PHYSICAL REVIEW VOLUME 128, NUMBER 5 DECEMBER 1, 1962

Ground-State Hyperfine Structure and Nuclear Magnetic Moment of Thulium-169

GEORGE J. RITTER
Division of Pure Physics, National Research Council, Ottawa, Canada

(Received July 2, 1962)

The hyperfine structure of the $4f^{13}6s^2\ {}^2F_{7/2}$ ground state of Tm^{169} has been studied by means of the atomic-beam magnetic-resonance method. By the use of multiple transitions, measurements were made from which the nuclear magnetic moment could be extracted without the use of approximate wave functions. Computer analysis of the experimental data gives the following results: $A=-374.1374\pm0.0016$ Mc/sec; $g_J=-1.14119\pm0.00004$; $g_I'=-(2.478\pm0.032)\times10^{-4}$ (with μ_I expressed in Bohr magnetons); whence $\mu_I=-(0.229\pm0.003)$ nm (corrected for the diamagnetic shielding effect of the electrons).

I. INTRODUCTION

THE electronic ground state of Tm has been determined by optical spectroscopy as $4f^{13}6s^2\ {}^2F$, with a separation of 8771.25 cm^{-1} between the ${}^2F_{7/2}$ and ${}^2F_{5/2}$ levels, the doublet being inverted.[1] The nuclear spin of Tm^{169} (natural abundance 100%), $I=\frac{1}{2}$, has been deduced optically first by Schüler and Schmidt.[2] Lindenberger's[3] recent investigation of the hfs of lines in the Tm II spectrum has confirmed this value. The latter author using the Goudsmit-Fermi-Segrè formula obtained from his data the value $-0.20_5\pm0.02$ nm for the nuclear magnetic moment of Tm^{169}.

Cabezas and Lindgren[4] studied the hfs of the neutron-rich radioisotopes, Tm^{170} and Tm^{171}, by means of the atomic-beam magnetic-resonance method. Apart from the nuclear spins of these nuclides they were also able to extract from their measurements an accurate value of the electronic g factor of the ${}^2F_{7/2}$ ground state. This value was found to differ significantly from the classical Landé value. By applying plausible corrections, they were able to account for the experimentally observed g_J. Further, using an improved radial wave function they recalculated the nuclear magnetic moment of Tm^{169} from Lindenberger's optical data and found it to be -0.25 nm.

In the following an atomic-beam magnetic-resonance investigation of the ground-state hyperfine structure of Tm^{169} is described. The measurements yield not only accurate values for the magnetic-dipole interaction constant and for the electronic g factor, but also the first directly measured value for the nuclear g factor of Tm^{169}. The present value of g_J confirms that of Cabezas and Lindgren.[4] Furthermore, the directly measured value of the nuclear g factor provides a test for the reliability of their assumptions concerning the wave function applicable to the Tm atom.

II. EXPERIMENTAL METHOD AND DATA

The atomic-beam magnetic-resonance apparatus and techniques used were the same as those described in a previous paper.[5]

Beams of Tm atoms were generated from tubular tantalum ovens heated to about 1000–1200°C (as measured with an optical pyrometer, uncorrected for emissivity) by the passage of high alternating current directly through the walls. The atoms were detected either by means of the electron-bombardment ionizer or by means of the tungsten surface-ionization detector, previously described.[5] The latter method was used for the greater part of the investigation. Efficient ionization was achieved by heating the tungsten ribbon to an apparent temperature of about 1750–1850°C (as measured with an optical pyrometer). Cs was used as the calibrating material for the homogeneous magnetic field. In the majority of runs, signal modulation was achieved by mechanical chopping of the beams.

Two series of measurements were made. The aim of the first series was to obtain accurate values for the magnetic-dipole interaction constant of the ${}^2F_{7/2}$ ground state, as well as for its electronic g factor. These were done at relatively low fields. The second series was aimed chiefly at a direct determination of the nuclear g factor of Tm^{169} and was done at relatively high fields.

