Test of Possible Variations of the Gravitational Constant by the Observation of White Dwarfs within Galactic Clusters

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An astronomical test is suggested for the hypothesis that the gravitational constant k varies throughout space-time according to the law $k=k_0(1+aV/c^2)$, where a is a dimensionless constant of the order of unity and V is the gravitational potential. If k varies, the gravitational self-energy of a body also varies. This brings about a correction in the gravitational force which, in the case of a very dense and massive body like a white dwarf, is not completely negligible. In the gravitational field of the Galaxy, the path of a white dwarf will be slightly different from that of a normal star. In consequence, a white dwarf in a very weakly bound cluster will escape; a white dwarf in a more strongly bound cluster will occupy an anomalous position. Testing these predictions seems to be within the limits of present-day possibilities.

1. INTRODUCTION

THE hypothesis that k may vary throughout space-time according to a law of the form

$$k=k(V)=k_0[1+a(V/c^2)] \tag{1}$$

has been considered by a number of people.[1–9] Here, a is a dimensionless constant of the order of unity and V is the gravitational potential. This simple law is supposed to hold only in the case in which one can neglect the variations of V throughout the region where the two interacting masses are placed and the contributions of the two masses to V. Actually, one should speak instead of possible variations of km_e^2/e^2, which is a dimensionless quantity.[10]

As far as we know, a practical method of testing the hypothesis has never been suggested.

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In the gravitational field of the Galaxy, the path of a white dwarf will be slightly different from that of a normal star. As a consequence, a white dwarf in a very weakly bound cluster will escape; a white dwarf in a more strongly bound cluster will occupy an anomalous position. Testing these predictions seems to be within the limits of present-day possibilities. Observations of white dwarfs within galactic clusters have been made in the past, in particular by Luyten,[11–12] and the results of these observations may already enable one to draw some provisional conclusions. However, reliable data, which may be available at the present time, concern only a very limited number of clusters; it will not be very satisfactory to have a fundamental question answered by such meager experimental evidence.

In several years time, it will be possible to establish the membership of white dwarfs to a larger number of (more-distant) clusters, by determining their proper motions from the Mount Palomar plates. Then an adequate test of the hypothesis will probably be possible.

From the absence of white dwarfs in Coma Berenices and from their presence in the Hyades, we deduce that the absolute value of a should lie between 0.9 and 6.6. One should not take these figures very seriously, and, for the time being, there is only a vague indication of the existence of a lower limit for a. On the other hand, the existence of an upper limit of that order of magnitude seems to be a reliable conclusion.[13]

2. THE ANOMALOUS FORCE ACTING ON A WHITE DWARF

Let us consider a star of mass M moving in the gravitational field of the Galaxy. If V is the galactic potential, the Newtonian force felt by the star is given by $-M(\partial V/\partial x_i)$. However, if k is not constant, we have to consider an additional force. Let Ω designate the negative gravitational self-energy and U the internal energy of the star. The total force felt by the star is given by $-M(\partial V/\partial x_i)-\partial\Omega/\partial x_i-\partial U/\partial x_i$.

In the deduction which follows, we assume that the Newtonian interaction between any two elements of the mass of the star is determined by the value $k(V)$ of the gravitational constant, V being the potential created by all other masses of the Galaxy in that region of the sky where the star is situated. For the sake of

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1 A. Einstein, *The Meaning of Relativity* (Princeton University Press, Princeton, New Jersey, 1950), p. 102.

2 D. W. Sciama, Monthly Notices Roy. Astron. Soc. **113**, 34 (1953).

3 P. Jordan, *Schwerkraft und Weltall* (Friedrich Viewig und Sohn, Braunschweig, Germany, 1955).

4 R. H. Dicke, Revs. Modern Phys. **29**, 363 (1957).

5 P. Jordan, Z. Physik **157**, 112 (1959).

6 C. H. Brans, Ph.D. thesis, Princeton University, 1961 (unpublished).

7 C. Brans and R. H. Dicke, Phys. Rev. **124**, 925 (1961).

8 C. Brans, Phys. Rev. **125**, 2194 (1962).

9 R. H. Dicke, Phys. Rev. **125**, 2163 (1962).

10 A. Finzi, Nuovo cimento **20**, 1079 (1961).

11 W. J. Luyten, Harvard College Observatory Annual Card No. 1202.

12 W. J. Luyten, Astrophys. J. **59**, 224 (1954).

13 The upper limit for the absolute value of a set by terrestrial experiments with a pendulum clock is at least one order of magnitude higher.

brevity the justification of this approximation is omitted.

