The levels in Pr^{142} are more closely grouped above 1.4-MeV excitation; only one $f_{7/2}$ group appears in Pr^{142}; the La^{140} doublet has no easily visible analog in Pr^{142}.

In summary, the level and spin assignments made in the experiment are internally consistent. The behavior of the cross section magnitudes is not always what might be expected, but the angular distributions are, for the most part, readily interpretable in terms of stripping theory.

ACKNOWLEDGMENTS

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Optical-Model Analysis of the Energy Dependence of Neutron Polarization near 1 MeV

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An optical-model analysis is made of the energy dependence of the polarization of neutrons elastically scattered from a number of medium-weight nuclei near 1 MeV. Attention is also paid to the energy variation of the total and reaction cross sections, so that the optical parameters chosen represent the average scattering over a range of nuclei and a range of energies in a meaningful fashion. Some angular distributions at 1 MeV are also plotted. The effect of compound-elastic scattering on these angular distributions and on the polarization near 1 MeV is found from a Hauser-Feshbach calculation.

1. INTRODUCTION

RECENTLY, Brown, Ferguson, and White[1] have measured the polarization of neutrons elastically scattered from Cu, Zn, Mo, and Cd as a function of energy in the range 300–1500 keV. The experimental resolution was ~40 keV, and the measurements revealed very marked fluctuations of the polarization with energy, with half-width of the order of 200 keV or less. At some energies there was evidence of possible sharp resonance effects of half-width ~50–100 keV.

The same authors have discussed the general trend of these observations, and attempted an optical-model fit to the polarization, using the Bjorklund-Fernbach potential. The present paper describes a more thorough investigation of the possibility of fitting the average neutron polarization and scattering data by the optical potential. It is clear that nothing more than the average trend of the data can be expected to follow from the optical model at energies near 1 MeV, if, indeed, that much is possible. However, any further discussion of the marked fluctuations and sharper resonance effects observed by Brown *et al.* will be the clearer if an idea is first got of the average strength of the single-particle amplitudes in the scattering, and this will be provided by the optical model.

With this purpose in view, a search has been made for optical parameters which will reproduce the energy variation of the polarization and the total and reaction cross sections near 1 MeV, as well as a number of angular distributions at various energies, over a range of nuclei from about $A=50$–100 for which the polarization data are available. This investigation is made in the early spirit of the optical model, in the expectation that any over-all fit that may be achieved—over a range of energy and of nuclei, as above—will be of some significance in regard to the relative strength of the single-particle amplitudes in the actual scattering process. It is not intended that a detailed A-by-A choice of parameters should be made, nor are the optical parameters finely adjusted as functions of energy. We are so near the credible limits of the optical model that such refinements would probably be illusory. As it is, the agreement with the average trend of the data which can be achieved is quite remarkable, and would appear to justify the intentions expressed above.

Peterson[2] has recently pointed out that the giant resonances observed in the total neutron cross sections, which have a typical width of many MeV, are the result of a nuclear Ramsauer effect, rather than of resonant single-particle excitations of the compound nucleus. Nevertheless, such single-particle resonances of the compound system may be present, and in certain circumstances may lead to fluctuations in the neutron total or reaction cross sections with a width of the order of as little as 100 keV. This idea has been put forward by

[1] D. Brown, A. T. G. Ferguson, and R. E. White, in *Proceedings of the International Symposium on Polarization* [Helv. Phys. Acta. Suppl. **6**, 291 (1960)].

[2] J. M. Peterson, Phys. Rev. **125**, 955 (1962).