The electronic configuration of Tm consists of completely filled shells minus one electron. In an external magnetic field such an atom having a nucleus with nuclear spin $I=\frac{1}{2}$ may be described by the following Hamiltonian

$$\mathcal{H}=A\mathbf{I}\cdot\mathbf{J}-g_J\mu_0\mathbf{J}\cdot\mathbf{H}-g_I'\mu_0\mathbf{I}\cdot\mathbf{H}. \tag{1}$$

Solution of the secular equation derived from this in either of the usual two representations leads to energy expressions similar to the well-known Briet-Rabi formula,[6] except that the I's and J's are interchanged. For example, in the low-field representation, the energy of the state characterized by the quantum numbers F and m is

$$W(F,m)=-\frac{\Delta W}{2(2J+1)}-g_J\mu_0 mH \pm\frac{\Delta W}{2}\left[\left(1+\frac{4m}{(2J+1)}x+x^2\right)\right]^{\frac{1}{2}} \tag{2}$$

where the $\pm$ signs refer to the $F=J\pm\frac{1}{2}$ levels; ΔW is the hfs separation; $x=(g_J-g_I')\mu_0H/\Delta W$; μ_0 is the

[1] W. F. Meggers, Revs. Modern Phys. **14**, 96 (1942).
[2] H. Schüler and T. Schmidt, Naturwissenschaften **22**, 838 (1934).
[3] K. H. Lindenberger, Z. Physik **141**, 476 (1955).
[4] A. Y. Cabezas and I. Lindgren, Phys. Rev. **120**, 920 (1960).
[5] G. J. Ritter, Phys. Rev. **126**, 240 (1962).
[6] G. Breit and I. I. Rabi, Phys. Rev. **38**, 2082 (1931).

TABLE I. Observed $\Delta F=0$ transition frequencies with their corresponding Cs calibrating frequencies.

ν_{Cs} (4, −3) ↔ (4, −4) (Mc/sec)	ν_{Tm} (3,0) ↔ (3, −1) (Mc/sec)
0.988 ±0.002	5.063 ±0.007
6.475 ±0.007	33.083 ±0.015
17.475 ±0.005	88.623 ±0.015
17.486 ±0.005	88.693 ±0.025
17.479 ±0.003	88.625 ±0.005
17.483 ±0.003	88.643 ±0.020
84.3000±0.0150	406.9920±0.0500
84.3097±0.0025	407.0865±0.0060
84.3150±0.0025	407.1130±0.0075
84.3280±0.0050	407.1950±0.0250
84.3500±0.0150	407.3510±0.0300
84.3765±0.0030	407.3965±0.0075
84.3768±0.0030	407.4035±0.0050
84.4105±0.0025	407.5445±0.0060
84.4200±0.0300	407.6300±0.0500
ν_{Cs} (4, −3) ↔ (4, −4)	ν_{Tm} (4,0) ↔ (4,1)
0.991 ±0.002	3.952 ±0.004
0.994 ±0.003	3.961 ±0.006
3.015 ±0.002	12.019 ±0.010
6.494 ±0.003	25.792 ±0.007
16.481 ±0.005	65.081 ±0.015
16.484 ±0.002	65.093 ±0.010
16.749 ±0.005	66.135 ±0.015
17.012 ±0.005	67.138 ±0.015
17.061 ±0.005	67.363 ±0.025
84.1690±0.0100	318.6100±0.0400
84.1998±0.0030	318.6602±0.0050
84.2220±0.0100	318.6900±0.0200
84.2480±0.0200	318.8160±0.0500
84.2710±0.0100	318.9000±0.0400
84.2934±0.0075	319.0080±0.0100
84.3092±0.0075	319.0060±0.0160
84.3165±0.0075	319.0800±0.0160
84.3310±0.0040	319.1090±0.0050

Bohr magneton; and $g_J=\mu_J/J$, $g_I'=\mu_I'/I$, where μ_J and μ_I' are in Bohr magnetons.

The energy levels obtained by means of the above expressions for the case of Tm[169] with $J=\frac{7}{2}$, $I=\frac{1}{2}$, are given schematically in Fig. 1. The levels are inverted, with the level with largest F value having the lowest energy, due to the nuclear magnetic moment being negative.[3]

With the focusing and analyzing magnets, A and B, respectively, in the "flop-in" arrangement only those atoms are refocussed on the detector which have undergone transitions between states with effective magnetic moments of opposite sign. That means that only a limited number of transitions allowed by the normal selection rules are observable. The broken arrows in Fig. 1 indicate the $\Delta F=\pm 1$ transitions observed in the present investigation; the observed low-frequency ($\Delta F=0$) transitions are indicated by double-barred arrows.