Then, we have

$$-\left(\frac{\partial\Omega}{\partial x_i}+\frac{\partial U}{\partial x_i}\right)=-\left(\frac{d\Omega}{dk}+\frac{dU}{dk}\right)\frac{dk}{dV}\frac{\partial V}{\partial x_i}$$

$$=-\left(\frac{d\Omega}{dk}+\frac{dU}{dk}\right)\frac{k_0 a}{c^2}\frac{\partial V}{\partial x_i}. \quad (2)$$

If the star were a rigid body, $\Omega(k)$ would be a linear homogeneous function of k, and therefore $d\Omega/dk$ would be equal to Ω/k; U would not depend on k. However, when k increases, the star contracts; there will be an additional variation of Ω on account of the positive work done by the gravitational forces, and a variation of U on account of the negative work done by the forces of pressure. In an equilibrium configuration these two forces are equal and opposite; therefore the net variation of $\Omega+U$ caused by the contraction vanishes. We find, in conclusion,[14]

$$d\Omega/dk+dU/dk=(1/k)\Omega.$$

a is supposed to be of the order of unity and k_0 differs very little from k. In the case of the sun $-\Omega/c^2$ is about 3×10^{-6} times the solar mass. Therefore, in this case (and in the case of any other star in the main sequence), the term given by (2) is completely negligible in comparison with $-M(\partial V/\partial x_i)$. However, in the case of a white dwarf, because of the much higher density, $-\Omega/Mc^2$ will be equal to a much larger fraction of the mass of the star—even hundreds of times larger. So it may be worth considering the effect of the term given by (2) on the motion of the star. In the case of a white dwarf, $-\Omega/Mc^2$ is a well-defined increasing function of M, and can be computed using the well-established theory of the structure of the white dwarfs.

Let us suppose, for instance, that a is positive. Then the effect of (2) will be equivalent to having the Newtonian force slightly weakened, by a factor $1+a(\Omega/Mc^2)$.

3. WHITE DWARFS IN THE GALACTIC FIELD

In a schematic representation of galactic motion, let us consider the stars of stellar population I in the solar neighborhood, moving in the galactic plane on circular orbits around the galactic center. In the case of the sun, the radius R_0 of the orbit is about 8200 parsecs $\simeq2.5\times10^{22}$ cm and the velocity is about 216 km sec^{-1}. For other stars in the solar neighborhood, the angular velocity is given by

$$\omega(R)=(B-A)+(2A/R_0)(R-R_0)+\cdots,$$

with

$$A=+6.3\times10^{-16}\text{ sec}^{-1}\text{ and }B=-2.2\times10^{-16}\text{ sec}^{-1}.^{15}$$

Such a motion can be accounted by a central force

$$F(R)=\alpha R_0-\beta(R-R_0)+\cdots, \quad (3)$$

with $\alpha=(B-A)^2=7.2\times10^{-31}$ sec^{-2} and $\beta=(B-A)^2+4A(B-A)=-14.2\times10^{-31}$ sec^{-2}. The fact that $-\beta$ turns out to be very nearly double the value of α shows that this force can be phenomenologically explained by the Newtonian attraction of a mass $\mathfrak{M}$, of about 9×10^{10} solar masses $\simeq1.8\times10^{44}$ g, placed in the center of the Galaxy. The period of revolution of a star is $\tau=2\pi[R^{3/2}/(k\mathfrak{M})^{1/2}]$.

If such a star is suddenly transformed into a white dwarf, it starts to feel a weakened gravitational force; as a consequence, the orbit will change. The major semiaxis of the new orbit will be given by $R[1-a(\Omega/Mc^2)]$. The new period of revolution will be larger:

$$\tau'=\tau[1-2a(\Omega/Mc^2)].$$

Clearly, one cannot compare directly the old trajectory of the star to the new. One can, however, test the predictions made above by considering the white dwarfs belonging to a galactic cluster and by comparing their position with that of the center of the cluster.[16]

So far we have well-established results from the search for white dwarfs in five galactic clusters: the nucleus of Ursa Major, Coma Berenices, the Hyades, the Pleiades, and Praesepe. No white dwarfs have been found in the nucleus of Ursa Major and in Coma, but one (Sirius B) has been found in the stream of Ursa Major. Seven white dwarfs have been found in the Hyades, one (probable) in the Pleiades, and four in Praesepe.[17] These figures should be compared with the theoretical predictions of Sandage,[18] based on the consideration of the birth-rate function: 9 in Coma, 23 in Hyades, 2 in the Pleiades, and 20 in Praesepe.

The white dwarfs discovered in the Hyades, the Pleiades, and Praesepe are all very blue stars (bluer than Sirius B); it would be rather difficult to discover white dwarfs of a later type in those clusters. On the other hand, a white dwarf having the mass of the sun cools to an effective temperature of 20 000°K in about 300 million years. By this time, the white dwarf already will be slightly less bright photographically than those discovered in the Hyades. An interval of time of 300 million years is short in comparison with the ages of the Hyades, Praesepe, and Coma, which are all about one billion years old. Therefore the white dwarfs which we have detected in those clusters must be those which have evolved recently.

