connected diagrams in which all particles interact.[13] If in some S-matrix element a subclass of particles do not interact with the rest of the particles, there will be additional momentum conservation δ functions for the subclass in N, so that it is not clear whether the inverse of D still exists.

The elements of N have left-hand cuts in energy which are due to the partial-wave projections only, other variables being fixed. Their discontinuity will again be given by a finite integral as before. One can therefore safely conclude that for relativistic model theories which sum connected diagrams only and have no crossing symmetry, the amplitude is meromorphic in the whole J plane. Singularities in l plane can arise only in connection with the disconnected S-matrix elements or through the spectral left-hand cuts due to crossed channels. In order to treat the crossing exactly, one has to write the unitarity in cross channels, say, again in terms of energy E' and total angular momentum J'. The relation of the variables in the crossed channels to that of the original channel, in the general case, is the important problem of the S-matrix theory yet to be solved.

[13] H. P. Stapp, "On the Masses and Lifetimes of Unstable Particles" (to be published).

ACKNOWLEDGMENTS

It is a pleasure to thank Geoffrey F. Chew, Henry P. Stapp, and Daniel E. Zwanziger for many stimulating discussions.

PHYSICAL REVIEW VOLUME 128, NUMBER 4 NOVEMBER 15, 1962

Intrinsic Parity from the S-Matrix Viewpoint

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(Received July 2, 1962)

The concept of intrinsic parity is developed as a construct of observables within the S-matrix framework, and ambiguities arising in more conventional approaches are avoided. It is shown that invariance of transition probabilities under spatial inversion implies the existence of a set of particle intrinsic parities that is real and unique to within a gauge transformation, provided all processes not forbidden by additive conservation laws do occur in nature. If certain of the conservation laws are multiplicative, then nonreal intrinsic parities are possible. The intrinsic parity of particle-antiparticle pairs is fixed by the general S-matrix postulates to be negative for fermions and positive for bosons, a result that parallels the one of field theory.

I. INTRODUCTION

THE question whether the sign of the intrinsic parity of particle-antiparticle pairs follows directly from the S-matrix postulates has served to focus attention on the more general question of the meaning of intrinsic parity in the S-matrix framework. In this paper the concept of intrinsic parity is systematically developed from observable S-matrix quantities. The approach is more general than the usual one, as the existence of a parity operation in the abstract space of field operators is not assumed; the only assumed invariance is that of the physically observed transition probabilities. The notion of an intrinsic parity for individual particles is not a manifest part of this assumption, but the concept can be established by construction, and the questions of existence and uniqueness completely answered. The problem of superselection rules, which clouds the conventional approach, causes no difficulty in this one. A more detailed comparison with the conventional approach is given in the final section.

* This work was done under the auspices of the U. S. Atomic Energy Commission.

II. INVARIANCE OF A PROCESS UNDER SPATIAL INVERSION

A *process* is taken to mean a reaction having specified initial and final particles. A process will be said to be *invariant* under spatial inversion (reflection) if and only if the transition probabilities associated with the process are invariant under an inversion through the origin of all polar three-vectors. The space parts of momentum-energy vectors are defined to be polar three-vectors, whereas spin vectors are not polar vectors. Thus, in the S-matrix framework, invariance under spatial reflection implies the equation[1,2]

$$|R(K_p)|=|R(K)|, \tag{2.1}$$

where K_p is the set of variables obtained from the set K by reversing the space parts of all the momentum-energy vectors k_i contained in K. This equation can also

[1] Henry P. Stapp, Phys. Rev. **125**, 2139 (1962) and *Lectures on S-Matrix Theory* (W. A. Benjamin Inc., New York, 1963). These references will be referred to as SI and SII, respectively.

[2] See SI, Appendix E.

be expressed in the form

$$P_0R(K)\equiv R(K_p)=R(K)\exp[i\alpha(K)], \qquad (2.2)$$

where $\alpha(K)$ is real for physical values of the k_i.