Measurements on the $\Delta F=\pm 1$ transitions were made at magnetic fields near zero (about 0.5 to 2.5 G); $\Delta F=0$ transitions were followed up to about 226 G. With the rf loops used (1.27-cm π loops and 2.54-cm σ loop), the Tm peaks had widths ranging from about 30–40 kc/sec to about 80–100 kc/sec, depending on the transition under consideration. The 30–40 kc/sec wide peaks

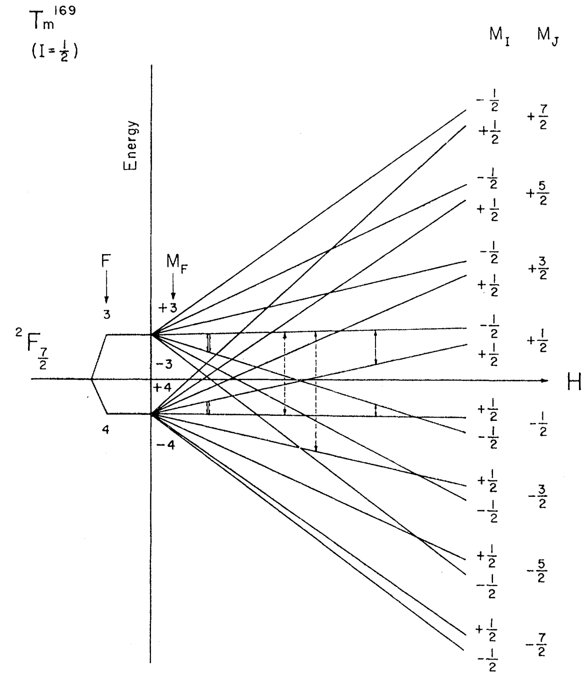

FIG. 1. Schematic energy-level diagram of the $^2F_{7/2}$ ground state of Tm[169] with the observed transitions indicated.

observed with the σ loop were superimposed on a wider peak of about 100–120 kc/sec. The Cs calibration peaks had widths varying from about 30 to 40 kc/sec over the field region used for the $\Delta F=0$ measurements. Signal-to-noise ratios for Tm using the hot-wire detector were of the order 15/1 to 50/1; for the electron-bombardment ionizer it was considerably smaller, 5/1 to 15/1.

The same procedure in making the measurements was employed as described in a previous paper.[5] For the first series of experiments the large number of measurements reduced to about 42 sets of data at various values of the magnetic field. These are given in Tables I and II.

In the second series of experiments the method of resonance in three consecutive rf loops[7–9] was used to render observable $\Delta m_I=\pm 1$, $\Delta m_J=0$ transitions in our atomic beam apparatus which had relatively short deflecting fields. In high enough magnetic fields, where the contribution of the term $g_I'\mu_0\mathbf{I}\cdot\mathbf{H}$ in the field-dependent part of Hamiltonian (1) becomes more important relative to the normally large term $g_J\mu_0\mathbf{J}\cdot\mathbf{H}$, one can obtain g_I' to a fairly high accuracy directly from such transitions. The ones observed in the present investigation are indicated by single bold-line arrows in Fig. 1.

For these observations there were two auxiliary rf loops on each side of the central C loop (1.27-cm π loop)

[7] G. K. Woodgate and P. G. H. Sandars, Nature **181**, 1395 (1958).
[8] G. O. Brink and W. A. Nierenberg, J. phys. radium **19**, 816 (1958).
[9] F. M. J. Pichanick, P. G. H. Sandars, and G. K. Woodgate, Proc. Roy. Soc. (London) **A257**, 277 (1960).

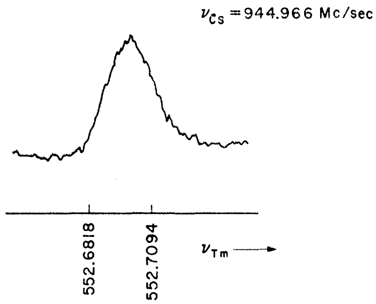

FIG. 2. A typical trace of the $(-\frac{1}{2},+\frac{1}{2})\leftrightarrow(+\frac{1}{2},+\frac{1}{2})$ line observed by means of the three-loop method at a C field of about 1633 G ($\nu_{Cs}=944.966$ Mc/sec). The frequencies of the $(3,0)\leftrightarrow(4,0)$ Tm transition excited in the two auxiliary loops were of the order 1520 Mc/sec.

in the gaps between the three main magnets. In principle the auxiliary loops could be operated in the stray fields of the main magnets. However, for some control over the field in the gaps, small auxiliary magnets were placed in these regions. In practice these were exicted in such a way as to cancel part of the stray fields from the neighboring main magnets.

