

Simple Nuclear Reactions of Indium with 30- and 2.9-GeV Protons*

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Cross sections have been measured for the following reactions of indium with 30- and 2.9-GeV protons: (p, p') , $(p, p\pi^+)$, and (p, pxn) ($x=1, 2, 4-6$). The cross sections were measured as a function of target thickness and the results corrected for production by secondaries. The cross sections for the $(p, p\pi^+)$ reaction are 0.12 and 0.18 mb at 30 and 2.9 GeV, respectively. For each of the other reactions the cross section at 30 GeV is approximately the same as that at 2.9 GeV. Upper limits of 6 and $0.3 \mu\text{b}$ were obtained for the $(p, p2\pi^+)$ reaction cross section at 2.9 and 30 GeV, respectively. The cross sections for the $(p, p\pi^+)$ reaction are in agreement with theoretical estimates.

I. INTRODUCTION

NUCLEAR reactions involving the emission of only a few particles have been the subject of considerable study in the last several years.¹ These so-called simple reactions have been investigated up to energies of several GeV. The recent availability of 30-GeV protons at the Brookhaven AGS makes it of interest to extend measurements on these reactions to this new energy range.

In the present study the cross sections for several simple reactions of indium with 30- and 2.9-GeV protons have been measured. These reactions may be divided into two categories. The first category consists of (p, pxn) reactions. A number of excitation functions in the low GeV region have recently been reported for reactions of this type²⁻⁴ and the cross sections have been found to be practically independent of bombarding energy. The second category consists of reactions involving no change in the mass number of the target nucleus, such as (p, p') , $(p, p\pi^+)$, and $(p, p2\pi^+)$. Reactions of this type have recently been considered by Ladenbauer and Winsberg³ and a summary of previous cross-section determinations is given in their paper. These reactions arise as the result of inelastic nucleon-nucleon collisions involving very low momentum, transfer,⁵ and considerable interest in them has developed recently because of their connection with the one-pion exchange process. Ericson *et al.*⁶ have thus calculated cross sections for reactions of this type on the basis of this process, and a comparison with the $(p, p\pi^+)$ cross sections reported in the present study may be made. Several mechanisms of a collective nature have recently been proposed for the (p, p') reaction at high energies in addition to the above

mechanism. These include both Coulomb⁷ and diffraction dissociation⁸ of the incident proton by the target nucleus.

Most of the reactions reported here have previously been investigated in the low GeV region by Nethaway and Winsberg.⁴ The cross sections given by these authors for 1-6-GeV protons are essentially independent of bombarding energy. In spite of the availability of the 1-6-GeV data, it was thought worthwhile to re-determine these cross sections at 2.9 GeV for a direct comparison with the 30-GeV results. It was indeed found that several of the measured cross sections at 2.9 GeV differed substantially from the values reported by Nethaway and Winsberg.⁴

II. EXPERIMENTAL

The irradiations at 30 GeV were performed in the circulating beam of the Brookhaven Alternating Gradient Synchrotron. The target foils were mounted on a frame which was then attached to an electromechanical flip mechanism. The frame was flipped into the beam at a time in the acceleration cycle corresponding to the desired bombarding energy. At other times the target was in a retracted position, shielded from spillout beam by the upstream magnets. The irradiations at 2.9 GeV were performed in the circulating beam of the Cosmotron. Two types of target assemblies, differing in thickness by factors of about 2 to 4, were used at both energies. The thin targets consisted of high-purity (99.99%) indium metal evaporated to a thickness of 4 mg/cm² onto high-purity 0.001-in. aluminum. The target stack also included a 0.001-in. aluminum monitor and several thin aluminum foils whose function was to stop backward indium recoils and compensate for sodium recoil loss from the monitor foil. The foils were arranged so that the indium was on the upstream side of the aluminum monitor. The over-all target thickness of this assembly was about 20-25 mg/cm². The thicker target assemblies consisted of high-purity (99.99%) indium foils with a thickness of approximately 15 mg/cm², and included a number of 0.001-in. or 0.003-in.-thick aluminum foils. The investigation of the effect

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¹ For a list of references see J. M. Miller and J. Hudis, *Ann. Rev. Nuclear Sci.* **9**, 159 (1959).

² N. T. Porile, *Phys. Rev.* **125**, 1379 (1962).