III. INTRINSIC PARITY OF A PROCESS

According to the S-matrix postulates given in previous papers,[1] interference effects between amplitudes referring to the same set of particles—i.e., differing only in momentum and spin labels—are to be regarded as observables.[3] Such amplitudes can be called compatible amplitudes. A consideration of transition probabilities associated with sums of compatible amplitudes shows that the phase factor $\alpha(K)$ must be independent of the momentum vectors of K. Therefore $\alpha(K)$ equals $\alpha(K_p)$ and

$$P_0{}^2R(K)=\{\exp[2i\alpha(K)]\}R(K)=R(K). \qquad (3.1)$$

Thus, the phase factor $\exp[i\alpha(K)]=\epsilon(T(K))$ is either plus or minus unity. This sign, $\epsilon(T(K))$, which depends only on the type of process $T(K)$ specified by K, will be called the intrinsic parity of the *process*.

IV. EXPERIMENTAL MEANING OF THE INTRINSIC PARITY OF A PROCESS

In an angular momentum representation K is replaced by the set (L,K^0,T), where L specifies angular momentum quantum numbers, K^0 specifies momentum magnitudes, and T specifies the type of process (type variables and energy signs). In this representation the operation of spatial inversion gives

$$P_0R(L,K^0,T)=(-1)^{\sum l(L)}R(L,K^0,T), \qquad (4.1)$$

where $\sum l(L)$ is the sum of the various angular momenta, both initial and final, specified by L. These angular momenta are usually taken to refer to relative momenta, the total momentum being fixed at zero.

Invariance of a process under spatial inversion implies, for any set (L,K^0,T) for which $R(L,K^0,T)$ is nonvanishing,

$$(-1)^{\sum l(L)}=\epsilon(T), \qquad (4.2)$$

where the intrinsic parity $\epsilon(T)$ of the process T depends only on the type of process. The number $(-1)^{\sum l(L)}$ is, in principle, an observable. This gives an experimental meaning to the intrinsic parity $\epsilon(T)$: The intrinsic parity of a process is not just an abstract phase factor; it has direct physical significance.

V. INTRINSIC PARITIES OF RELATED PROCESSES

If a process is invariant under spatial reflection it has a well-defined intrinsic parity. In S-matrix theory several related processes are described by a single analytic function. In this section it will be shown that, because of the analyticity requirement, all related processes described by a single function are invariant under spatial reflection if any one is, and the intrinsic parities of all these processes are fixed by the intrinsic parity of any one. The general rule is that the intrinsic parity of the process is reversed by switching an initial fermion to a final antifermion, or vice versa, but is unchanged if the particle switched is a boson. These relationships are derived as follows.

By definition the intrinsic parity of a process invariant under spatial reflection is the sign ϵ in the equation

$$R(K_p)=\epsilon R(K), \qquad (5.1)$$

where K_p is obtained from K by reversing all the momentum three-vectors. It was shown in appendices E and F of SI that the above equation for $R(K)$ is equivalent to the equation

$$M(K_p)=\epsilon(P^u\cdot\tilde{\sigma})M(K)(P^d\cdot\tilde{\sigma}), \qquad (5.2)$$

where $P^\alpha\cdot\tilde{\sigma}$ stands for the product

$$P^\alpha\cdot\tilde{\sigma}=\textstyle\prod_i p_i{}^\alpha\cdot\tilde{\sigma}^{(i)}/m_i. \qquad (5.3)$$

Here the product runs over all the (lower) undotted indices for $\alpha=u$ and (lower) dotted indices for $\alpha=d$. The p_i is the *physical* momentum-energy vector of the particle associated with the ith spinor index, and m_i is its mass. The physical momentum-energy vectors are here denoted by p_i, to distinguish them from the mathematical ones, k_i, which are minus the physical ones for initial particles.