For values of C fields up to about 1700 G, the C magnet (with a low-voltage, high-current winding) was excited by a stabilized high-current supply based on the design of Garwin *et al.*[10]

The Tm transition, $(3,0)\leftrightarrow(4,0)$ in the low-field representation, was excited in the two auxiliary loops (1.5-cm long σ loops). Two separate oscillators were used to provide incoherent radiation in the two loops. Proper coupling and loading of rf power into the loops were imperative for optimum "two-loop" effect. When this was realized, the "three-loop" effect showed up strongly. Figure 2 shows a trace of the $(-\frac{1}{2},+\frac{1}{2})\leftrightarrow(+\frac{1}{2},+\frac{1}{2})$ transition (m_I, m_J representation) observed in this way at a field of about 1633 G. The frequencies of the $(3,0)\leftrightarrow(4,0)$ Tm transition excited in the two auxiliary loops were in this case of the order 1520 Mc/sec—as they were in the majority of runs.

The hot-wire detector was used exclusively in this

TABLE II. Observed $\Delta F=\pm1$ transition frequencies with their corresponding Cs calibrating frequencies

ν_{Cs} $(4,-3)\leftrightarrow(4,-4)$ (Mc/sec)	ν_{Tm} $(3,0)\leftrightarrow(4,0)$ (Mc/sec)
0.3144±0.0025	1496.5554±0.0100
0.3958±0.0030	1496.5546±0.0040
0.8239±0.0030	1496.5552±0.0050
ν_{Cs} $(4,-3)\leftrightarrow(4,-4)$	ν_{Tm} $(3,0)\leftrightarrow(4,-1)$
0.1923±0.0020	1497.3154±0.0030
0.1993±0.0020	1497.3380±0.0020
0.3490±0.0015	1497.9440±0.0100
0.3520±0.0020	1497.9540±0.0070
0.5665±0.0020	1498.8060±0.0060
0.5800±0.0030	1498.8600±0.0080

[10] R. L. Garwin, D. Hutchinson, S. Penman, and G. Shapiro, Rev. Sci. Instr. **30**, 105 (1959).

series of experiments. The signal-to-noise ratio was usually of the order 15/1 to 30/1. The Cs calibration peaks had widths ranging from about 100 to 150 kc/sec, whereas the linewidths for Tm varied from about 25 kc/sec to about 40 kc/sec, depending slightly on the field region; the higher the magnetic field the narrower the peaks, because at high fields the field dependence of the $\Delta m_I=\pm1$, $\Delta m_J=0$ transitions is reduced, as is illustrated in Fig. 3. This figure shows the variation of the $(-\frac{1}{2},+\frac{1}{2})\leftrightarrow(+\frac{1}{2},+\frac{1}{2})$ and $(-\frac{1}{2},-\frac{1}{2})\leftrightarrow(+\frac{1}{2},-\frac{1}{2})$ transition frequencies with H, as observed experimentally.

Accurate measurements were made of these two transitions, at fields in the region of 1100 and 1650 G. Table III gives a summary of the data. The ultimate accuracy of these measurements was limited by the stability of the C field.

From the above measurements both the sign and magnitude of the nuclear g factor could be deduced, since it was possible to measure with sufficient accuracy the relatively small contribution to the magnitude of the Zeeman splitting which arises from the direct interaction of the nuclear magnetic moment with the applied field. It is also possible, however, to determine the sign of A, and hence of g_I, independently by examining the trajectories of the atoms under special refocusing conditions, in a manner outlined by Davis, Feld, Zabel, and Zacharias.[11] Apart from the slight differences in energy values already mentioned, a change of sign of the nuclear magnetic moment will mean additional even more drastic changes in the energy-level diagram given in Fig. 1: for $A>0$, that is, $\mu_I>0$, the level with largest F value will have the highest energy. This inversion is accompanied in any given transition by changes of sign of the associated magnetic quantum numbers, as is shown below. Since the direction of the forces on the atoms in an inhomo-