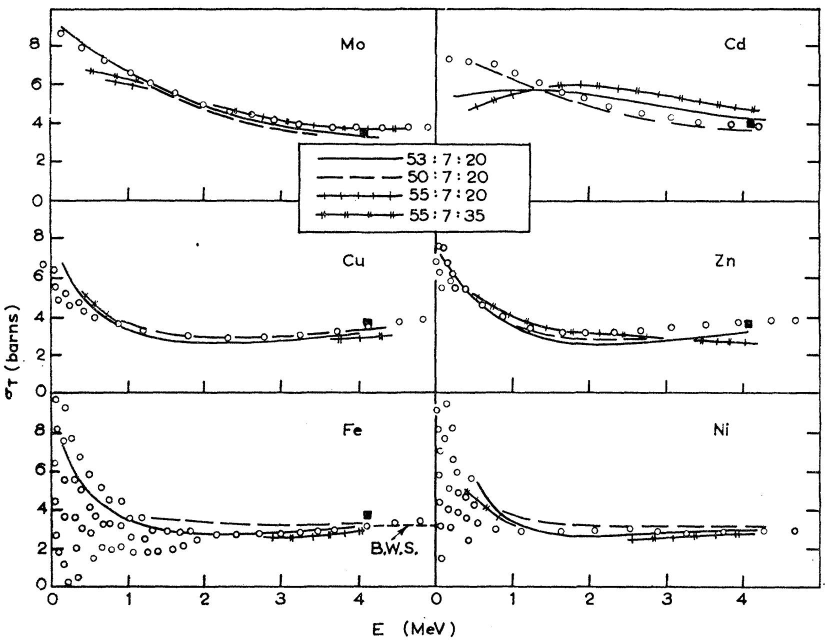

FIG. 1. Neutron total cross sections at a few MeV. The experimental curves (lines of circles) are taken from the Brookhaven compilations; they exhibit strong resonance behavior below 1 MeV. Optical total cross sections are shown for the parameters listed. Except for Cd, the energy variation of σ_T near 1 MeV is given best by the parameters 53:7:20 (see text). The optical calculation of Beyster, Walt, and Salmi (reference 10) is shown for Fe beyond 4 MeV; cross sections calculated by Bjorklund and Fernbach (reference 9) at 4.1 MeV are shown thus: ■.

Kenny and Caro[3] to explain resonances observed in the (p,n) reaction cross section from Cu^{63} at 5–11 MeV, and is a result of the deformation of the target nucleus. Similar single-particle excitations may be responsible for some of the fluctuations in the polarization data considered here. The further explanation of the sharp resonances in the data lies outside the scope of this paper, and may arise from detailed compound-nuclear effects. The work of Ericson[4] on coherent fluctuations in the statistical model of compound-nuclear scattering may be important in this connection.

It is also well known that the relative strengths of the partial waves in scattering through an optical potential vary very strongly with energy, even in the range of a few hundred kilovolts. Even though resonances may not occur in the partial waves, the very strong fluctuations in amplitude can lead to quite rapid variation with energy of the total and absorption cross sections—the well-known maxima at zero energy are a case in point. More particularly, the change in the relative strengths of the partial waves can lead by interference to strong variations *with energy* of the polarization over a range of a few hundred keV. This will be evident in the optical polarization curves given below, and is sufficient to follow broadly the average trend of the data.

2. THE OPTICAL CALCULATIONS

The optical calculations have been made in collaboration with the late Frank Bjorklund, using the Bjorklund-Fernbach optical code. A number of choices for the potential depths are listed in the figures in the following way:

$$V_{CR}(\text{MeV}):V_{CI}(\text{MeV}) :V_{SR} \text{ (ratio to Thomas term)}. \quad (1)$$

The remaining parameters used in all cases are $V_{SI}=0$, together with

$$a=0.6\text{ F};\qquad b=1.0\text{ F};\qquad R_0=1.25A^{1/3}\text{ F}. \quad (2)$$

The notation is that of Bjorklund.[5]

The parameters used are the result of a fairly extensive search of parameter space, and were chosen for the reasons given below. They form a reasonable interpolation between the optical parameters needed to correlate scattering and bound-state data at zero incident energy (see the work of Ross, Mark, and Lawson,[6] Khanna and Tang,[7] and Green[8]), and the parameters used by Bjorklund and Fernbach[9] at 4.1 MeV. The use of the particular set of shape parameters (2) is in line with the most successful optical potentials previously used at low energies (see Bjorklund[5]). It is not suggested that the parameters used are precisely the best set—such a statement has little meaning in any event, and certainly none in connection with the average energy trend of various data taken for a number of nuclei,

[3] M. J. Kenny and D. E. Caro, Austral. J. Phys. **14**, 242 (1961).