The general solution of the above equation satisfying also the spinor-transformation property of M functions is

$$M(K)=V(K)+\epsilon P^u\cdot\sigma V(K_p)P^d\cdot\sigma, \qquad (5.4)$$

where

$$V(K)=\sum_{\mu_i} F^{\mu_1\mu_2\cdots\mu_n}(K)\sigma^{\mu_1}(1)\otimes\sigma^{\mu_2}(2)\otimes\cdots\otimes\sigma^{\mu_n}(n). \qquad (5.5)$$

Here F is a tensor of rank n given by

$$F^{\mu_1\mu_2\cdots\mu_n}(K)=\tfrac{1}{2}(\tfrac{1}{2})^n\operatorname{Tr}[\sigma^{\mu_1}(1)\otimes\cdots\otimes\sigma^{\mu_n}(n)M(K)], \qquad (5.6)$$

where n is half the number of spinor indices. These have been grouped in pairs, one lower dotted and one lower undotted, corresponding to the right- and left-hand indices of the σ's. (The spinor index types can be converted, if necessary, so that the number of dotted and undotted indices are equal.)[1] The expression in (5.4) satisfies (5.2) by virtue of the relations $P\cdot\sigma P\cdot\tilde{\sigma}=P\cdot\tilde{\sigma}P\cdot\sigma=1$. To write $M(K)$ as an analytic function one must replace the physical momentum vectors p_i by the corresponding mathematical ones k_i. Then $M(K)$ takes the form

$$M(K)=V(K)+\epsilon'(K^u\cdot\sigma)V(K_p)(K^d\cdot\sigma), \qquad (5.7)$$

[3] This is a slight modification of the postulate C given in SI. It was proposed and used in SII. It is the amplitudes specified by the R functions or M functions discussed in SI and SII that are referred to by this assumption. Specific phase assignments are implied for these by the analyticity requirement.

where $\epsilon'=\epsilon(-1)^{N_I}$, and N_I is the number of spinor indices associated with initial particles.

The expression for $M(K)$ in Eq. (5.7) is the analytic function that describes the processes in all channels. Since ϵ' is independent of K over the physical region, it is an absolute constant. The intrinsic parities ϵ defined in (5.1) are given, therefore, by

$$\epsilon=\epsilon'(-1)^{N_I}. \tag{5.8}$$

The immediate consequence of this equation is that the intrinsic parities of two processes are opposite if one is obtained from the other, changing a fermion in the initial configuration to its antiparticle in the final configuration. That is, if the sum over both initial and final particles of the angular momentum quantum numbers is even for some process, it must be odd for the process obtained by switching an initial fermion to a final antifermion, or vice versa. For bosons the intrinsic parity of the process is not changed by such a switch.

To understand the essential idea involved here one may consider the simple case of the scattering of a spin $-\frac{1}{2}$ particle by a spin-zero particle. If the process is invariant under spatial inversion the M function takes the form

$$M(K)=v(K)\cdot\sigma-\epsilon(k^a\cdot\sigma)[v(K_p)\cdot\sigma](k^b\cdot\sigma), \tag{5.9}$$

where the k are *mathematical* momentum-energy vectors, and the mass of the fermion has been taken to be unity; v is a combination of the four-vectors of the problem. Going to the nonrelativistic limit, $k\cdot\sigma$ becomes plus one for final particles and minus one for initial particles. Thus in the scattering process the surviving contribution is

$$[v(K)+\epsilon V(K_p)]\cdot\sigma, \tag{5.10}$$

and only the part even or odd under $k\rightarrow k_p$ contributes for ϵ plus one or minus one, respectively. But if one analytically continues the momentum-energy vector, for, say, the initial fermion from the physical region where $k_i^0<0$ to the physical region where $k_i^0>0$, in order to obtain the function representing pair production, the nonrelativistic limit of $k\cdot\sigma$ is changed from minus one to plus one; the nonrelativistic limit for pair production is

$$[v(K)-\epsilon v(K_p)]\cdot\sigma. \tag{5.11}$$

Thus if the even part of $v(K)$ contributes in the scattering, the odd part contributes in the pair production, and vice versa.

For a scattering process, in which the incoming and outgoing particles are the same, the intrinsic parity is plus one, since it is plus one for the no-scattering part of this process. That is, if the sum of the initial orbital angular momenta is even (odd), then the sum of the final ones is also even (odd). By virtue of the result shown above, this connection will be reversed in the production of a fermion particle-antiparticle pair by a boson particle-antiparticle pair; even orbital states will go to odd orbital states, and vice versa. Also, a fermion-antifermion pair must be in an odd orbital state if it is to be emitted in a reaction without changing the orbital states of the remaining particles. These relations are just what is meant by the statement that the intrinsic parity of a fermion-antifermion pair is -1.

