasma frequencies are all large compared to the incident frequency, but the sample is small enough that the incident field penetrates uniformly. If we can also assume that even if the carriers have opposite sign the first term in the radical is large compared to the second, we can expand the radical in Eq. (10). We then obtain for the two solutions for H_0:

$$\begin{aligned}H_{01}&=-\frac{m_1^*c}{e_1}\left(\frac{\omega_{p1}^2}{\omega}+\mu f\frac{\omega_{p2}^2}{\omega}\right)\\ &=-\frac{e_1cL}{(1+\chi L)\omega}(N_1+fN_2),\\ H_{02}&=\frac{m_2^*c\omega}{e_2}\left(\frac{\omega_{p1}^2+\omega_{p2}^2}{\omega_{p1}^2+\mu f\omega_{p2}^2}\right)\\ &=\frac{m_2^*\omega c}{e_2}\left[\frac{N_1/N_2+m_1^*/m_2^*}{N_1/N_2+f}\right].\end{aligned} \tag{11}$$

We see that one resonance line moves out to high magnetic field as the carrier concentrations increase. The magnetic field for this resonance is independent of the carrier masses in this approximation as in the case of the plasma resonance for a single carrier type. The second resonance is independent of the carrier concentration and sample shape if the ratio of the concentrations and the ratio of the masses does not change. In the further approximation that one plasma frequency is much greater than the other, the second resonance is at the cyclotron field of the carrier with the smaller plasma frequency.

It is tempting to interpret the lack of polarization effects in the transverse Azbel'-Kaner effect as the cancellation of the plasma fields of the "effective" carriers.[5] This problem has only been treated in the case where the skin depth is small compared to the mean free path, i.e., the number of "effective" carriers is extremely small compared to the number of ineffective ones. In the case where the skin depth is comparable to the mean free path, the number of "effective" carriers is comparable to the number of "ineffective" carriers (actually the concept loses much of its value in this case). One might now expect the resonances to be at a field other

[10] There will, in general, be n resonance lines for n different carriers.

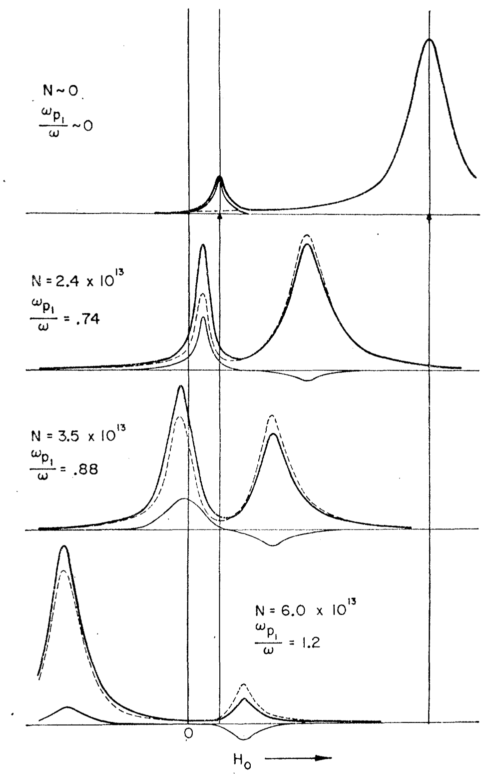

FIG. 1. Calculated microwave power absorption as a function of magnetic field for various carrier concentrations N. The dashed line is the power absorption due to the heavy holes, the light solid line that due to the light holes, and the bold line is the sum.

than the cyclotron field in analogy to H_{02} in Eq. (11). Note that a small amount of shift in a "cyclotron resonance" line due to plasma effects is not necessarily toward lower magnetic field as it would be for a single carrier.

One can readily solve the actual inhomogeneous Eqs. (8) including finite scattering frequencies on a computing machine. This solution yields the actual absorption spectra $\mathcal{P}=\mathrm{Re}[(N_1e_1\dot{\mathbf{r}}_1+N_2e_2\dot{\mathbf{r}}_2)^*\cdot\mathbf{E}_0/1+\chi L]$ and thus gives the linewidths and intensities. It turns out that even when the scattering rate is rather large ($\omega/\rho\sim 1$) the peak of the resonant absorption falls very close to the solutions of the inhomogeneous equations.

We have solved this problem on a computer for the ratio of masses and plasma frequencies for the light and heavy holes in germanium (1:8 and 1:1.6, respectively).[11] The power absorption as a function of magnetic field for several values of the total number of holes per cubic centimeter, N, is shown in Fig. 1. We chose $\omega\tau$ equal to 5 and 10 for the light and heavy holes, respectively, independent of N. (These values are somewhat higher than have been obtained in our experiments on germanium but are chosen here to better illustrate the phenomena involved.) The light solid line is the absorption by the light holes, the dashed line is that by the heavy holes, and the bold line is the sum. Both resonances shift to low field; the resonance line originally due to the light holes increases in intensity, passes through zero field, and moves to arbitrarily large negative field as the carrier concentration increases. The resonance line that was originally due to the heavy holes decreases in intensity and asymptotically approaches a field equal to 4/3 the light-hole cyclotron resonance field. If the plasma frequency of the heavy holes were much larger than the plasma frequency of the light holes, this line would approach the light-hole cyclotron field and indeed be a "light-hole cyclotron resonance." One certainly can not say in this case that the light-hole cyclotron resonance line is due to power absorption by the light holes. Whenever plasma effects are present, the absorption is a cooperative phenomena involving all of the carriers. One notices from Fig. 1 that the motion of the carriers at the small resonance at high concentrations is such as to cancel the polarization effects. Thus while the main plasma line results from a mode of oscillation where the two types of carriers move in phase, the small cyclotron resonance line arises from a mode in which they move out of phase. The shift of these resonances as a function of carrier concentration is also shown in Fig. 3.