[4] T. Ericson, Phys. Rev. Letters **5**, 430 (1960); *Advances in Physics*, edited by N. F. Mott (Taylor and Francis, Ltd., London, 1960), Vol. 9, p. 425; see also *Proceedings of the International Conference on Nuclear Structure, Kingston* (University of Toronto Press, Toronto, Canada, 1960), p. 697.

[5] F. Bjorklund, *Proceedings of the International Conference on the Nuclear Optical Model, Tallahasee* (Florida State University Studies No. 32, Tallahassee, Florida, 1959), p. 1.

[6] A. A. Ross, H. Mark, and D. Lawson, Phys. Rev. **102**, 1613 (1956).

[7] F. C. Khanna and Y. C. Tang, Nuclear Phys. **15**, 337 (1960).

[8] A. E. S. Green, reference 5, p. 44.

[9] F. Bjorklund and S. Fernbach, Phys. Rev. **109**, 1295 (1958).

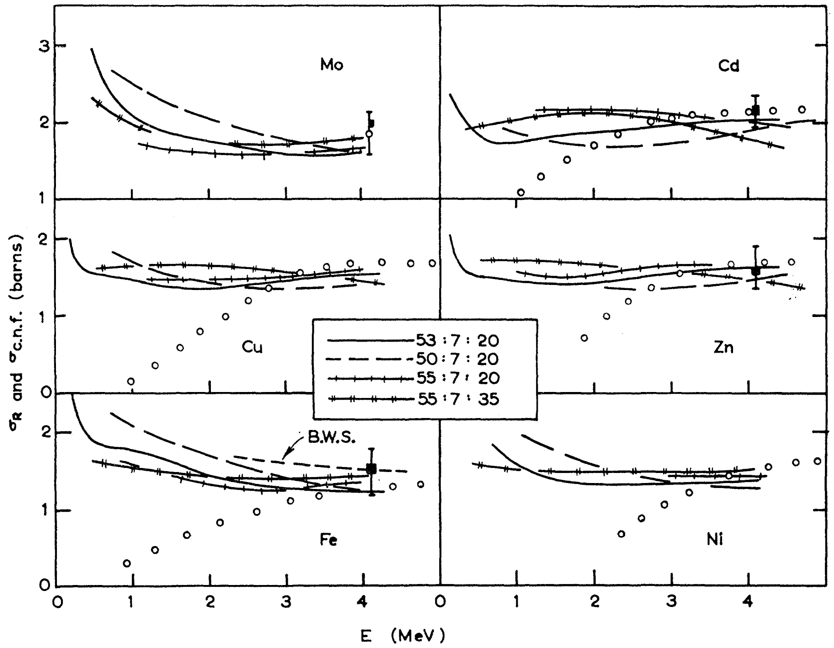

FIG. 2. Neutron reaction cross sections and optical cross sections for compound-nucleus formation at a few MeV. The experimental curves (lines of circles) are taken from the Brookhaven compilations. At 1 MeV the reaction cross section is very small, and $\sigma_{c.n.f.}$ can be equated to the cross section for compound-elastic scattering. The optical cross section $\sigma_{c.n.f.}$ as calculated by Beyster, Walt, and Salmi (reference 10) is shown for Fe; the points marked ■ are $\sigma_{c.n.f.}$ as calculated by Bjorklund and Fernbach (reference 9) at 4.1 MeV. All the curves drawn agree with the data at 4 MeV within the experimental errors, as indicated for Fe, Zn, Mo, and Cd at 4.1 MeV.

which is the matter of interest here. Nevertheless, it will be seen from the figures and from the following remarks that the potentials which *will* reproduce the average energy trends are fairly well restricted.