VI. INTRINSIC PARITY OF A PARTICLE

Suppose all processes are invariant under spatial reflection. The intrinsic parities $\epsilon(T)$ of all processes T are then well defined. If there is a set of numbers ω_t, one for each particle type t, such that the intrinsic parity of every process is the product of these ω_t's, extended over the particles participating in the process, the set of ω_t will be called a (possible) set of *particle intrinsic parities*. Specifically, we require of a set of particle intrinsic parities ω_t, for all physically occurring processes T, that

$$\epsilon(T)=\prod_t \omega_t^{N_t(T)}=\prod_{[T]} \omega_t, \tag{6.1}$$

where $N_t(T)$ is the number of particles of type t occurring in the process T, and $[T]$ is the set of particles in the process T.

For generality we may consider, in these formulas, that the intrinsic parities of initial and final particles are represented by relative reciprocals (inverses), since this condition is already imposed by the no-scattering case in which $\epsilon(T)=1$. Specifically, in (6.1) the ω_t will be considered to be replaced by ω_t^{-1} for initial particles. This added generality becomes effective only if the ω_t are different from plus or minus unity, a possibility we mean to include.

It is not obvious that a set of particle intrinsic parities can be found. And if their existence can be demonstrated the question of their uniqueness arises.

If there are conservation laws, such as charge conservation, that forbid the occurrence of certain processes, then the particle intrinsic parities, if they exist, are certainly not unique. An additive conservation law requires, for occurring processes, that

$$\sum_{[T]} q_t\equiv\sum q_t=0, \tag{6.2}$$

where q_t is the number of units of the conserved quantity carried by particles of type t. The contributions associated with initial particles are to appear in (6.2) with reversed signs. But then all M functions are invariant under multiplication by $\exp(2\pi i\sum q_t\alpha)$, for arbitrary α. This implies that if $\omega=\{\omega_t\}$ is one set of particle intrinsic parities then $\omega'=\{\omega_t'\}$, given by

$$\omega_t'=\omega_t\exp(2\pi i q_t\alpha),$$

is another set, since

$$\prod_{[T]} \omega_t = \prod_{[T]} \omega_t' \tag{6.3}$$

for any occurring process T.

For the case in which there are several independent additive conservation laws restricting the class of occurring processes, let A be the matrix whose integer elements A_{tg} are the units of the conserved quantity labeled by g and carried by particles of type t. It will always be possible to use integers, since noncommensurate units give independent conserved quantities and any common factor can—and will—be factored out. The requirement that the conservation laws be independent means that the N_g columns of A_{tg} are linearly independent. The number of particle types N_t will be assumed finite, and N_t must be at least as large as N_g. Generally, A will have more rows than columns.

The direct extension of the above agrument shows that if $\omega = \{\omega_t\}$ is a set of particle intrinsic parities, then $\omega' = \{\omega_t'\}$, given by

$$\omega_t' = \omega_t \exp(2\pi i \textstyle\sum_g A_{tg}\alpha_g), \tag{6.4}$$

is another set.

There is an N_g-fold ambiguity in the assignment of particle intrinsic parities associated with the gauge transformation (6.4). The question now posed is whether a set of particle intrinsic parities exists and, if it does exist, whether it is unique to within this gauge transformation. It will be assumed, for the moment, that all processes not forbidden by the additive conservation laws do occur. Multiplicative conservation laws will be discussed later.

It is clear that if the process intrinsic parities are completely arbitrary one can never find a set of particle intrinsic parities. Specifically, if the particles occurring in some process are the sum of the particles occurring in a set of other processes, then the intrinsic parity of the first process must be the product of the intrinsic parities of the processes of the set. Otherwise one would obtain from (6.1) an immediate contradiction. However, it is a basic assumption of S-matrix theory (the decomposition law ansatz[4]) that the R functions contain terms that are products of the R functions for the various separate processes that can occur. This implies the following composition requirement for process intrinsic parities: The intrinsic parity of a process must equal the product of the intrinsic parities of any set of processes that, combined, have the same set of initial and final particles, since the R function for the former contains a term that is a product of the R functions of the latter. This composition requirement also guarantees the consistency of the assignments of process intrinsic parities with the unitarity relations.