3. Anisotropic Masses

If the effective masses in our material are not isotropic, the m^* in Eq. (1) or Eqs. (8) must be replaced by the appropriate effective mass tensors.[12] In this case the double use of the complex notation for both the out-of-phase component and the y direction does not lead to any simplification of the solution of the equations. However, one can readily solve for the complex magnetoplasma conductivity for a single spheroidal mass (two principal masses equal). Since this solution exhibits several of the features of the actual four-ellipsoid problem of n-type germanium, an examination of it is particularly interesting. For an isotropic mass (and constant ρ), the conductivity for the electric field in the magnetic-field direction is independent of the magnetic field. This is not so in the case of an anisotropic mass. We therefore treat two cases: First, the transverse case with $\mathbf{E}$ perpendicular to $\mathbf{H}_0$, and second, the longitudinal case with $\mathbf{E}$ parallel to $\mathbf{H}_0$.

We assume that $\mathbf{M}^*$ is a tensor with two principal masses equal. We choose the arbitrary direction of the dc magnetic field as the y axis and the electric field (and the x axis) perpendicular to this direction in such a way that the z axis is perpendicular to the figure axis of the spheroidal mass tensor. This minor sacrifice of

[11] We have assumed that both the light and heavy holes can be approximated by isotropic masses with average values of the actual masses.

[12] We do not consider effective masses more complex than tensor. The actual masses of the holes in germanium would be such a more complex case.

generality greatly simplifies the algebra. In Eq. (1), with

$$\mathbf{M}^*=\begin{Vmatrix} M_{xx} & M_{xy} & 0 \\ M_{xy} & M_{yy} & 0 \\ 0 & 0 & M_{zz} \end{Vmatrix},$$

we assume $E_x=E_{0x}e^{i\omega t}$ and $\dot{\mathbf{r}}=\mathbf{v}e^{i\omega t}$. We may now solve the resulting algebraic equations for v_x:

$$\sigma_{xx}\equiv(1+L\chi)\frac{Ne^2v_x}{E_{0x}}=\frac{Ne^2}{M_{xx}}\frac{(i\omega_y'+\rho)(i\omega_z'+\rho)}{(i\omega_y'+\rho)[(i\omega_x'+\rho)(i\omega_z'+\rho)+\omega_{cy}^2]-(M_{xy}^2/M_{xx}M_{yy})(i\omega_z'+\rho)(i\omega+\rho)^2}, \tag{12}$$

where

$$\omega_i'=\omega-LNe^2\omega^{-1}/(1+\chi L)M_{ii}$$

and

$$\omega_{cy}=eH_0/c(M_{xx}M_{zz})^{1/2},$$

the usual cyclotron frequency for $\mathbf{H}_0$ in the y direction. The separation of this expression into real and imaginary parts is rather tedious and not very rewarding. We can readily find the magnetic field for resonance (for $\rho/\omega\ll1$) by setting the denominator equal to zero for $\rho=0$. This yields

$$\omega_{cy}=\pm\left(\omega_x'\omega_y'-\frac{M_{xy}^2}{M_{xx}M_{yy}}\frac{\omega_z'}{\omega_y'}\omega^2\right)^{1/2}. \tag{13}$$

We note that, unlike the case of an isotropic mass, there are certain frequencies ω at which the system cannot be brought into resonance by the application of a magnetic field, i.e., those which make the quantity in brackets negative. As in the isotropic case, when the plasma frequencies become large compared to ω the position of the resonance becomes independent of the masses.

For the longitudinal case we assume that both $\mathbf{E}$ and $\mathbf{H}_0$ are both in a single arbitrary direction and we choose it as the x axis. The z axis is chosen perpendicular to this direction and also perpendicular to the figure axis.

$$\sigma_{xx}=\frac{Ne^2}{M_{xx}}\frac{[(i\omega_y'+\rho)(i\omega_z'+\rho)+\omega_{cx}^2]}{(i\omega_x'+\rho)[(i\omega_y'+\rho)(i\omega_z'+\rho)+\omega_{cx}^2]-(M_{xy}^2/M_{xx}M_{yy})(i\omega_z'+\rho)(i\omega+\rho)^2}. \tag{14}$$

We note that the expressions in the square brackets in the numerator and the denominator of Eq. (14) are identical. Thus, if $M_{xy}=0$, i.e., the magnetic and electric fields are along a principal axis, the terms in square brackets cancel, and one merely has the high-frequency conductivity of a plasma with no magnetic-field dependence. There are three other conditions in which we can readily see that the longitudinal magnetoplasma resistance tends to disappear: at high carrier concentrations, at high magnetic fields and when $\omega_z'=0$ for $\rho/\omega\ll1$. The longitudinal magnetoplasma resistance for the actual case of n-type germanium exhibits similar features.