Within the interpolation stated, the parameters were further selected so as to give the best over-all fit to the energy dependence of the polarization data, regard also being paid to the energy variation of the total cross section. Any optical parameters which reproduce angular distributions or polarizations will usually give the total cross section correctly at that particular energy. The same is not always true in regard to the energy dependence of the polarization and total cross section. Optical parameters previously used at 4.1 MeV by Bjorklund and Fernbach[9] and by Beyster, Walt, and Salmi[10] could not be used successfully to give the energy variation of the total cross section at lower energies, nor of the polarization near 1 MeV.

Figure 1 shows a number of total cross sections (experimental data from the Brookhaven compilations[11]). The curves marked 50:7:20 and 53:7:20 [cf. (1) above] show the best agreement over this range of targets; these parameters were also found to give on the whole the best estimates of the polarization. It will be noticed that the ratio to the Thomas term (20, which corresponds in the potential 53:7:20 to $V_{SR}=5.3$ MeV) is less than the value (35) used by Bjorklund and Fernbach.[9] This is necessary to fit the polarization data, and is similar to the reduction in spin-orbit strength needed to fit proton scattering data below 14 MeV—see Bjorklund *et al.*[12] It is possible that this difference from the higher energy optical fits is evidence for a more peaked spin-orbit potential than the Thomas form function, since the neutrons at a few MeV (and protons near the Coulomb barrier) explore only the tail of the potential. A stronger potential, varying smoothly to higher energy, could then be used.

The other parameters are included in Fig. 1 as a matter of interest, to show how they differ in predicting the total cross section, when considered as a function of energy. The earlier calculation by Beyster, Walt, and Salmi[10] is shown for Fe above 4 MeV. At lower energies their calculation gives a similar curve to that marked 50:7:20; neither reproduce the total cross section well for Fe near 2–3 MeV. A number of values of σ_T as calculated by Bjorklund and Fernbach[9] are also shown at 4.1 MeV.

The energy variation of the reaction cross section does not become relevant until 3–4 MeV, well beyond the range of the polarization data. Nevertheless, some reference was made to the reaction cross sections in choosing the parameters, to see if it were possible to cover the full range from 1–4 MeV. The optical absorption cross sections are plotted in Fig. 2, together with the experimental reaction data.[11] The parameters required for the polarization near 1 MeV do, in fact, yield reaction cross sections at the higher energy in reasonable agreement with the data.[13] This agreement is as good

[10] J. R. Beyster, M. Walt, and E. W. Salmi, Phys. Rev. **104**, 1319 (1956).

[11] *Neutron Cross Sections*, compiled by D. J. Hughes and R. Schwartz, Brookhaven National Laboratory Report, BNL-325 (U. S. Government Printing Office, Washington, D. C., 1958), 2nd ed.; Suppl. No. 1, 1960, compiled by D. J. Hughes, B. A. Magurno, and M. K. Brussel.

[12] F. Bjorklund, G. Campbell, and S. Fernbach, in reference 1, p. 432.

[13] The absorption cross section represents compound-elastic scattering almost entirely at 1–2 MeV, but by 3–4 MeV or beyond should be approximately equal to the reaction cross section, since sufficient other channels with appreciable cross section are then open to make compound-elastic scattering unimportant.

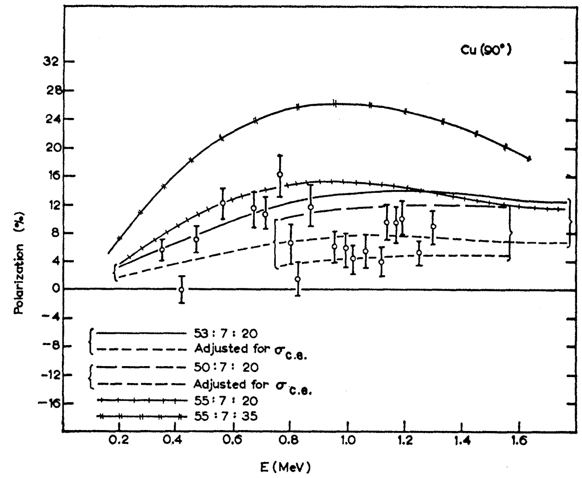

FIG. 3. Polarization of neutrons scattered from copper at 90°.