The mathematical problem may now be framed. The processes allowed by the conservation laws can be characterized by vectors C_p with integer components C_{pt} giving the number of particles of type t occurring in the process. Initial particles will be represented by negative numbers. The conservation law requirement is, then,

$$\textstyle\sum_t C_{pt}A_{tg} = 0 \quad \text{for all} \quad p, g. \tag{6.5}$$

For every C_p there is a given intrinsic parity ϵ_p. These are subject to the composition requirement: If there is a set of integers b_p such that

$$\textstyle\sum_p b_p C_{pt} = 0 \quad \text{for all} \quad t, \tag{6.6a}$$

then

$$\prod_p \epsilon_p^{b_p} = 1. \tag{6.6b}$$

The positive b_p represent the number of times the corresponding processes p occurs in one set of processes, and the negative b_p represent the number of times processes p occur in another set. If the two sets combine to give the same set of particles, then the products of the two sets of intrinsic parities must be equal. In constructing this formulation of the composition requirement the fact is used that the process intrinsic parity must be unchanged if the same particle is added both initially and finally. This is a part of the composition requirement that follows from the positiveness of the intrinsic parity of the no-scattering parts of scattering functions. A single C_p actually represents the whole class of processes generated from one by adding the same sets of initial and final particles. The problem is to show the existence of a set of ω_t satisfying

$$\prod_t \omega_t^{C_{pt}} = \epsilon_p \quad \text{for all} \quad p, \tag{6.7}$$

and the uniqueness of this solution to within the gauge transformation

$$\omega_t' = \omega_t \exp(2\pi i \textstyle\sum_\alpha A_{tg}\alpha_g). \tag{6.8}$$

This algebraic problem may be solved as follows. Note first that it is not necessary to stick to the original conserved quantities. Linear combinations of conserved quantities are also conserved, and one can choose any set of N_g linearly independent ones. It is possible to choose the conserved quantities in such a way that for each conserved quantity g there is a particular linear combination of particles, specified by integer coefficients M_{gt}, such that this combination carries one unit of this conserved quantity and zero units of all the others. (The proof of this statement will be deferred until the end of the argument.) The intrinsic parity of this group of particles, defined by

$$\epsilon_g = \prod_t \omega_t^{M_{gt}}, \tag{6.9}$$

is transformed by the gauge transformation into

$$\begin{aligned} \epsilon_g' = \prod_t (\omega_t')^{M_{gt}} &= \epsilon_g \exp(2\pi i \textstyle\sum_t M_{gt}A_{tg'}\alpha_{g'}) \\ &= \epsilon_g \exp(2\pi i \alpha_g), \end{aligned} \tag{6.10}$$

[4] See SI, Appendix I.

where the last line follows from the property of M,

$$\sum_t M_{gt}A_{tg'}=\delta_{gg'}. \tag{6.11}$$

According to (6.10) the ϵ_g can be fixed arbitrarily by means of the gauge transformations. Conversely, the specification of the ϵ_g completely removes the ambiguity in the ω_t associated with the gauge transformation, since the α_g and the A_{tg} in (6.4) are both integers.

For every particle there is an allowed process involving only this particle and the multiples of the groups of particles constructed above. In particular, for a particle of type s the process represented by

$$C_{st}=\delta_{st}-\sum_g A_{sg}M_{gt} \tag{6.12}$$

will be an allowed process, since, by virtue of (6.11),

$$\sum_t C_{st}A_{tg}=A_{sg}-A_{sg}=0. \tag{6.13}$$

(A and M are generally nonsquare.) Thus, there will be an equation from the set (6.7) that reads

$$\begin{aligned}\epsilon_s&=\prod_t \omega_t^{C_{st}}\\ &=\omega_s\prod_t \omega_t^{-\Sigma_g A_{sg}M_{gt}}\\ &=\omega_s\prod_g \epsilon_g^{-A_{sg}},\end{aligned} \tag{6.14}$$

which can be solved to give

$$\omega_s=\epsilon_s\prod_g \epsilon_g^{A_{sg}}. \tag{6.15}$$

This shows that if a solution exists it is unique, aside from the ambiguity given by the gauge transformation.