The set of coupled vector equations for the motion of the carriers in a multivalley semiconductor can be written in the following compact form:

$$\mathbf{D}_j\cdot\mathbf{M}^*\cdot\mathbf{D}_j^{-1}\cdot(\ddot{\mathbf{r}}_j+\rho_j\dot{\mathbf{r}}_j)+Le^2/(1+\chi L)\times\textstyle\sum_j N_j\mathbf{r}_j-(e/c)\dot{\mathbf{r}}_j\times\mathbf{H}_0=e\mathbf{E}, \tag{15}$$

where $\mathbf{r}_j$, ρ_j, N_j are the average position, the scattering rate and the number of carriers in the jth valley, respectively. For n-type germanium

$$\mathbf{M}^*=m_0\begin{Vmatrix} 0.082 & 0 & 0 \\ 0 & 0.082 & 0 \\ 0 & 0 & 1.64 \end{Vmatrix},\quad \mathbf{D}_{1,3}=\begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{3} & \pm\sqrt{2}/\sqrt{3} \\ 0 & \mp\sqrt{2}/\sqrt{3} & 1/\sqrt{3} \end{Vmatrix},\quad \mathbf{D}_{2,4}=\begin{Vmatrix} 1/\sqrt{3} & 0 & \pm\sqrt{2}/\sqrt{3} \\ 0 & 1 & 0 \\ \mp\sqrt{2}/\sqrt{3} & 0 & 1/\sqrt{3} \end{Vmatrix},$$

where m_0 is the free electron mass and the [1$\bar{1}$0], [110], and [001] axis in the crystal are chosen as the x, y, and z directions, respectively.

The differential equations are reduced to algebraic equations by the substitution of an $e^{i\omega t}$ time dependence for $\mathbf{E}$ and $\mathbf{r}_j$. The problem was then programmed for a computer to give the $\dot{\mathbf{r}}_j$ as the parameters $\mathbf{E}$, $\mathbf{H}_0$, ρ_j, N_j were changed. It was assumed that ρ_j and N_j were the same for each valley. We have computed the magnetoplasma resonance absorption for $\mathbf{H}_0$ in the [001] direction and $\mathbf{E}$ in the [010] direction for $\omega_p^2/\omega^2=7$ as a function of ω_c/ω. We here define the plasma frequency as

$$\omega_p=\left[\frac{Le^2}{1+\chi L}\frac{\sum N_j}{m_{100}^*}\right]^{1/2}$$

and

$$\omega_c=eH_0/m_{100}^*c,$$

where $m_{100}{}^*$ is the cyclotron mass for the magnetic field in the [100] direction.[13] In this direction, all the ellipsoids have the same cyclotron mass and only a single line is seen in cyclotron resonance. If the behavior were that of an isotropic carrier with $m^*=m_{100}{}^*$ the resonance would occur at $\omega_c/\omega=(\omega_p{}^2/\omega^2-1)=6$. We find that magnetoplasma resonance occurs at $\omega_c/\omega=5.8$. The position of the resonance was checked at two other values of $\omega_p{}^2/\omega^2$ and the line appears to move almost as if it were due to an isotropic carrier with mass $m_{100}{}^*$.

Most interesting is the appearance in the computed results of a weak resonance at twice the cyclotron field for both $\omega_p{}^2/\omega^2=28$ and 56. It was presumably masked by the baseline of the main line at $\omega_p{}^2/\omega^2=7$. It was about three orders of magnitude weaker than the main line. This resonance is presumably analogous to the residual resonance near the light-hole cyclotron resonance field at high plasma frequencies in p-type germanium in that it arises from a mode of oscillation of this complex plasma in which plasma forces are largely cancelling and the magnetic forces tend to dominate.[14] We have not studied other orientations of the magnetic field in the transverse case since the computing machine time involved can be fairly large. It might be in order to add that the existence of resonances of this type which do not shift with carrier concentration but are at fields other than the cyclotron field might justify considerable caution in the interpretation of cyclotron resonance measurements where plasma effects cannot be excluded or accounted for.

For the longitudinal case where the electric and magnetic fields are parallel we have obtained solutions for the fields parallel to the [001] axis. The high symmetry in this situation allows considerable simplification of the problem. The results of these computations are in excellent agreement with the experimental observations shown as the dashed lines in Fig. 6. The longitudinal magnetoplasma effect at low concentration is almost identical to the transverse resonance but saturates at a finite conductivity instead of falling to zero as $H_0{}^2$. At $N\approx6.4\times10^{13}$/cc the longitudinal effect almost disappears in a fairly abrupt manner. This is the value of N such that the number of electrons in each ellipsoid ($\frac{1}{4}N$) is just that amount to make $\omega_z'=0$ for each ellipsoid. At higher values of N the variation of conductivity with applied field reappears but now has the form shown in the last section of Fig. 6. As the carrier concentration increases the strength of longitudinal effect decreases relative to the transverse effect. We can thus find all the results deduced by inspection of Eq. (14) for the longitudinal effect for a single ellipsoid.