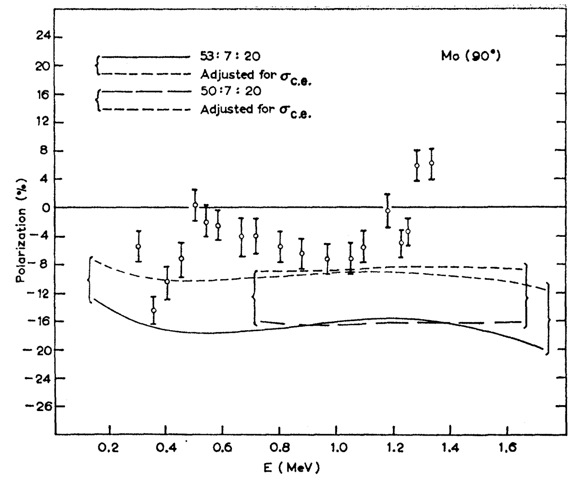

FIG. 6. Polarization of neutrons scattered from molybdenum at 90°.

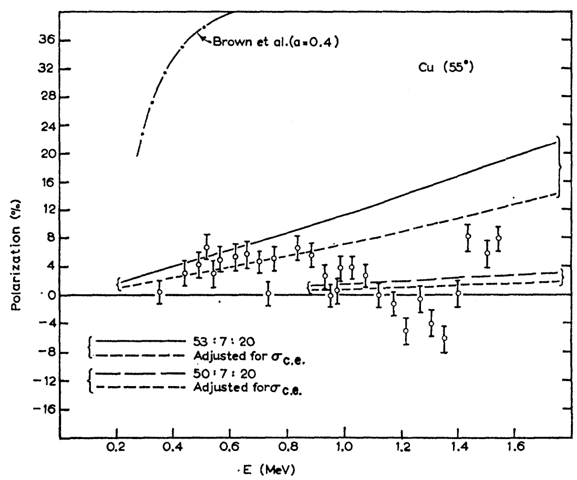

FIG. 4. Polarization of neutrons scattered from copper at 55°. The polarization calculated by Brown *et al.* (reference 1) is also shown.

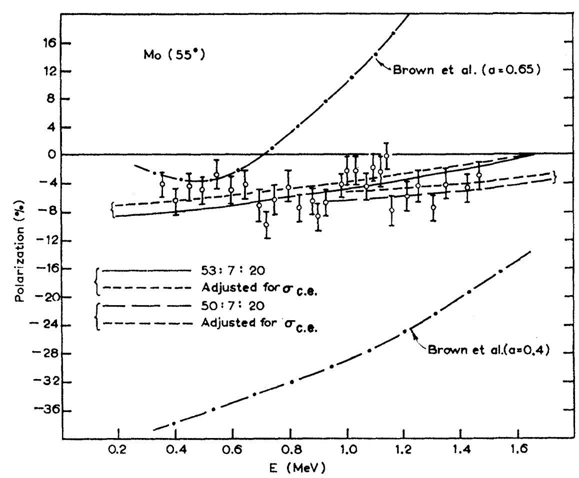

FIG. 7. Polarization of neutrons scattered from molybdenum at 55°. Two curves calculated by Brown *et al.* (reference 1) are also shown.