To show that the solution exists one must confirm that the solution of the particular equations from (6.7) used above will also satisfy the remaining infinite number of equations in (6.7). Also, one must show that the solution (6.15) for the ω_s is consistent with the values of the ϵ_g, which are products of ω_s.

To verify the consistency of (6.15) with (6.9) one must show that

$$\prod_s [\epsilon_s \prod_{g'} \epsilon_{g'}^{A_{sg'}}]^{M_{gs}}=\epsilon_g. \tag{6.16}$$

In virtue of (6.11) this is equivalent to the condition

$$\prod_s \epsilon_s^{M_{gs}}=1, \tag{6.17}$$

which is a special case of (6.6), with the C_{st} given by (6.12). The self-consistency is thus demonstrated.

To show that this solution will satisfy all the equations (6.7), consider an arbitrary process, represented by C_{pt}. The left-hand side of (6.7) is

$$\prod_t \omega_t^{C_{pt}}=\prod_s [\epsilon_s\prod_g \epsilon_g^{A_{sg}}]^{C_{ps}}, \tag{6.18}$$

which in virtue of (6.5) becomes

$$\prod_t \omega_t^{C_{pt}}=\prod_s \epsilon_s^{C_{ps}}. \tag{6.19}$$

The C_{pt} can be written as

$$C_{pt}=\sum_s C_{ps}C_{st}', \tag{6.20}$$

where C_{st}' is the particular set of C's given by (6.12), and (6.5) is used again. Thus Eq. (6.6) can be invoked to give

$$\epsilon_p=\prod_s \epsilon_s^{C_{ps}}, \tag{6.21}$$

which, when combined with (6.19), gives the desired (6.7). Thus, the solution of the system of Eqs. (6.5) through (6.7) exists and is unique aside from gauge transformations (6.8). If the ϵ_g are chosen to be real, the particle intrinsic parities will be real.

It remains to be demonstrated that one can choose the conserved quantities in such a way that for each conserved quantity, labeled by g, there is a linear combination of particles, with integer coefficients M_{gt}, that carries one unit of the conserved quantity g and zero units of the other conserved quantities.

The original conservation laws are represented by the matrix A. Factors common to all elements of a column can be divided out. Consider the first column. It is possible to find a linear combination, with integral coefficients, of the particle types such that this combination bears one unit of the first conserved quantity. Starting with any two elements of the first column of A one can find, as is well known,[5] integral coefficients such that the corresponding linear combination of these two elements is the greatest factor common to them. This combination may then be combined with any other element, using integral coefficients, to give the greatest factor common to these. And one can continue. Since the greatest common factor of all the elements of the first column is unity, the procedure must produce after a finite number of steps a linear combination of rows corresponding to a single unit of the first conserved quantity. The remaining conserved quantities can now be changed by subtracting off appropriate integer multiples of the first conserved quantity so that this particular combination bears zero units of all other conserved quantities. Considering these operations as matrix operations on A, this matrix is now transformed to a form with one in, say, the first row of the first column, and zeros in the rest of the first row. Common factors may again be divided out and the procedure applied to the second column. The greatest common factor, unity, can be moved to the second row and appropriate multiples of the second column subtracted from all other columns. Repeated application of the procedure gives the desired result.

[5] See, for instance, A. A. Albert, *Introduction to Algebraic Theories* (University of Chicago Press, Chicago, Illinois, 1941), pp. 9, 118.

The linear independence of the original columns of A ensures that these operations never lead to a column of zeros.