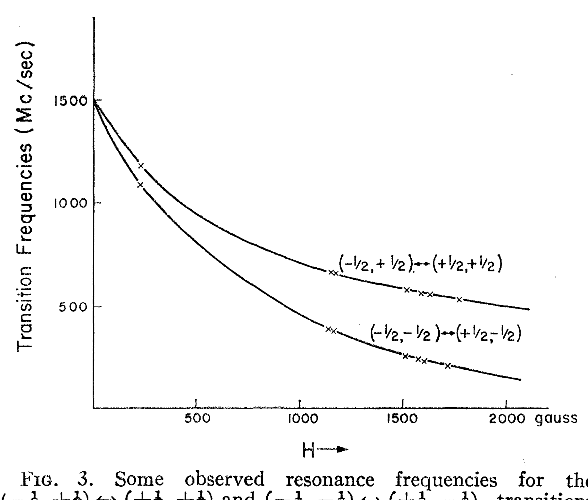

FIG. 3. Some observed resonance frequencies for the $(-\frac{1}{2},+\frac{1}{2})\leftrightarrow(+\frac{1}{2},+\frac{1}{2})$ and $(-\frac{1}{2},-\frac{1}{2})\leftrightarrow(+\frac{1}{2},-\frac{1}{2})$ transitions plotted as a function of magnetic field, H.

[11] L. Davis, B. T. Feld, C. W. Zabel, and J. R. Zacharias, Phys. Rev. **76**, 1076 (1949).

TABLE III. Observed ($\Delta m_J=0$; $\Delta m_I=\pm 1$) transition frequencies with their corresponding Cs calibrating frequencies.

ν_{Cs} (4, −1) ↔ (4, −4) (Mc/sec)	ν_{Tm} (3, −1) ↔ (4,0) (Mc/sec)
555.710±0.060	383.420±0.030
555.790±0.040	383.400±0.020
555.840±0.040	383.390±0.020
555.950±0.040	383.270±0.060
561.710±0.070	379.718±0.015
562.980±0.030	378.920±0.030
563.060±0.050	378.860±0.030
563.225±0.040	378.814+0.020
563.230±0.040	378.808±0.020
563.230±0.050	378.788±0.020
563.675±0.020	378.525±0.015
563.690±0.040	378.485±0.020
918.595±0.050	233.310±0.020
918.660±0.050	233.305±0.020
1021.600±0.080	207.720±0.020
1021.980±0.080	207.640±0.020
ν_{Cs} (4, −3) ↔ (4, −4)	ν_{Tm} (3,0) ↔ (4,1)
565.590±0.030	669.510±0.030
904.880±0.040	561.285±0.015
904.920±0.050	561.280±0.020
910.930±0.050	559.955±0.020
921.916±0.020	557.570±0.010
921.934±0.020	557.568±0.010
923.240±0.040	557.290±0.015
923.436±0.020	557.246±0.010
923.480±0.030	557.235±0.015
923.712±0.030	557.192±0.015
926.156±0.035	556.654±0.010
928.326±0.040	556.200±0.010
928.460±0.020	556.165±0.010
928.484±0.020	556.169±0.010
930.377±0.035	555.759±0.010
944.930±0.030	552.710±0.015
944.966±0.020	552.700±0.010
945.090±0.020	552.676±0.010
945.190±0.020	552.654±0.007
945.200±0.020	552.650±0.007
945.228±0.025	552.647±0.007
945.262±0.015	552.640±0.010
945.310±0.015	552.630±0.010
945.480±0.050	552.580±0.020

geneous magnetic field depends on the sign of m_J, the trajectory for a given transition will therefore depend upon the sign of the nuclear magnetic moment.

In the present investigation, two obstacle wires at the end of the B magnet were used[12]; one was used to stop unwanted atoms in the beam of low effective moments or of high velocity or both, which were only slightly deflected by the A and B fields. The second wire was used to explore the beam trajectories.

In order to refocus the atoms which have undergone transitions from $m_J=\pm\frac{1}{2}$ to $\mp\frac{3}{2}$, the B field was lowered to about one-third to about one-quarter that of A. The two observable transitions under these conditions denoted by both low-field and high-field quantum numbers (F, m_F, m_I, m_J) are characterized by $(3, 0, -\frac{1}{2}, +\frac{1}{2}) \rightarrow (4, -1, +\frac{1}{2}, -\frac{3}{2})$ and $(4, 0, +\frac{1}{2}, -\frac{1}{2}) \rightarrow (3, 1, -\frac{1}{2}, +\frac{3}{2})$ for μ_I assumed negative, or by $(3, 0, +\frac{1}{2}, -\frac{1}{2}) \rightarrow (4, 1, -\frac{1}{2}, +\frac{3}{2})$ and $(4, 0, -\frac{1}{2}, +\frac{1}{2}) \rightarrow (3, -1, +\frac{1}{2}, -\frac{3}{2})$ for μ_I assumed positive. For both μ_I negative and μ_I positive the transition involving the (3,0) level will have a lower frequency than the one involving the (4,0) level. These two lines in either case correspond to opposite trajectories down the apparatus.