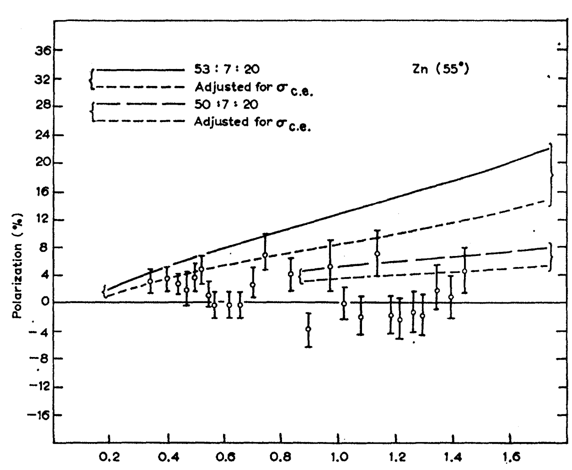

FIG. 5. Polarization of neutrons scattered from zinc at 55°.

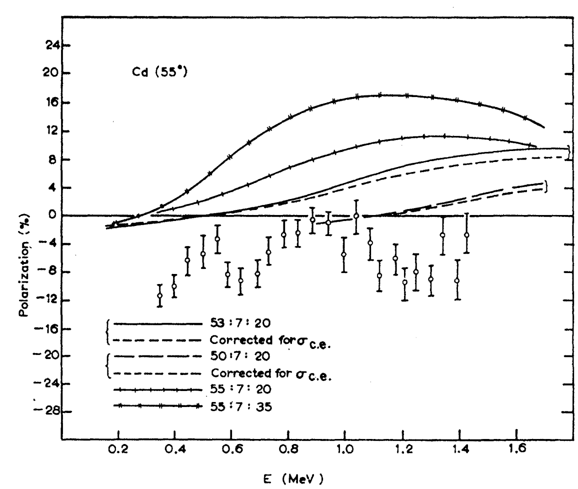

FIG. 8. Polarization of neutrons scattered from cadmium at 55°.

as that obtained by the earlier workers referred to above; however, our parameters do not yield a good fit to the angular distributions at 3–4 MeV.

The polarization data[14] of Brown *et al.*[1] is plotted in Figs. 3–8, together with the optical polarization for the parameters 53:7:20 and 50:7:20.

On the whole, the parameters 53:7:20 give a reasonable fit to the average trend of the data, at least when allowance is made for compound-elastic scattering (see Sec. 3 below). The parameters 50:7:20, when so corrected, make the polarization rather too small.

It should be noted that the effect of multiple scattering has not been allowed for in detail in the data. Brown *et al.*[1] state that their polarizations should be increased by about 30%. The correction may show some variation with energy, but this will not be rapid, and will not be sufficient to invalidate our comparison with the optical model. The correction rather favors the parameters 53:7:20, as stated above.

Curves are also shown using the other parameters listed in Figs. 1 and 2 for the two cases of Cu at 90° (Fig. 3) and Cd at 55° (Fig. 8). In Figs. 4–7 these parameters would give a poorer fit than the two curves shown, though with the correct general trend (and sign). They are included for Cu at 90° to demonstrate the typical range of variation, and for Cd at 55° to show that for none of the generally suitable parameters is the calculation successful. Possibly the data have the wrong sign, or it may be that this target is too heavy to be encompassed by the same parameters.

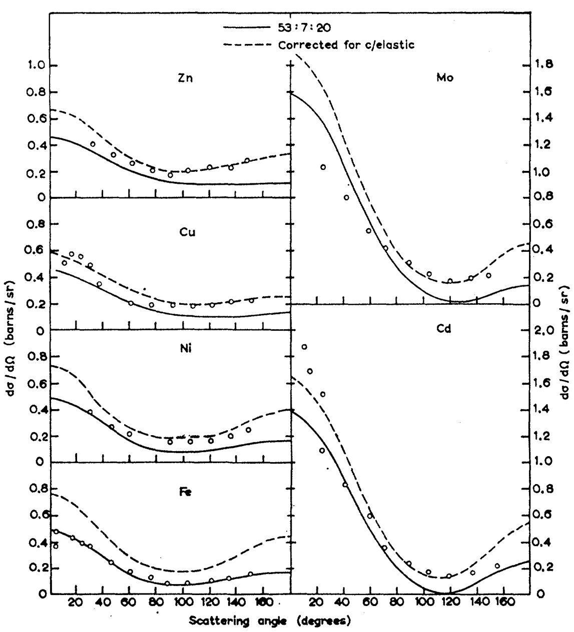

FIG. 9. Neutron angular distributions at 1 MeV. The data are taken from the Brookhaven compilations, where references are given. The optical curve for parameters 53:7:20 is the full line; the same corrected for compound-elastic scattering is shown dashed.