It is possible to take intrinsic parities of all particles either plus or minus unity, and there is no reason to do otherwise. Starting from the originally real set, a gauge transformation would generally take the particle intrinsic parities to complex values. One could contemplate adding a certain gauge transformation for each of the conservation laws. If the various α_g were taken to be incommensurate with each other and with unity, then the absolute physical requirement that the intrinsic parity of all processes be real would, by itself, entail the existence of all the conservation laws. One might try, therefore, to claim that the conservation laws were a consequence of invariance under spatial inversion. Such a terminology must be regarded as illogical. The question of whether invariance under spatial reflection is maintained or not is simply the question of whether $|R(K_p)|$ equals $|R(K)|$. Any restriction more stringent than this is not an expression of invariance under spatial inversion alone.

To get a simple picture of the freedom available in the assignment of intrinsic parities, it is helpful to use the notion of a null group. A set of particles will be called a *null group* if it carriers zero units of all constants of the motion. According to the above analysis, the product of the intrinsic parities of the particles of a null group is specified uniquely, since it is invariant with respect to the gauge transformations. This product will be called the intrinsic parity of the null group. Since all processes not forbidden by the conservation laws are assumed to occur, one can consider for every occurring process A a related occurring process A_n that differs from A by the presence of the null group n of final particles. The intrinsic parity of the null group is then

$$\epsilon_n=\epsilon(A_n)/\epsilon(A), \qquad (6.22)$$

the quotient of the two observable process intrinsic parities. That this quotient is independent of A is ensured by the above analysis. It can also be seen directly from the composition requirement, which implies that

$$\epsilon(A_n)\epsilon(B)=\epsilon(B_n)\epsilon(A), \qquad (6.23)$$

since the sum of the initial and final particles of the set of A_n and B and of B_n and A are the same. [Recall that $\epsilon^2(T)$ is unity.] Null groups of four or more particles are associated with occurring processes; one can switch certain of the final particles to initial antiparticles. The relationship between the intrinsic parities of the null group and the various associated processes obtained in this way is fixed by the results of Sec. V. Particle-antiparticle pairs are simple null groups whose intrinsic parities are plus or minus one for bosons or fermions, respectively. Self-conjugate particles are the simplest type of null group.

The discussion, so far, has been based on the simplifying assumption that all processes not forbidden by additive conservation laws actually occur. Of course, selection rules may also be expressed by multiplicative conservation laws. However, these generally do not affect the analysis. As mentioned before, the processes represented by any C_p constitute a whole class of processes that differ from one another by arbitrary numbers of particles occurring both initially and finally. Multiplicative conservation laws generally forbid only certain of these. But the occurrence of any one is sufficient for the analysis, since the corresponding ϵ_p is then physically determined.

There is one type of multiplicative law that does forbid the occurrence of all processes of classes represented by certain C_p's. These are multiplicative conservation laws that can be represented by an additive conservation law that is valid to within multiples of some modulus.

The selection rule forbidding processes involving an odd number of fermions is a multiplicative law of this type. In actual practice this particular law is evidently not needed, since the selection rule is already guaranteed by the additive conservation laws of baryon and lepton number, but it would be necessary if, for instance, there were self-conjugate fermions.

To extend the analysis to include additive "quasi-conservation" laws, that are valid, say, modulo m, a type of fictitious momentumless particle carrying m units of the quasi-conserved quantity, can be introduced. These fictitious particles can be considered to supply the missing units of the quasi-conserved quantity, which can then be considered exactly conserved. The previous analysis is then applicable. The intrinsic parity of the fictitious particle can be fixed at plus one, by choice of gauge, and simply ignored. The results are, therefore, the same as before except that the gauge transformation will introduce multiples of $(\omega_f)^{1/m}$ into the intrinsic parities of the physical particles, ω_f being the original real intrinsic parity of the fictitious particle. If ω_f is minus one the new intrinsic parities of the real particles may no longer be real. An alternative procedure would be to keep the intrinsic parities real, but simply add the negative unit of intrinsic parity for each missing m units of the quasi-conserved quantity. This second procedure departs somewhat from the original program, but it is just as useful a method for cataloging the angular momentum selection rules.