It was found that the line with the lower frequency was blocked out when the second obstacle wire was moved across the beam direction towards the side closest to the concave pole of the B magnet. Similarly, the transition with higher frequency was blocked out when the obstacle wire was moved in the direction of increasing field. This implied that A must be negative, confirming the result obtained by the other method.

The experiment was repeated at about the same C field with the A and B magnets set to refocus atoms which have undergone transitions from $\pm\frac{3}{2}$ to $\mp\frac{1}{2}$. In this case, the transition with the higher frequency was blocked out when the stop wire was on the side of the neutral-beam position closest to the concave pole of the B magnet, again confirming that A is negative.

III. RESULTS

By plotting the experimental data, given in Table II, for the two $\Delta F=\pm 1$ transitions on large-scale graph paper and extrapolating to zero field, the hfs interval was found to be 1496.555±0.010 Mc/sec.[13] This gave the following preliminary value for the hfs interaction constant: $|A|=374.1388\pm 0.0025$ Mc/sec. The value $g_J=-1.1410\pm 0.0003$ was obtained by solution of the Breit-Rabi type formula (2) using only some of the data in Table I.

In order to extract the maximum amount of information with the highest accuracy from the experimental data given in Tables I, II, and III, these were finally analyzed using the computer facilities of the University of California. A least-squares fit was made of the parameters A, g_J and g_I'. The fundamental constants used were taken from Cohen, Crowe and DuMond[14]; the calibrating Cs data from Ramsey[15] and Kusch and Hughes.[16] The following results were obtained:

$$A=-374.1374\pm 0.0016 \text{ Mc/sec},$$
$$g_J=-1.14119\pm 0.00004,$$
$$g_I'=-(2.478\pm 0.032)\times 10^{-4}.$$

IV. THE NUCLEAR MAGNETIC MOMENT

A. Present Measurement

The g_I' quoted above yields

$$\mu_I=g_I' I\times 1836.12$$
$$=-(0.227_5\pm 0.003) \text{ nm}.$$

[12] H. Lew, Phys. Rev. **91**, 619 (1953).

[13] G. J. Ritter, Fifth Molecular Beam Conference, Brookhaven, L. I., New York, 1960 (unpublished).

[14] E. R. Cohen, K. M. Crowe, and J. W. M. DuMond, *Fundamental Constants of Physics* (Interscience Publishers, Inc., New York, 1957).

[15] N. F. Ramsey, *Molecular Beams* (Oxford University Press, New York, 1956).

[16] P. Kusch and V. W. Hughes, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1959), Vol. XXXVII/1.

It should be pointed out again that this value does not involve the use of any assumed wave functions and is equivalent to the one obtained by nuclear magnetic-resonance methods (if NMR were applicable to Tm). To correct for the diamagnetic shielding effect of the electrons, this value has to be divided by $(1-\sigma)$, where $\sigma=0.0079$ for $Z=69$.[15] The corrected value for the nuclear moment of Tm^{169} is then

$$\mu_I(\mathrm{Tm}^{169}) = -(0.229\pm0.003)\ \mathrm{nm}.$$

This value is the most accurate to date. It is consistent with the theoretical value of about -0.26 nm obtained by Mottelson and Nilsson[17] in a discussion of the nuclear properties of Tm^{169} in terms of the unified nuclear model.[18,19]

B. Comparison with Values Derived from A

There are a number of theoretical relations connecting the magnetic dipole interaction constant, A, with the nuclear magnetic moment, μ_I, or the nuclear gyromagnetic ratio, g_I. These relations involve varying assumptions about the field in which the electrons move. In the case of Tm, where the electronic configuration consists of completely filled shells minus one electron, the magnetic moment may be derived from the observed hfs interaction constant by means, for example, of the following expression, provided an estimate of the radial quantity, $\langle 1/r^3\rangle$, is available:

$$A = g_I' \frac{\mu_0^2}{h\times 10^6}\left[\frac{2L(L+1)}{J(J+1)}\right]\mathfrak{F}(J,Z_i)\langle 1/r^3\rangle\ \mathrm{Mc/sec.} \quad (3)$$

In this expression μ_0 is the Bohr magneton; $\mathfrak{F}(J,Z_i)$ is a relativistic correction factor, very close to unity for f electrons and hence discarded here.

In order to estimate $\langle 1/r^3\rangle$, one needs some approximate radial wave function. It has been customary to use hydrogenic wave functions. However, it has been shown recently, especially in the rare-earth region,[4,20–22] that this is a rather crude approximation.