³ I. M. Ladenbauer and L. Winsberg, *Phys. Rev.* **119**, 1368 (1960).

⁴ D. R. Nethaway and L. Winsberg, *Phys. Rev.* **119**, 1375 (1960).

⁵ G. Cocconi, A. N. Diddens, E. Lillethun, and A. M. Wetherell, *Phys. Rev. Letters* **6**, 231 (1961).

⁶ T. Ericson, F. Selleri, and R. T. Van de Walle, *Nuclear Phys.* **36**, 353 (1962).

⁷ M. L. Good and W. D. Walker, *Phys. Rev.* **120**, 1855 (1960).

⁸ M. L. Good and W. D. Walker, *Phys. Rev.* **120**, 1857 (1960).

of target thickness on the measured cross sections was of particular importance in this study because some of the reactions in question are very sensitive to low-energy secondaries. The formation of In^{115m} from In^{115} thus occurs with a cross section of several hundred millibarns for low-energy neutrons⁹ and with a cross section of 2–40 mb for medium-energy charged particles.¹⁰ The irradiations of the thin targets were performed in duplicate, while a single bombardment of the thicker targets was performed at each energy. The effect of low-energy particles on the formation of In^{115m} was also investigated by exposing indium foils to the stray radiation present inside the accelerator vacuum chamber. The activity of the foils was found to be negligibly small compared with the In^{115m} activity obtained in the actual bombardments at both accelerators.

Following the irradiations, a stack of foils was punched out of the target about 2 mm back from the leading edge. The number of protons striking the target disk was determined by means of the $\text{Al}^{27}(p,3pn)$ reaction. The cross sections for this reaction were taken as 8.6 and 9.1 mb at 30 and 2.9 GeV, respectively.¹¹ The activity of Na^{24} in the monitor foil was determined by assay of the foil with a calibrated beta-proportional counter.

The target foil was subjected to radiochemical analysis, and indium, cadmium, and silver were separated by standard procedures.^{12,13} The main steps in the indium separation were bromide extraction into diethyl ether and TTA extraction into benzene. The main steps in the cadmium separation were an anion exchange separation and precipitation of the sulfide. Silver was separated from cadmium as soon after the end of bombardment as possible by precipitation of AgCl . The precipitate was purified from cadmium by dissolution in NH_4OH and reprecipitation. The sample was then filtered, weighed, and allowed to stand until 21-min Ag^{115} had completely decayed to Cd^{115} . Cadmium was then separated and decontaminated as outlined above. The chemical yields were determined after completion of the activity measurements by spectrophotometric and flame photometric methods.

The activity measurements were performed with a calibrated 3-in. by 3-in. NaI crystal connected to a 256 channel pulse-height analyzer as well as with beta-proportional counters. The decay of the various indium nuclides formed in this experiment gave rise to a rather complicated gamma-ray spectrum so that considerable

TABLE I. Counting and calibration procedures.

Nuclide	Counting procedure	Calibration procedure	Calibration radiation
4.3-h In^{109}	gamma	gamma	0.205-MeV γ , 70%
4.9-h In^{110m}	gamma	gamma	0.66-MeV γ , 100%
2.84-day In^{111}	gamma	gamma	0.172-MeV γ , 86%
			0.247-MeV γ , 94%
104-min In^{113m}	gamma	gamma	0.393-MeV γ , 65%
49-day In^{114m}	beta	gamma	0.191-MeV γ , 18%
4.5-h In^{115m}	gamma	gamma	0.335-MeV γ , 48%
53-h Cd^{115}	beta	gamma	0.335-MeV γ , 48%
43-day Cd^{115m}	beta	beta	β , 100%
21-min Ag^{115}	$\text{Ag}^{115} \rightarrow \text{Cd}^{115}$, 91%

care was required in the analysis of the data. In order to facilitate the analysis, pure sources of several indium nuclides were prepared by alpha bombardment of silver and 10-MeV proton bombardment of indium. The gamma-ray spectra of these nuclides were used to unfold the spectra resulting from the sources produced in the high-energy bombardments into their various components. Decay curve analysis of the individual gamma-ray peaks was also carried out. The gamma-ray spectra were usually measured with the scintillation counter subtending a large solid angle. A calibration experiment was performed in which spectra were obtained with the use of a much larger sample-scintillator distance in order to evaluate the reduction in peak intensity resulting from gamma-gamma coincidence summing. In some instances this effect was found to be as large as 35%.