4. Linewidth and Resolution

In many magnetic-resonance experiments the sharpness and the resolution of the lines can be improved by making the observations at a higher experimental frequency. This is not true in the present case. If the sharpness of a magnetoplasma line is defined by the ratio of the resonant field divided by the half-width, it is equal to $\omega'\tau$. This quantity is zero when $\omega_p=\omega$. For the present purposes we are interested in having the maximum sharpness in the region where $\omega_p\sim2\omega\sim\frac{2}{3}\omega'$. We thus wish to find the conditions for a maximum $\omega_p\tau$. We will assume that the relaxation time due to ionized impurity scattering $\tau_I=aT^{-3/2}/N_I$, and the relaxation time due to phonon scattering $\tau_L=bT^{3/2}$, where N_I is the density of ionized impurities and T is the temperature. We further assume that the resultant scattering rate $(1/\tau)$ is just the sum of these rates, and that the only impurities in the sample are the ionized donors. We can set $\omega_p=cN^{1/2}$, and then

$$\omega_p\tau=\left[\frac{1}{ac}\left(\frac{N}{T^3}\right)^{1/2}+\frac{1}{bc}\left(\frac{N}{T^3}\right)^{-1/2}\right]^{-1}. \tag{16}$$

We find by setting the derivative of $\omega_p\tau$ with respect to N/T^3 equal to zero that $\omega_p\tau$ is a maximum for $N/T^3=a/b$ and $(\omega_p\tau)_{\max}=c(ab)^{1/2}/2$.

We note that the condition for maximum sharpness is independent of the operating frequency. The optimum value of N/T^3 is attained by properly doping the sample. For n-type germanium in the temperature and impurity region of interest here, $a=28$, $b=2.3\times10^{-9}$, $c=4.1\times10^4$, and $(\omega_p\tau)_{\max}=5$. This yields an $(\omega'\tau)_{\max}=7.5$. For samples with approximately the above optimum value of N/T^3 we experimentally obtained values for $\omega'\tau$ about one-third of this value. The difference is probably due in large part to ionized impurities in the sample in excess of the number of ionized donors. This limit on $\omega'\tau$ exists only because we insisted that $\omega_p\sim2\omega$. For $\omega_p\gg\omega$, $\omega'\tau$ can be considerably larger.

EXPERIMENTAL

To verify the theoretical approach discussed above we measured the microwave conductivity as a function of magnetic field of small, approximately ellipsoidal, samples of both n- and p-type germanium doped with about 10^{15} donors or acceptors per cc. The carrier concentrations were sufficiently high that magnetoplasma effects were important ($\omega_p\gtrsim\omega$). In particular, we checked the dependence of the magnetoplasma resonances on carrier concentration and sample shape and orientation. The carrier concentration was controlled by adjusting the temperature of suitably doped samples over the 10 to 30°K range. The concentration was determined as a function of temperature by Hall meas-

[13] When ω_p is not much larger than ω_c, there is some ambiguity in the definition of a plasma frequency. The above definition is the one which almost allows the transverse magnetoplasma resonance in the present case to be treated as if it resulted from a single isotropic mass. The mass used when $\omega_p\gg\omega_c$ would be the "conductivity mass."

[14] A similar resonance with considerably greater intensity exists for a model of two equivalent ellipsoids at right angles to each other with the magnetic field directed along their common principal axis.

urements. In order to minimize the error due to inhomogeneities in doping impurity concentration, the samples used for the plasma experiments were cut from between the potential arms of the Hall sample after the Hall measurements were completed.

1. Hall Measurements

We attempted to make a fairly accurate determination of the carrier concentration as a function of temperature. Two sets of potential arms were used to insure that there were no large carrier concentration inhomogeneities in the sample; in all samples actually used, the difference in concentration between the arms did not exceed 2%. The contacts were soldered to the sample with a suitably doped solder, and were Ohmic to within 5%, even at 15°K. Unsoldering and resoldering the contacts had no effect on the measurements. The Hall voltages were measured potentiometrically. The samples were glued to a sapphire block with GE varnish No. 7031 which was in turn glued to a copper block containing two germanium resistance thermometers. This assembly was placed in the can shown in Fig. 2 which will be described in more detail below.

The n-type samples were so oriented that the current was in the [100] direction. In this case errors in Hall measurements due to selective population of the valleys[15] would not exist since all of the valleys are equivalent. All of the Hall measurements were made in a magnetic field of about 8000 Oe. At this magnetic field $\omega_c\tau \sim 10$ and we are in the high-field region, where the Hall constant depends only on the number of carriers. This was checked by determining the Hall constant as a function of field and noting that it was constant to within 1% over the range 4000 to 9000 Oe.

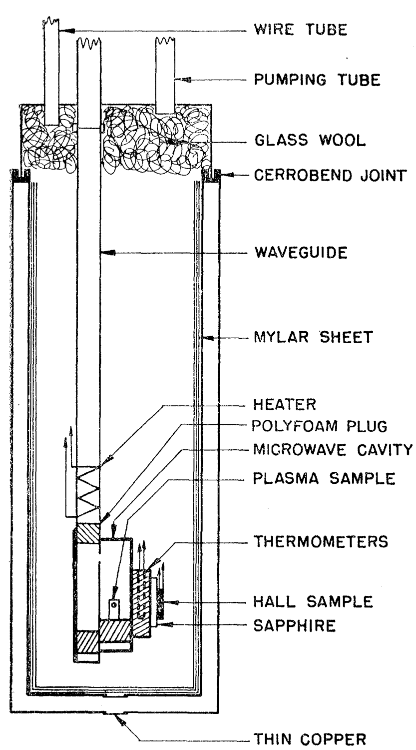

FIG. 2. Schematic drawing of the can used for controlling the temperature of the sample during Hall and plasma measurements.