Cabezas and Lindgren[4] have derived $\langle 1/r^3\rangle$ for Tm by assuming a modified hydrogenic wave function; adjusted to give, together with the Thomas-Fermi potential, the experimental value for the spin-orbit coupling constant $\zeta=2506\ \mathrm{cm}^{-1}$.[1] They have shown that $\langle 1/r^3\rangle=10.5$ atomic units (a.u.) is a much more reliable value than the value 13.1 a.u. obtained by the use of a pure hydrogenic wave function. Judd and Lindgren[20] have estimated that this value should not be in error by more than 5%. Lindgren[21] has recently recalculated $\langle 1/r^3\rangle$ along the same lines and finds it to be 10.73 a.u. Freeman and Watson[22] using Hartree-Fock wave functions suggest a value $\langle 1/r^3\rangle=12.25$ a.u.

TABLE IV. Summary of values of the nuclear magnetic moment (in nm) of Tm^{169} obtained by different methods.

(1) Present direct value	-0.229 ± 0.003
(2) Calculated from the present A and various $\langle 1/r^3\rangle$ using the following wave functions:	
(i) pure hydrogenic	-0.20
(ii) modified hydrogenic (Lindgren)	-0.24 ± 0.01
(iii) Hartree-Fock (Freeman and Watson)	-0.21
(3) From Tm II spectrum (Lindenberger)	$-0.20_5\pm0.02$
(4) By paramagnetic resonance (Hayes and Twidell)	$(\pm)0.24\pm0.01$

The μ_I's calculated by means of expression (3), using the present value for A and the above $\langle 1/r^3\rangle$ values, are summarized in Table IV. The quoted limits of error on the value obtained from Lindgren's[21] $\langle 1/r^3\rangle$ take into account a possible variation of 5% in this radial quantity.[20] They are also large enough to include diamagnetic corrections. Of the μ_I's obtained from the present A using the various $\langle 1/r^3\rangle$ values, this value is in closest agreement with the directly measured value $\mu_I=-(0.229\pm0.003)$ nm.

In Table IV the values of the nuclear magnetic moment as obtained by other workers using other experimental methods are also given. Lindenberger[3] arrived at his value by three independent methods from the hfs constants for the $4f$ and $6s$ electrons. These he obtained from a high-resolution optical investigation of a number of lines in the Tm II spectrum. Hayes and Twidell[23] observed the Tm III spectrum in a paramagnetic resonance study of x-irradiated Tm-doped CaF_2 crystals. They obtained $\langle 1/r^3\rangle$ for Tm^{2+} from the list of $\langle 1/r^3\rangle$ values given by Judd and Lindgren[20] by interpolation and used it in the formula given by Elliot and Stevens[24] to obtain the value for μ_I given in Table IV. Their quoted error does not take into account possible errors in the calculated values of $\langle 1/r^3\rangle$.

V. ATOMIC g FACTOR

The atomic g factor, $g_J=-1.14119\pm0.00004$, obtained in the present investigation, is in excellent agreement with the value $g_J=-1.14122\pm0.00015$, obtained by Cabezas and Lindgren.[4] These values both differ significantly from the classical Landé value, -1.14286, or even the value -1.14319 obtained from the latter by taking into account the anomalous magnetic moment of the electron.

Judd and Lindgren[20] in a discussion of the Zeeman effect of the rare earths have pointed out that there are a number of other corrections to be made to the simple Landé formula for the g values of ground-term levels of configurations of the type $4f^n$, before these can be

[17] B. R. Mottelson and S. G. Nilsson, Z. Physik **141**, 217 (1955).
[18] A. Bohr, Kgl. Danske Videnskab. Selskab, Matt.-fys. Medd. **26**, Nr. 14 (1952).
[19] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Matt.-fys. Medd. **27**, No. 16 (1953).
[20] B. R. Judd and I. Lindgren, Phys. Rev. **122**, 1802 (1961).
[21] I. Lindgren, Nuclear Phys. **32**, 151 (1962).
[22] A. J. Freeman and R. E. Watson (private communication, 1962).
[23] W. Hayes and J. W. Twidell, J. Chem. Phys. **35**, 1521 (1961).
[24] R. J. Elliot and K. W. H. Stevens, Proc. Roy. Soc. (London) **A218**, 553 (1953).

compared with the experimental values. These include, besides the Schwinger correction for the anomalous moment of the electron already mentioned, a correction to allow for deviations from pure *LS* coupling; a relativistic correction, directly related to the kinetic energy of the electrons; and a diamagnetic correction, depending on the electron density of the core.