The activity of the various samples was also determined with beta proportional counters. The disintegration rates obtained from measurements with these detectors were based on previous calibrations of sources with approximately the same beta-ray energy by means of 4π beta measurements. In those cases where γ rays of reasonably well-known abundance are present, calibration was also based on γ -ray measurements. The counting and calibration procedures used in the determination of the activity of the individual nuclides, as well as the assumed branching ratios, are summarized in Table I. The branching ratios are based on the NRC compilation.¹⁴

III. RESULTS

The experimental cross sections obtained at 30 and 2.9 GeV are summarized in Table II. This table lists the results of the two thin-target experiments as well as that of the thick-target experiment at each energy. In most cases there is no systematic difference between the results of these two types of experiments and the cross sections were averaged. For In^{115m} , Cd^{115} , and Cd^{115m} , however, the cross sections obtained in the thick target experiment were larger than those ob-

⁹ H. C. Martin, B. C. Divek, and R. F. Taschek, *Phys. Rev.* **93**, 199 (1954).

¹⁰ N. T. Porile, *Phys. Rev.* **121**, 184 (1961).

¹¹ J. B. Cumming, J. Hudis, A. M. Poskanzer, and S. Kaufman (to be published).

¹² D. N. Sunderman and C. W. Townley, *The Radiochemistry of Indium*, National Academy of Sciences, National Research Council (U. S. Government Printing Office, Washington, D. C., 1960).

¹³ J. R. DeVoe, *The Radiochemistry of Cadmium*, National Academy of Sciences, National Research Council (U. S. Government Printing Office, Washington, D. C., 1960).

¹⁴ *Nuclear Data Sheets*, National Academy of Sciences, National Research Council (U. S. Government Printing Office, Washington, D. C., 1960).

TABLE II. Experimental cross sections at 30 and 2.9 GeV.

Reaction ^a	Nuclide	σ thin (mb)	30 GeV σ thick (mb)	$\bar{\sigma}$ (mb)	σ thin (mb)	2.9 GeV σ thick (mb)	$\bar{\sigma}$ (mb)	$\sigma_{30}/\sigma_{2.9}$	σ -2 GeV (from reference 4) (mb)
$(p,p6n)$	In ¹⁰⁹	7.0	7.7	7.4	7.9	7.9	7.9	0.94±0.08	9.7
		7.5			8.0				
$(p,p5n)$	In ^{110m}	9.4	9.5	9.5	8.5	9.1	8.8	1.08±0.09	15
		9.6			8.8				
$(p,p4n)$	In ¹¹¹	13.2	13.7	12.9	14.7	13.4	14.4	0.90±0.08	18
		11.2			15.1				
$(p,p2n)$	In ^{113m}	4.0	2.8	3.3	3.8	4.0	3.8	0.87±0.08	2.2
		3.2			3.6				
(p,pn)	In ^{114m}	39	42	40	40	41	40	1.00±0.09	43
		40			39				
		σ extrapolated			σ extrapolated				
(p,p')	In ^{115m}	2.0	2.2	1.7	2.1	2.5	1.9	0.89±0.13	3.6
		1.8			2.0				
	Cd ¹¹⁵	0.028	0.030	0.023	0.059	0.079	0.048	0.48±0.11	0.05
		0.024			0.055				
$(p,p\pi^+)$	Cd ^{115m}	0.18	0.23	0.10	0.18	0.22	0.13	0.77±0.39	0.13
		0.14			0.14				
	Cd ¹¹⁵ +Cd ^{115m}			0.12			0.18	0.69±0.30	0.18
$(p,p2\pi^+)$	Ag ¹¹⁵		≤0.0003		≤0.006				

^a The reactions are listed for an In¹¹⁵ target.

tained in the thin-target experiments at both energies, indicating the effect of low-energy secondaries. The final cross sections for the formation of these nuclides were obtained by a linear extrapolation to zero thickness. While this procedure is not strictly correct it is a good approximation for relatively short extrapolations involving thin-target data. For the purpose of the extrapolation, the target thickness was taken as that of the entire stack. While the division of the experimental cross sections into the above two categories involves a certain degree of arbitrariness it is consistent with the notion that secondary effects are most important in those instances where the cross section for the secondary reaction is much larger than that for the primary reaction.