If one uses the original procedure, in which the process intrinsic parities are factorized into contributions from the participating particles, the intrinsic parity of a self-conjugate fermion is forced to be purely imaginary. This follows immediately from the result of Sec. V, since the intrinsic parity of a pair of these particles is minus one. Indeed, if the only selection rule were the one requiring the total number of fermions—initial plus final—to be even, then the intrinsic parities

of bosons would necessarily be purely real and those of fermions purely imaginary. As this selection rule follows from the basic requirement of Lorentz invariance, whereas the baryonic and leptonic and the other conservation laws are not required by our general postulates, it may be sensible to choose the intrinsic parity assignments consistent with the possibility that the former is the only absolute conservation law—to take the intrinsic parities of bosons either plus or minus unity and the intrinsic parities of fermions either plus or minus the imaginary unit. The intrinsic parities of each particle would then become the same as that of its antiparticle, and self-conjugate combinations would have well-defined intrinsic parities. This condition removes most of the ambiguity associated with the gauge transformations, and is perhaps to be preferred over the arbitrary convention that the intrinsic parities be chosen real, since the latter precludes the possibility of self-conjugate fermions. That it is possible to choose the particle intrinsic parities in this way follows from the above analysis, provided the restriction to even numbers of fermions is the only selection rule represented by a nonadditive conservation law that forbids the occurrence of all the processes represented by any C_p.

VII. COMPARISON WITH OTHER TREATMENTS

The point of view regarding intrinsic parities developed here grows naturally out of an S-matrix philosophy, but is presumably not restricted to this approach. It is, in fact, essentially the view that has always been favored by the author. It differs somewhat, however, from what seems to be the prevailing view.

The standard approach[6] is to start with the idea that there may be a certain transformation on the field operators that can be called the parity operation. This operation must express the field operators at each point in space-time in terms of the field operators at the point obtained by inversion through the origin in space, with time unchanged. If this operator takes all observable quantities into observable quantities, and if the transformation of these observables is consistent with the classical physical meaning of spatial inversion, then the operation may be called a possible parity operation. It is specified that particles are to be considered as transforming into themselves, not their antiparticles, under spatial inversion, this being a convention that distinguishes a "parity" operation from its product with antiparticle conjugation.

Divergences between this conventional attitude and the one adopted in the present paper may be noted: First, the existence of particle intrinsic parities is assumed right at the start of the usual approach. The transformation on each field will contain a possible phase factor and the states constructed using these operators will, under the transformation, be multiplied by these factors, which are, then, essentially the particle intrinsic parities. These quantities are therefore introduced at the outset; their existence is never in question. In the present approach the starting point is a possible invariance of *transition probabilities* under spatial inversion. The notion of a particle intrinsic parity emerges only after analysis, by a construction based on observable quantities; the discussion is not predicated on the supposition that the physically observed invariance is a manifestation of some corresponding symmetry operation, of a particular form, on the field operators of an abstract Hilbert Space.

A second difference revolves about the question of what quantities are observables. For Wick, Wightman, and Wigner[6] this rather abstruse question is the key to the entire situation. Since the parity operation is required, from their point of view, to give well-defined and physically permissible values to all observable quantities, the question of what quantities are observable becomes an essential one. This leads to the question whether one can measure the relative phases between states corresponding to different values of good quantum numbers. For z components of angular momentum the relative phases of the various eigenstates are clearly observable. But for different charge states the question is unresolved. Yet questions regarding intrinsic parities devolve to this perhaps insoluble problem of whether such phases are, or are not, measurable.

In the S-matrix approach developed here particle intrinsic parities are nothing more or less than a factorization of the observable "angular momentum parity defect" $(-1)^{\Sigma l}$. Questions like whether or not all observables are properly transformed under some formal operation do not enter; particle intrinsic parities are merely a convenient device for cataloging the angular momentum selection rules implied by an invariance of transition probabilities under spatial inversion.

[6] G. C. Wick, A. S. Wightman, and E. Wigner, Phys. Rev. 88, 101 (1952). This reference will be taken as the standard in the field. Some elaboration of what these authors say has been made, however, so the reader should go to the reference for an exact statement of their position.