[14] The author is much indebted to Professor D. Brown for the private communication of these data prior to publication.

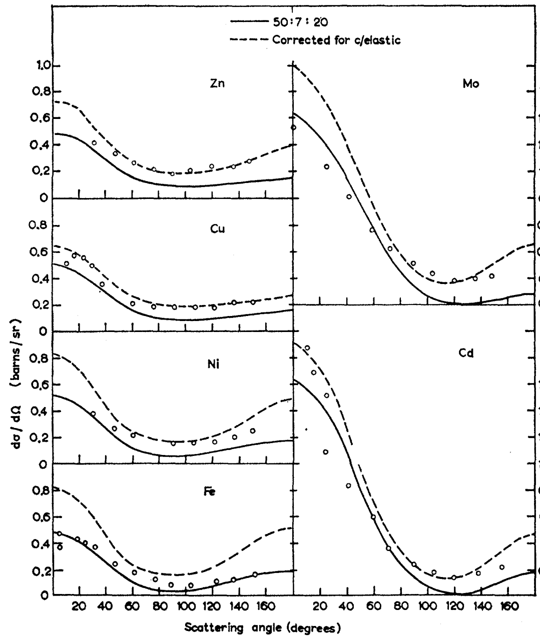

FIG. 10. Neutron angular distributions at 1 MeV. The data are taken from the Brookhaven compilations, where references are given. The optical curve for parameters 50:7:20 is the full line; the same corrected for compound-elastic scattering is shown dashed.

Brown *et al.*[1] themselves made an optical calculation, using a deeper well of approximately the same shape as here. Their curves for Mo at 55° are shown for comparison in Fig. 7, and a curve for Cu at 55° in Fig. 4.[15]

The fluctuations in the averaged polarization data which remain after an optical fit has been made are clearly very significant, and merit further study.

A number of angular distributions at 1 MeV are plotted in Figs. 9 and 10 (data from the Brookhaven compilations,[16] where references are given) using the parameters 53:7:20 and 50:7:20, respectively. The agreement is very good, considering that the parameters were not chosen specifically to represent angular distributions. The dashed curves represent the effect of compound-elastic scattering, as outlined in the next section.

3. COMPOUND-ELASTIC SCATTERING

It is clear from the well-known behavior of the neutron total cross sections near 1 MeV (see Fig. 1 above) that no comparison of the averaged elastic, reaction, or polarization data with the optical model will be of any significance unless some estimate is made of com-

[15] The polarization shown in Fig. 8 of Brown *et al.* (reference 1) is plotted with the wrong sign. In both their Figs. 8 and 9, the lower part of the figure should be positive.
[16] D. J. Hughes and R. S. Carter, Brookhaven National Laboratory Report BNL-400, 1956 (unpublished).

pound-elastic scattering. Compound-elastic scattering has therefore been calculated, using the Hauser-Feshbach method, with the penetration factors obtained from the optical absorption cross section itself in the usual way. Partial waves up to $l=6$ are included, and allowance is made for nonzero target spin ($Cu^{63,65}$: spin 3/2). It is assumed that only the entrance channel is open, which will be a good approximation at 1 MeV and a fair approximation even up to 2–3 MeV, as will be seen from the reaction cross sections quoted in Fig. 2.

The effect of compound-elastic scattering on the angular distributions at 1 MeV is shown by the dashed lines in Figs. 9 and 10. The cross section is increased by a factor of the order 1.5–2.0. Considering that the optical parameters have not been directly chosen to fit these angular distributions in detail, the corrected curves fit the data surprisingly well. The errors in the data (not shown) may be of the order of 10% or more (including multiple-scattering corrections).