The ratio of the formation cross sections obtained at 30 and 2.9 GeV is listed in column 9 of Table II. The error associated with this quantity includes a 7% error in the ratio of monitor cross sections¹¹ and a 5% error based on the average reproducibility of the results. In the case of Cd^{115m} a 25% experimental error was included because of difficulties in decay curve resolution. An additional error ascribable to the extrapolation to zero thickness was included for the (p,p') and $(p,p\pi^+)$ reaction cross sections. An uncertainty of 100% in the difference between the thin-target and extrapolated cross sections was assumed. The individual cross sections have an additional error of 5 to 20% due to uncertainties in the decay schemes and to systematic errors in the analysis of the gamma-ray spectra. The cross sections for the formation of In^{114m}, In^{115m}, Cd¹¹⁵, Cd^{115m}, and Ag¹¹⁵ have been corrected for the isotopic abundance of In¹¹⁵ (0.957). All other products can be formed from both In¹¹⁵ and In¹¹³. All cross sections except those for the formation of In¹⁰⁹ and In¹¹¹ represent

independent yields. The latter include part of the yield of the corresponding (p,xn) reaction which, however, is expected to be small compared to that of the isobaric (p,pxn) reaction.²

The most important feature of the results is the absence of significant changes in going from 2.9 to 30 GeV. The cross sections for the (p,pxn) and (p,p') reactions are essentially the same at both energies. The (p,pxn) reactions of copper have recently been investigated at 2.9 and 30 GeV.¹⁵ The ratios of cross sections at 30 and 2.9 GeV obtained in that study were about the same as the present values. These results are perhaps not surprising since the cross sections for (p,pxn) reactions are already independent of energy in the low GeV region.

The cross section for the $(p,p\pi^+)$ reaction is substantially lower at 30 than at 2.9 GeV but unfortunately the error in this particular cross-section ratio is large. The cross section for the formation of Cd¹¹⁵ does, however, show a significant decrease in going from 2.9 to 30 GeV. The relative yield of the ground and isomeric states of Cd¹¹⁵ is of some interest. It is seen in Table II that $\sigma_{\text{Cd}^{115m}}/\sigma_{\text{Cd}^{115}}$ is approximately 3 and 4 at 2.9 and 30 GeV, respectively. The spins of In¹¹⁵, Cd¹¹⁵, and Cd^{115m} are (9/2+), (1/2+), and (11/2-), respectively,¹⁴ so that the state with spin closest to that of the target nucleus appears to be favored. The attempt to detect the $(p,p2\pi^+)$ reaction was unsuccessful and only an upper limit could be obtained. This limit includes about 70% of the yield of¹⁴ 20-sec Ag^{115m} as well as the entire yield of 21-min Ag¹¹⁵. If the unobserved yield

¹⁵ J. Hudis, I. Dostrovsky, G. Friedlander, J. R. Grover, N. T. Porile, L. P. Remsberg, R. W. Stoenner, and S. Tanaka, Phys. Rev. (to be published).

of Ag^{115m} is neglected, it is seen that the ratio of $(p, p\pi^+)$ to $(p, p2\pi^+)$ reaction cross sections is greater than 30 at 2.9 GeV and greater than 400 at 30 GeV. An attempt to detect a $(p, p2\pi^+)$ reaction has also been made by Ladenbauer and Winsberg³ who report an upper limit of 0.02 mb for 6-GeV protons on I^{127} .

The results of the present study at 2.9 GeV may be compared with the results of Nethaway and Winsberg⁴ for 2-GeV protons on a 97 mg/cm² thick indium target. The results of these authors, adjusted to a monitor cross section of 9.1 mb,¹¹ are listed in column 10 of Table II. It is seen that the agreement ranges from excellent to poor. The average difference between the reported cross sections is 36%. These discrepancies are probably mainly due to different considerations of the effect of secondary production and to errors in the analysis of the gamma-ray spectrum. The uncertainties in the gamma-ray branching ratios can contribute very little because essentially the same branching ratios were used in both studies.