[15] S. Koenig, Helv. Phys. Acta 8, 795 (1961).

2. Plasma Resonance

When the Hall measurements were completed, the sample for the plasma measurement was cut from the Hall sample. For disks the Hall sample was ground to a thickness of 0.010 in. and a disk about 0.070 in. in diameter cut with an ultrasonic cutter. The disk was then generally thinned to about 0.005 in. with a CP No. 8 etch. After plasma measurements on the disks they were cut into "needles" about 0.070×0.005×0.005 in. The corners of the needles were then rounded by etching. Spheres and "ellipsoids" were formed by cutting appropriately shaped rectangular parallelopipeds from the Hall sample. These were then agitated with an air jet in a cylinder lined with a fine emery paper. The resulting samples were approximate ellipsoids with about the same axial ratios as the initial parallelopipeds. These were usually further reduced in size by etching.

For the plasma measurements the sample was placed at an electric field maximum at the center of a TE_{01n} cavity resonant at about 23 kMc. The cavity was in one of the symmetrical arms of a magic tee. The power absorption by the sample was determined as a function of magnetic field by a measurement of the power reflected from this cavity. Neither light modulation nor field modulation was used and the change in the reflection from the undercoupled cavity was thus proportional to the absorption.

In order to accurately control the carrier concentration, considerable care had to be taken in the control and determination of the sample temperature. The determination of the absolute temperature was not actually required, but the temperature scale had to be consistent for the Hall and the microwave measurements to within 0.1°K. This presented no real problem for the case of the Hall measurements where the sample can readily be placed in good thermal contact with the thermometers. However, for plasma resonance measurements of significant absolute accuracy, the sample could not be close to any significant volume of material with dielectric constant much different from unity. The sample was therefore suspended in the center of the cavity by glueing it with a thin layer of GE No. 7031 to a 1-mil-thick Mylar strip. Thermal contact to the thermometers was made with helium gas. The apparatus for the temperature control and measurement is shown schematically in Fig. 2.

The can, which was largely double walled, was submerged in liquid helium. The space between the walls

contained air, which froze and formed a vacuum. The inside wall of the can was insulated with several layers of Mylar sheet and the upper cup with glass wool. To avoid a dangerous explosion in the event that the can developed a slight leak and was subsequently removed from the helium, a hole was drilled and covered with a 1-mil copper foil. This section of the can was soldered to the upper cup with Cerrobend. A thin-walled stainless steel waveguide extended into the can, where it joined a brass waveguide of 10-mil wall thickness.

A heater coil of Manganin wire was wound around this guide, and a dc current was adjusted to give the proper temperature. The copper microwave cavity was in contact with a copper block containing two Texas Instrument germanium resistance thermometers. A plug of polyfoam prevented the sample from being in more intimate contact with the heater than were the thermometers. Wires from the heater, the thermometers, and the Hall sample were brought out through a stainless steel tube, the vacuum seal being made at room temperature. The wave guide and the cavity were terminated with adjustable shorts so the frequency and coupling of the cavity could be adjusted. A pumping tube was provided to adjust the pressure of helium in the can. Before transferring liquid helium approximately one atmosphere of helium was introduced into the can. After the transfer this was pumped to about 10^{-2} mm Hg. After the heater current was set it required about 15 min for the system to come to equilibrium. The temperature at which the system came to equilibrium did depend on the pressure of helium gas in the can, but the plasma frequency vs thermometer temperature curve obtained was independent of pressure for a pressure variation of more than an order of magnitude in each direction about the operating pressure. This gives confidence that the sample and the thermometers were indeed at the same temperature. We always operated at a sufficiently low microwave power that the results were independent of the power.

All of the measurements were taken with linearly polarized microwaves. In some cases—especially that of the light and heavy holes—the results would have been more impressive with circular polarization, however, the added complications connected with making adjustments on the cavity over this temperature range to insure accurate circular polarization at the sample were sufficient to discourage us from attempting this. In order to do the curve fitting comparison (necessitated by the low $\omega\tau$) between theory and experiment for the light and heavy holes a fairly exact knowledge of degree of polarization would be required.

The curves of absorption vs magnetic field were traced on a pen recorder. The magnetoplasma resonance field was taken as the field for maximum reflection from the undercoupled cavity with a correction for the finite observed $\omega'\tau$. It was assumed in this correction that the line shape was Lorentzian although this was only approximately true in some cases.

Since our samples were only approximate ellipsoids, the electric field was not strictly uniform over the entire sample. An experiment was performed to check whether this nonuniformity gave any appreciable contribution to the broadening, or changed the resonant magnetic field. The plasma resonance of an approximate ellipsoid made by the blower-grinder technique was measured at several temperatures (carrier concentrations). A flat face was then ground in a direction not perpendicular to any principal axis; about one-third of the crystal was removed. The plasma measurements were then repeated with essentially the same results. The concept of a depolarization factor therefore seems to be a moderately good approximation even for samples far from ellipsoidal.

Although our volume to surface ratio was quite small, one would still not expect surface effects to change the average carrier concentration. The absence of this effect was checked by etching the same sample first to give an n-type surface and then to give a p-type surface. The results of the plasma measurements were the same in both cases. The absence of any such effect is further confirmed by the consistency of our results with samples of different volume to surface ratios.