As an estimate of the average[17] effect of compound-nuclear processes on the polarization, the unpolarized cross section (which appears in the denominator) has been corrected to allow for the compound-elastic scattering. As an approximation, we take the ratio of corrected to uncorrected angular distributions at 55° and 90° at 1 MeV from Figs. 9 and 10, and apply this ratio uniformly throughout the energy range 200–1600 keV. The polarization, corrected in this way for compound-elastic scattering, is plotted as the dashed lines in Figs. 3–8. It is clear that the correction is quite significant.

[17] Compare with possible coherent statistical effects in the compound nucleus, as mentioned in the introduction.

ACKNOWLEDGMENTS

The author very gratefully acknowledges the generous collaboration of the late Frank Bjorklund, and of others of his colleagues at the Lawrence Radiation Laboratory who have assisted in performing these calculations or in discussing the results. Thanks are also due to E. A. Christiansen for assistance with the compound-elastic calculations, and to R. W. Humphrey for much useful discussion.

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High-Energy Gamma Rays and Low-Energy Protons and Deuterons from $C^{12}+p$ for $E_p=14$–20 MeV*

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The 90° yield of gamma rays from proton bombardment of C^{12} was studied for proton energies between 14 and 20 MeV. Gamma rays are observed corresponding to the ground-state decay of the C^{12} 4.43-, 12.7-, and 15.1-MeV levels and from the $C^{12}(p,\gamma_0)N^{13}$ reaction. Three resonances are observed in the yield of the 15.1-MeV gamma ray. These resonances correspond to N^{13} levels at 18.1, 18.65, and 19.8 MeV. The N^{13} 18.1-MeV level has a width of 330±100 keV in the center-of-mass system. The other two levels have widths less than 200 keV. The yields of the 4.43- and 12.7-MeV gamma rays reveal little or no structure. The excitation function of the $C^{12}(p,\gamma_0)N^{13}$ reaction also shows little structure. Angular distributions and absolute integrated cross sections for the $C^{12}(p,p')C^{12}$ reaction are given for the C^{12} 12.7-MeV level with $E_p=17.5$, 19.5, and 20 MeV and for the C^{12} 15.1-MeV level with $E_p=19.5$ MeV. The latter is approximately symmetrical about 90° which suggests possible contribution from the compound nucleus reaction mechanism. Comparison of the cross sections for $C^{12}(p,p'\gamma)C^{12}$ and $C^{12}(p,p')C^{12}$ gives $\Gamma_\gamma/\Gamma=0.027\pm0.007$ for the C^{12} 12.7-MeV state and 1.15±0.3 for the C^{12} 15.1-MeV state. The yield of the $C^{12}(p,d)C^{11}$ reaction was measured by the stacked foil technique from threshold ($E_p=17.85$ MeV) to 19.8 MeV. There is no evidence for structure. Angular distributions are given for the $C^{12}(p,d)C^{11}$ reaction for $E_p=19.3$, 19.5, and 20.0 MeV. The results are compared to previous work on the $B^{11}(d,n)C^{12}$ reaction and it is concluded that the contribution of the compound nucleus reaction mechanism to either reaction is most likely small for the bombarding energies in question.

I. INTRODUCTION

Although extensive experimental data on the inelastic scattering of 15–20 MeV protons from light nuclei have become available in recent years,[1] there are very few data on the scattering from levels above 10-MeV excitation energy. Scattering from levels with excitation energies which are a large fraction of the bombarding energy should show some distinctive features because of the low energy of the outgoing protons. For instance, it is expected that the direct-interaction cross section would be relatively low for such reactions since the high values of outgoing orbital angular momentum (and thus incoming orbital angular momentum) which contribute the major part of the

* This work was supported by the U. S. Atomic Energy Commission, the Higgins Scientific Trust Fund, and the Air Force Office of Scientific Research.

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[1] G. Schrank, E. K. Warburton, and W. W. Daehnick, Phys. Rev. **127**, 2159 (1962); and references therein.