The less massive white dwarfs cool much more quickly, because they have a larger surface and a smaller thermal capacity. The white dwarfs discovered in the clusters should be, therefore, on the average, rather massive. The expression $-\Omega/Mc^2$ should take relatively

[14] The author has been informed that a previous calculation of this anomalous force acting on a star has been made by J. Peebles (private communication).

[15] J. H. Oort, Astrophys. J. **116**, 233 (1952).

[16] White dwarfs in globular clusters do not seem, on the other hand, to provide favorable conditions for the test of our hypothesis.

[17] W. J. Luyten (private communication).

[18] A. R. Sandage, Astrophys. J. **125**, 422 (1957).

large values, and the dispersion of these values should be rather small. We shall adopt therefore the simplifying assumption (really not essential) that $-\Omega/Mc^2$ takes the same value, 3×10^{-4}, for all the white dwarfs under consideration. This is, roughly, a hundred times the value which $-\Omega/Mc^2$ takes in the case of the sun.

4. WHITE DWARFS IN WEAKLY BOUND CLUSTERS

If a cluster is bound so weakly that the internal forces are not able to hold the white dwarf against the disruptive effect of the anomalous force, the white dwarf will be lost in space after a relatively short time. Among the clusters mentioned above, the nucleus of Ursa Major and Coma are the most weakly bound; the fact that no white dwarf has been found there can be perhaps considered a confirmation of our hypothesis.

If a star evolved into a white dwarf and escaped from Coma only a few tens of millions of years ago, we should be able to discover it, still in the neighborhood of the cluster, but already out of its gravitational influence. This white dwarf, which should be particularly hot, would share the proper motion of the cluster. From its position, it would be possible to tell immediately whether a is positive or negative.

The stream of Ursa Major seems to be very weakly bound, and yet, as mentioned above, a white dwarf has been found there. To explain this fact one can mention, first of all, that Sirius is a double star: the masses of Sirius A and Sirius B are, respectively, 2.33 and 0.98 solar masses. The acceleration of the system due to the anomalous force is only 98/331 of the one to which Sirius B would be subjected if it were a single star.

Secondly, the linear dimensions of the stream are very large; accordingly, the time needed to escape is very long. Taking into account the fact that Sirius B must have evolved into a white dwarf many hundreds of millions of years ago, one finds that its presence in the stream is compatible with (1), if the diameter of the stream is of the order of some hundreds of parsecs. Astronomical evidence seems to show that this is actually the case.

Luyten has searched for white dwarfs in some ten additional clusters; the proper motions of these white dwarfs have, however, not yet been determined. Under these conditions a certain percentage of spurious members will be included inevitably in the clusters. By very careful statistical considerations, it still may be possible to use his results in order to show that the weakly bound clusters are, on the average, poorer in white dwarfs.

5. WHITE DWARFS IN STRONGLY BOUND CLUSTERS

If the attractive forces of the cluster are sufficiently strong, the white dwarf will not escape and will continue to move under the joint influence of the galactic forces, the forces of the cluster, and the anomalous force.

The following elementary treatment of this complicated mechanical problem, though obviously not rigorous, should lead nevertheless to a reasonably correct estimate of the main effect which we expect to observe.

Let us suppose, first of all, although the hypothesis is not really quite justified, that the influence of the cluster may be treated as a field of force. It will be a central force which vanishes in the center of the cluster.

Slightly changing the notations, let us now designate by R_0 the distance of the center of the cluster from the center of the Galaxy and, accordingly, by $B-A$ the angular velocity of the cluster and by $\alpha R_0-\beta(R-R_0)$ the radial component of the galactic field in the neighborhood of the cluster. However, we will use the numerical values valid in the case of the sun for these constants, since the clusters which we want to consider are all relatively near. Further, let $2l$ be the diameter of the cluster.

We are interested only in the component of the force of the cluster directed towards the center of the Galaxy and, for the sake of an easier treatment, we make the rough hypothesis that, within the cluster, this component is a linear function of the distance R (from the center of the Galaxy), which takes the value $-f$ at $R=R_0-l$ and the value $+f$ at $R=R_0+l$. This component is given therefore by $f(R-R_0)/l$.