RESULTS

1. p-Type Germanium

The resonances observed were interpreted with the assumption that both hole masses are independent of energy and momentum. While this is an excellent approximation for the light holes, the heavy holes have a momentum dependence which results in an anisotropy in the cyclotron mass of approximately 30%.

In Fig. 3 the positions of the resonance lines as calculated from Eqs. (8) are plotted as a function of the total carrier concentration N. We have used the appropriate cyclotron mass of the heavy holes for $\mathbf{H}_0$ in the [110] direction, which was its experimental orientation. We determined the ratio of the number of heavy to light holes ($=20$) from the density of states mass. The arrows show the position of the lines in the absence of plasma effects, when the carrier concentration is low. The thickness of the lines is a qualitative indication of the relative peak intensity of the two resonances at a given carrier concentration. The figure is plotted assuming the microwave field were circularly polarized. For the experimental linear polarization, the two directions of $\mathbf{H}_0$ are equivalent and the figure has reflection symmetry about the abscissa. The motion of these two resonances as a function of N has been discussed in some detail above. The circles in Fig. 3 are experimental determinations of the resonance maxima positions for a spheroid with an axial ratio of about 2. The discrepancy in the initial motion of the heavy-hole line is most probably due to the assumption of isotropic mass. The dashed lines indicate what would be the resonant fields if there were no coupling between the two plasmas. A

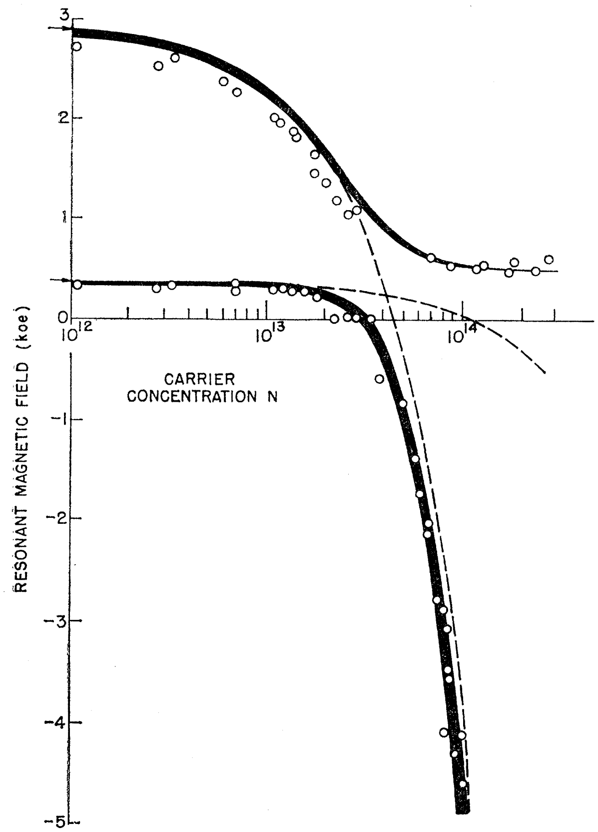

FIG. 3. The calculated resonant magnetic fields for the magnetoplasma resonances vs carrier concentration for p-type germanium. The thickness of the lines is approximately proportional to the relative intensity. The dashed lines indicate what would be the resonant fields if there were no coupling between the plasmas. The circles are experimental points.

figure comparing the actual experimental and theoretically calculated line shapes for this case appears in reference 2.

Although Fig. 3 indicates reasonably good agreement with the theory, it was not a very critical test of the dependence of the resonant field on depolarization or sample shape factor L. For germanium $\chi=1.2$, and therefore ω_p is not strongly dependent on L for $L>1$. For the spheroid of Fig. 3, $L\approx 5.3$. As a better verification of the shape dependence, thin disks and needles were studied. Figure 4 shows these results. The graph represents only that range where $\omega_p>\omega$ and we show only the more intense (and the more shape dependent) of the two resonances. The curves are theoretical and were calculated from Eqs. (8) with L calculated from the measured sample dimensions. There are no adjustable parameters. The top curve corresponds to the disk with the electric field in its plane and $\mathbf{H}_0$ normal to the plane. The data is actually for two samples of the same dimensions (diam=0.072 in., thickness=0.001 in.) cut from the same Hall sample. For this curve $L=1.1$. The lowest curve is for a needle (length=0.062 in., diam=0.005 in., $L=6.2$) cut from one of the disks.

In both of the above cases $\mathbf{H}_0$ was oriented along the symmetry axis of the sample, so that the elements of the depolarization tensor, L_{xx}, L_{yy}, were equal. If the magnetic field (and therefore the z axis) is perpendicular to the symmetry axis they are no longer equal, although, of course, the tensor is still diagonal. The middle curve of Fig. 4 shows the results for the same disks as shown in the upper curve but with $\mathbf{H}_0$ in the plane of the disk, though still normal to the microwave electric field and still in a [110] direction. The theoretical curve for this two carrier system with anisotropy of $\mathbf{L}$ in the plane perpendicular to $\mathbf{H}_0$ was calculated using Eqs. (8) with a tensor depolarization factor. The solution for a simple plasma with an anisotropic $\mathbf{L}$ can be obtained readily[1] and is instructive in considering the shape of this curve. The resonance occurs in this case when

$$\omega_c=\pm\left[\left(\omega-\frac{\omega_{px}^2}{\omega}\right)\left(\omega-\frac{\omega_{py}^2}{\omega}\right)\right]^{1/2}. \tag{17}$$

where $\omega_{pi}=[L_{ii}Ne^2/(1+\chi L_{ii})m^*]^{1/2}$. Thus the resonant field is zero when either of the two factors is zero. We see in Fig. 4 that the magnetic field for resonance becomes zero at the same carrier concentration as that for the disk in the isotropic $\mathbf{L}$ configuration. In the carrier concentration range where the two factors in Eq. (17) have the opposite sign, there is no H_0 which will bring the system into resonance. The absorption maximum in this nonresonant case occurs at $H_0=0$.