Nethaway and Winsberg⁴ compared their results at 2 GeV with a Monte Carlo cascade-evaporation calculation. They found that the experimental cross sections for the (p, pxn) reactions were uniformly higher than the calculated values. While the present cross sections tend to be lower than the corresponding values reported by these authors, particularly if no adjustment for a difference in monitor cross section is made, the same conclusion holds. The present experimental (p, pxn) cross sections are thus larger than the calculated values by factors ranging from 3 to greater than 10. Reasons for discrepancies of this type have been given elsewhere.²

IV. DISCUSSION

The cross section for the $(p, p\pi^+)$ reaction at high energies has recently been estimated by Ericson *et al.*⁶ by use of the impulse approximation. The reduced excitation function given by these authors is independent of mass number and may be compared with experimental results in the GeV region. In order to make this comparison the experimental cross sections have to be multiplied by E/AS^2 , where E is an enhancement factor discussed below, A is the target mass number, and S is the effective separation energy of the most loosely bound particle in the product nucleus of the reaction. The effective separation energy was taken as equal to the separation energy in the case of neutron emission and as equal to the separation energy plus an effective Coulomb barrier energy in the case of charged particle emission. The enhancement factor arises from the fact that the calculated cross sections are too large because the interaction of the incident and outgoing particles with other target nucleons than the one leading to the reaction in question is not taken into account. Ericson *et al.*⁶ give a prescription for estimating this correction factor based on either a uniform or a Gaussian nuclear-density distribution. These two distributions represent fairly extreme assumptions about the nuclear-density

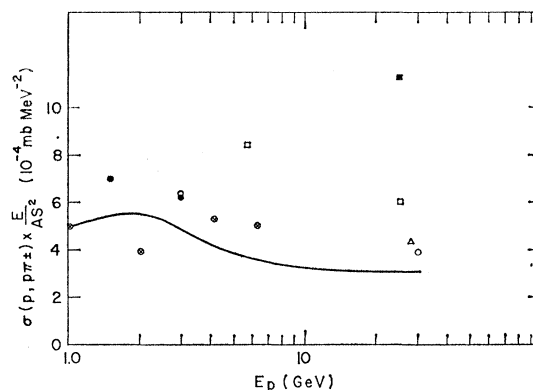


FIG. 1. Calculated and experimental cross sections for $(p, p\pi^\pm)$ reactions at GeV energies. The reduced cross section is defined in the text. The solid curve is from reference 6. The points stand for the following experimental cross sections, where open symbols refer to $(p, p\pi^+)$ reactions and closed symbols to $(p, p\pi^-)$ reactions: \circ — In^{115} , this work; \square — In^{115} , reference 4; \triangle — Al^{27} , reference 16; \blacksquare and \blacksquare — Cu^{65} , references 17 and 18; \bullet — Ga^{69} , reference 2.

distribution and in the present comparison the enhancement factor has been taken as the average of the enhancement factors obtained from the above two models. The enhancement values range from about 7 for aluminum to 15 for indium.

The calculated and experimental cross sections for $(p, p\pi^+)$ reactions are shown in Fig. 1. Cross sections for $(p, p\pi^+)$ reactions at GeV energies have also been measured for indium at 1–6 GeV,⁴ for aluminum at 28 GeV,¹⁶ and for Cu^{65} at¹⁷ 5.7 and 24 GeV.¹⁸ The reduced cross sections for these reactions are also included in Fig. 1 for comparison with the calculated excitation function. The calculated excitation function is also expected to hold for the $(p, p\pi^-)$ [or (p, n)] reaction and the cross sections for this reaction on Ga^{69} with 1.5 and 2.9 GeV protons² and on Cu^{65} with 24 GeV protons¹⁸ are also included. It is seen that the experimental cross sections agree to within a factor of about 1 to 3 with the calculated values and the agreement is about the same irrespective of energy or mass number. It is also seen that practically all the experimental values lie above the calculated curve. Ericson *et al.*⁶, in fact, indicate that their calculated values may be too low by roughly a factor of 2 because the contributions of events involving pion production at the vertex of the target nucleus are neglected.

The formation of In^{115m} from In^{115} can occur through a variety of mechanisms. These include nucleon-nucleon scattering of the type considered for the $(p, p\pi^+)$ reaction, nucleon-nucleon elastic scattering, and Coulomb excitation. At very high energies two additional collective mechanisms become possible, namely, the dis-

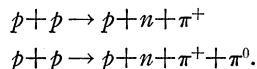
¹⁶ J. B. Cumming, G. Friedlander, J. Hudis, and A. M. Paskanzer, *Phys. Rev.* **127**, 950 (1962).