For a normal star moving on a galactic circular orbit of radius R_0, the attractive force and the centrifugal force must be equal and opposite; therefore,

$$\alpha R_0=(B-A)^2R_0. \qquad (4)$$

In a galactic system of reference rotating with the cluster, the time average of the radial acceleration of a white dwarf must vanish. Remembering (3) and neglecting terms of higher order, one finds, therefore,

$$\alpha R_0+\beta\langle R-R_0\rangle_{\text{av}}-(B-A)^2\langle R\rangle_{\text{av}}+(f/l)\langle R-R_0\rangle_{\text{av}} +a(\Omega/Mc^2)\alpha R_0=0. \qquad (5)$$

On the left-hand side, we have omitted a term $-2(B-A)\langle v_{\text{rot}}\rangle_{\text{av}}$, where v_{rot} represents the velocity of the white dwarf in the rotating system of reference and in the direction of the galactic rotation. It is immediately seen that $\langle v_{\text{rot}}\rangle_{\text{av}}$ vanishes.

One deduces from (4) and (5) that

$$(\beta-\alpha+f/l)\langle R-R_0\rangle_{\text{av}}=-a(\Omega/Mc^2)\alpha R_0 \qquad (6)$$

or, remembering that $\beta-\alpha=4A(B-A)$,

$$[4A(B-A)+f/l]\langle R-R_0\rangle_{\text{av}}=-a(\Omega/Mc^2)\alpha R_0. \qquad (6')$$

$[4A(B-A)+f/l]\langle R-R_0\rangle_{\text{av}}$ is the radial acceleration felt by a normal star in the cluster. This acceleration is directed toward the center of the cluster only if $4A(B-A)+f/l$ is positive. At the end of Sec. 2 the hypothesis was made that a is positive. Under these conditions (6′) shows that the white dwarfs of the cluster will, on the average, be displaced in the direction opposite to the galactic center. For a cluster which is not very strongly bound, $4A(B-A)+f/l$ can be small

in comparison with f/l. In this case, the displacement of the white dwarfs can be large, although the anomalous force is rather weak.

Let us estimate the effect in the case of the Hyades. We shall choose the radius $l=15$ light years$\simeq$4.9 parsecs and the mass $M=150$ solar masses.[19] One must realize, however, that the radius of a cluster is hard to define exactly and the mass is difficult to estimate. We have, in the present case, $f/l=5.8\times10^{-30}$ sec^{-2} and $4A(B-A)+f/l=3.6\times10^{-30}$ sec^{-2}. From (6′) we get

$$\langle R-R_0\rangle_{\text{av}}=a\times6\times10^{-5}R_0=a\times4.9 \text{ parsecs.}$$

If we assume that the white dwarfs necessarily escape when $\langle R-R_0\rangle_{\text{av}}$ is larger, say, than $\frac{2}{3}$ of the radius of the cluster, then, from the presence of white dwarfs in the Hyades, we deduce that the absolute value of a cannot be larger than 6.6.

We would like, of course, to observe the displacement of the white dwarfs in a cluster. If R_i $(i=1, 2\cdots n)$ are the radial coordinates of the n white dwarfs which we observe, the fluctuation of their average $(1/n)\sum R_i$ around $\langle R\rangle_{\text{av}}$ will be of the order of $l/\sqrt{n}$. Remembering that the white dwarfs discovered so far in the Hyades are seven in all, we realize that it will not be easy to tell whether the effect is really there. The situation would be slightly better if we could theoretically predict the sign of a. It would also be slightly better if more white dwarfs could be discovered in the Hyades.

The Hyades are situated approximately in the direction of the galactic anticenter; therefore, the displacement of the white dwarfs along the galactic radius cannot be observed directly on a plate. It will perhaps be possible to observe it in the following way. The linear dimensions of the Hyades, 9.8 parsecs, are of the order of magnitude of their distance, 40 parsecs; the system moves essentially as a rigid body, since the internal motions are very slow. As a consequence, the proper motions of the member stars which are farther from us and from the galactic center are slower than those of the member stars which are nearer.

Let us make a similar calculation for the case of Coma Berenices, already considered in Sec. 4. We can take for its radius $l=5$ parsecs and for its mass $M=70$ solar masses. Then we have $f/l=2.6\times10^{-30}$ sec^{-2} and $4A(B-A)+f/l=0.5\times10^{-30}$ sec^{-2}.

$$\langle R-R_0\rangle_{\text{av}}=a\times4.3\times10^{-4}R_0=a\times3.53 \text{ parsecs.}$$

From the experimental fact that no white dwarfs have been found in Coma, and, using again the assumption that the white dwarfs escape when $\langle R-R_0\rangle_{\text{av}}>\frac{2}{3}l$, we deduce that the absolute value of a must be larger than 0.9.

Praesepe seems to be even more strongly bound than the Hyades. Since in the case of the Hyades the anomalous force already is not strong enough to pull the white dwarfs out of the cluster, the effect in Praesepe will probably be completely undetectable.

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[19] B. J. Bock, Sky and Telescope **10**, 239 (1951).