The Longitudinal Case

When $\mathbf{H}_0$ is parallel to $\mathbf{E}$ one would expect no change in the conductivity with $\mathbf{H}_0$ for a plasma of two isotropic carriers. When this was tried in the case of p-type ger-

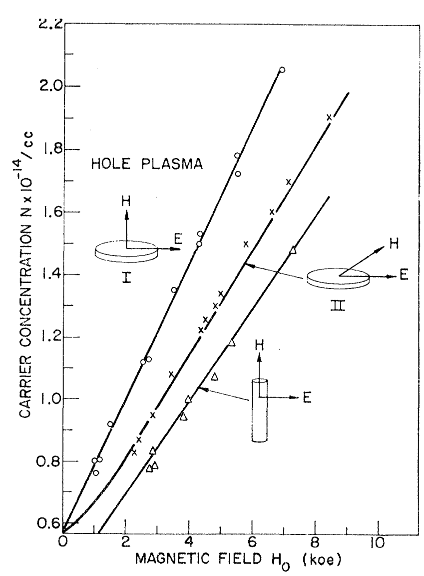

FIG. 4. Magnetic field for resonance vs carrier concentration for p-type germanium. The lines give the theoretical values for the sample shapes and field configurations shown. The points are experimental.

manium a longitudinal magnetoplasma effect was observed with a strength of about one-tenth that of the transverse effect in the region of $\omega_p \approx \omega$. The form of the effect had a great similarity to that calculated for ellipsoidal masses and observed in n-type germanium. The effect went through an approximate "null point" at about the same carrier concentration for which this happens in n-type germanium. Because of the complex nature of the masses we have not attempted to calculate this effect.

Sample in Nonuniform Field

In most of the experimental work the samples were placed at a maximum in the microwave electric field (antinode), and since the samples were small the uniform applied field produced a uniform field in the sample. When p-type samples with carrier concentrations somewhat in excess of 10^{14}/cc were moved towards a node, where the electric field goes through zero, the main plasma line decreased much more rapidly than the small residual "cyclotron resonance" line. When the sample was at the node the two lines had about the same peak intensity. The theoretical treatment discussed above cannot be applied without considerable modification to a situation where the applied field changes a great deal over the sample. A simple physical argument shows that one would expect some enhancement of the coupling to the mode of oscillation corresponding to the cyclotron resonance line if the microwave electric field varied over the orbit of the motion in the dc magnetic field. This situation is somewhat similar to the transverse Azbel'-Kaner effect, where the nonuniform field is created by the skin depth. When n-type samples were placed at a node, the maximum absorption remained close to $H_0=0$ regardless of carrier concentration.

Modes of oscillation of a plasma where the electric field is not uniform over the sample have been observed in InSb by Cardona and Rosenblum and will be discussed in a future publication.

2. n-Type Germanium

In Fig. 5 are shown the results for an n-type sample. This was again a disk of approximately the same dimensions as the p-type sample above. Curves are shown for both the case where $L_{xx}=L_{yy}$ and $L_{xx}\neq L_{yy}$. The theoretical curves are plotted assuming that we have a single isotropic mass equal to the cyclotron mass for $\mathbf{H}_0$ in the [100] direction, which was the experimental orientation of $\mathbf{H}_0$. Computation with the actual tensor equations, Eq. (15), for this four-ellipsoid problem indicated that this was an approximation valid to a few percent. These calculations used sufficient machine time to prevent us from actually plotting the theoretical curves in this manner. Agreement in the $L_{xx}\neq L_{yy}$ case is only good to about 10% in carrier concentration. Other n-type samples had discrepancies which were not always quite this large. The source of this error is not known. It may be due to the effect of the anisotropic L on the anisotropic masses; we never computed this case with the tensor equations. Some typical curves of the magnetoplasma resonance in n-type germanium are shown by the solid lines in Fig. 6. When $\mathbf{H}_0$

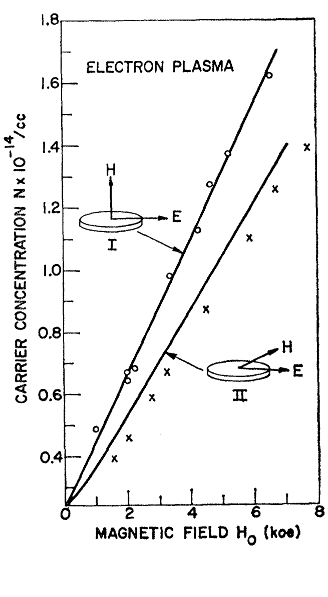

FIG. 5. Magnetic field for resonance vs carrier concentration for n-type germanium. The lines give the theoretical values (for an appropriate isotropic mass) for the field configurations shown. The points are experimental.