¹⁷ D. W. Barr, University of California Radiation Laboratory Report UCRL-3793, 1957 (unpublished).

¹⁸ G. Rudstam, E. Bruninx, and A. C. Pappas, *Phys. Rev.* **126**, 1852 (1962).

sociation of the incident proton by the Coulomb field of the nucleus⁷ or by diffraction scattering from the nucleus.⁸ In either case the reactions are of the type $p \rightarrow p + \pi^0$ and $p \rightarrow n + \pi^+$. If the energy of the incident proton is much greater than its rest mass and than that of the dissociation products, then a virtual dissociation transition leading to virtual states of rather long lifetime becomes possible. Good and Walker⁷ point out that in this case the dissociation products can be observed directly by means of a very small momentum transfer to the incident proton. This transfer can be accomplished by the Coulomb field of the nucleus leaving the latter with very little excitation energy. The threshold for this process for an indium target is estimated as about 10.4 GeV and the cross section at 30 GeV as about 2 mb. Diffraction scattering of the proton by the indium nucleus can also lead to the dissociation reaction and the threshold for this process is also about 10.4 GeV. The cross section for this process is more difficult to estimate, however. In view of the magnitude of the estimated cross section, it is considered unlikely that the above mechanisms are of much importance in the formation of In^{115m} . This conclusion follows from the fact that the change in the angular momentum of the struck nucleus is likely to be quite small so that preferential de-excitation to the isomeric state will not be probable. The experimental results are consistent with this conclusion. The fact that the cross section for the (p, p') reaction is so much larger than that for the $(p, p\pi^+)$ reaction is consistent with the much larger number of nucleon-nucleon processes that can lead to this reaction.

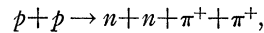
The cross section for the $(p, p2\pi^+)$ reaction has been found to be a factor of at least 400 smaller than that for the $(p, p\pi^+)$ reaction. This fact may be attributed to the following factors. The $(p, p\pi^+)$ reaction can arise from either one of the following nucleon-nucleon reactions if only single and double production reactions are considered:



Ladenbauer and Winsberg³ have pointed out that the $(p, p2\pi^+)$ reaction, on the other hand, requires at least two successive nucleon-nucleon collisions. One possible mechanism for this reaction consists of two successive P - P interactions of the type considered for the $(p, p\pi^+)$ reaction. This mechanism is analogous to plural quasi-elastic scattering which has recently been observed at

high energies.¹⁹ It has been estimated²⁰ that the latter process occurs with a frequency of about 0.3 relative to single quasi-elastic scattering for a high- Z target. While a similar frequency for plural relative to single inelastic scattering of the type considered here is not unreasonable, one would actually expect the ratio of $(p, p2\pi^+)$ to $(p, p\pi^+)$ reactions to be much smaller than this value. This is primarily due to the fact that the residual excitation following the emission of the prompt particles must be less than about 8 MeV if either reaction is to occur. This is obviously a more difficult condition to fulfill when two scattering processes are required.

The $(p, p2\pi^+)$ reaction can also occur following a reaction of the type



provided that the energetic neutron interacts further in order to eject a proton from the nucleus. The cross sections for the above three nucleon-nucleon reactions have recently been measured at²¹ 2 GeV and found to be 16, 4, and 0.6 mb, respectively. The cross section for the $(p, p\pi^+)$ reaction would thus be favored by a factor of 33 on this basis although this factor is undoubtedly different at higher energies. The excitation energy restriction discussed above applies, in addition, to this particular mechanism as well.

Summarizing the results of the present study, it appears that the cross sections for $(p, p\pi^n)$ reactions of indium are equal at 2.9 and 30 GeV. The energy dependence and magnitude of the $(p, p\pi^+)$ reaction cross section are in agreement with theoretical estimates and the cross sections for the (p, p') and $(p, p2\pi^+)$ are consistent with the mechanisms believed to be responsible for these reactions.

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¹⁹ G. Cocconi, A. N. Diddens, E. Lillithum, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Wetherell, *Phys. Rev.* **126**, 277 (1962).

²⁰ R. Karplus and Y. Yamaguchi, *Nuovo cimento* **22**, 588 (1961).

²¹ E. Pickup, D. K. Robinson, and E. O. Salant, *Phys. Rev.* **125**, 2091 (1962).