Cabezas and Lindgren[4] in their work on Tm^{170} have shown that since the ground state of Tm is essentially a single-electron state, the effect of configuration interaction caused by spin-orbit coupling is quite negligible. By assuming the same modified hydrogenic wave function as used for obtaining the $\langle 1/r^3 \rangle$ given in Sec. IV(B), they calculated the corrections to the g factor due to relativistic and diamagnetic effects. The final calculated g value, -1.14121, obtained in this way, is in excellent agreement with the experimental value.

ACKNOWLEDGMENTS

The author wishes to thank Dr. Hin Lew for his interest and encouragement in this work. He is also greatly indebted to Dr. Howard A. Shugart and his associates at the University of California for the use of their computer facilities.

PHYSICAL REVIEW VOLUME 128, NUMBER 5 DECEMBER 1, 1962

Charge Transfer between Positive Cesium Ions and Cesium Atoms*

LAWRENCE L. MARINO, A. C. H. SMITH,† AND E. CAPLINGER
Space Science Laboratory, General Dynamics/Astronautics, San Diego, California

(Received July 16, 1962)

The cross section for charge transfer between cesium ions and cesium atoms has been determined as a function of primary ion energy from 50 to 4000 eV. At each energy the cross section was obtained from measurements of the number of slow ions formed along a known length of the fast primary ion beam in passing through cesium vapor, the primary beam current, and the number density of cesium atoms inside the interaction region. The measured cross sections are higher than those of other investigations within the same energy range in which different methods were used, but are consistent with those of an investigation in the energy range 5 to 25 keV. The result of a least-mean-squares fit indicates that the cross section may be represented between 50 eV and 4000 eV by the expression $Q^{1/2}\times 10^8 = 26.8 - 1.46 \ln V$, where Q is the cross section (cm²) and V is the primary ion beam energy (eV).

INTRODUCTION

CONSIDERABLE diversity exists between the results of a number of investigations of the resonance charge transfer reactions of the alkali metals. In this paper, results are presented for charge transfer between cesium ions and cesium atoms. These data are thought to be more reliable than those of earlier investigations within the same energy range.

Interest in cesium-ion propulsion for space vehicles and in the cesium diode as a thermoelectric device has emphasized the need for study of reactions involving cesium atoms and ions. Resonance charge transfer may be an important process occurring in both these devices. The process is represented by

$$Cs^{+}(1)+Cs(2) \rightarrow Cs(1)+Cs^{+}(2). \qquad (1)$$

In the encounter the incident ion (1) captures an electron from the atom (2) with negligible momentum transfer.

* Supported in part by the Advanced Research Projects Agency (Project Defender) through the Air Force Cambridge Research Laboratory.

† Present address: General Dynamics/General Atomic, San Diego, California.

In cases of resonance charge transfer, theory[1–3] predicts that the cross section at a given relative velocity increases with decrease of the ionization potential of the atom. Because of the low ionization potential of cesium, the charge transfer cross section is expected to be very large at low relative velocities of the particles. Resonance charge transfer in cesium has been investigated by Kushnir *et al.*[4,5] (KPS and KB), Bukhteev and Bydin[6] (BB), Chkuaseli *et al.*[7] (CNG), and Speiser and Vernon[8] (SV). The results of these investigations are in considerable disagreement with each other. In addition,

[1] Yu. N. Demkov, Scientific Notes, Leningrad State University, No. 146, Issue 8, 74 (1952).

[2] O. Firsov, J. Exptl. Theoret. Phys. (U. S. S. R.) 21, 1001 (1951).

[3] I. Flaks and E. Solov'ev, Soviet Phys.—Tech. Phys. 3, 564 (1958).

[4] R. Kushnir, B. Palyukh, and L. Sena, Bull. Acad. Sci. U. S. S. R. 23, 995 (1959).

[5] R. Kushnir and I. Buchma, Bull. Acad. Sci. U. S. S. R. 24, 989 (1960).

[6] A. Bukhteev and Y. Bydin, Bull. Acad. Sci. U. S. S. R. 24, 966 (1960).

[7] D. Chkuaseli, U. Nikoleishvili, and A. Guldamashvili, Bull. Acad. Sci. U. S. S. R. 24, 972 (1960).

[8] R. Speiser and R. Vernon, American Rocket Society Space Flight Report to the Nation, Report No. 2068-61, 1961 (unpublished).