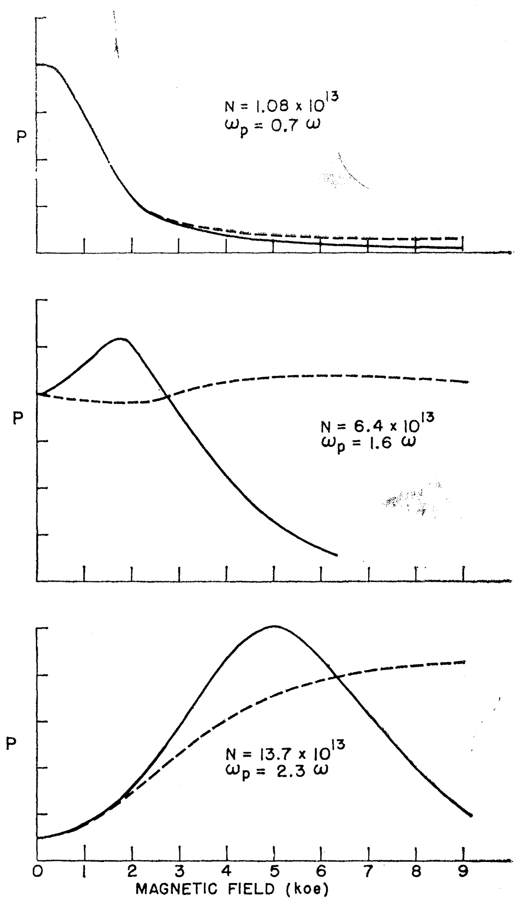

FIG. 6. Experimental curves of microwave power absorption vs magnetic field for three carrier concentrations N. The solid lines give the results when $\mathbf{H}_0$ is perpendicular to $\mathbf{E}$ and the dashed lines are for $\mathbf{H}_0$ parallel to $\mathbf{E}$.

is oriented parallel to the microwave electric field but still in a [100] direction, data of the form shown by the dashed curves in Fig. 6 is obtained. The experimental data agrees very well with the results calculated with the four ellipsoid model and discussed above. The small line which was calculated to exist at twice the cyclotron field was never observed. This is not surprising in view of its small intensity and the low $\omega\tau$ when the temperature was raised to obtain a high carrier concentration.

Hall effect and plasma measurements on a small sphere of n-type indium antimonide also have shown good agreement with theory. In this case we have an isotropic mass and, as expected, no longitudinal magnetoplasma resistance was observed.

CONCLUSIONS

These measurements have demonstrated the validity of the simple drift-velocity treatment of magnetoplasma effects in small samples, even when more than one type of carrier is present and when tensor masses are involved. In particular, it has been demonstrated that when more than one type of carrier is present a mode of oscillation exists wherein the carriers can move in such a way that the majority carrier tends to short out the plasma effects of the minority. When the sample size becomes comparable with the wavelength of the radiation in the sample, or when the sample is placed in a nonuniform field, the simple theory does not explain the phenomena. In such cases other resonances can be observed.[16] Since resonances may be observed which are independent of carrier concentration or sample shape over a large range, and yet are not at the cyclotron field of any carrier in the material, the usual checks to assure that cyclotron resonances are unperturbed by plasma effects may in some cases be insufficient.

The very good correspondence between the carrier concentrations determined by Hall measurements and by plasma measurements provide an additional check on the Hall effect as a means of measuring carrier concentration. Indeed, one can imagine certain circumstances where the measurement of carrier concentration by microwave magnetoplasma methods may be more convenient than the usual Hall effect techniques.

ACKNOWLEDGMENTS

We are very grateful to Professor E. Burstein and to many of our colleagues at RCA Laboratories for helpful and stimulating conversations, to S. Kahng for programming this problem for the computer, and to H. Hanson for his assistance throughout the course of the experiment.

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Recombination Luminescence in Alkali Halides*

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A new type of luminescence in nominally pure alkali halide crystals was studied. At room temperature the excitation was most efficient in the wavelength region that has been associated with band-to-band transitions. It is suggested that the emission results from the capture of a hole by an F center. In agreement with this model the emission (430 mμ in KCl) is the same as that which is observed when a crystal containing α centers is irradiated in the α band at low temperature. From the thresholds of the excitation spectra the following band gaps are derived: NaCl, 8.0 eV; KCl, 8.1 eV; KBr, 7.3 eV; and KI, 5.8 eV. A comparison with external photoelectric thresholds as obtained by other investigators yields the electron affinities: NaCl, 0.8 eV; KCl, 0.6 eV; KBr, 0.9 eV; and KI, 1.6 eV.

INTRODUCTION

THE optical absorption spectra of the alkali halides have a peculiar shoulder or step[1] about one electron volt above the absorption edge. No photoconductivity has been observed at the absorption edge itself and the entire region of strong absorption between the edge and the step is photoelectrically inert. It seems to be associated with the production of excitons and will be called the exciton region in this paper. Above the step, it has been suggested by Taft and Philipp[2] from a study of the external photoemission, band-to-band transitions should begin to take place.

Recently we reported briefly on a new type of luminescence in pure alkali halides at room temperature.[3] It

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