

Study of the K^+ Decay*

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Some properties of the various decay modes of K^+ mesons have been studied with hodoscope scintillation counters. 122 $K_{\mu 3}$ events were identified in the high-energy end of the muon energy spectrum from 115 ± 3 MeV to 134 MeV which yielded $(0.42 \pm 0.10)\%$ as the relative branching ratio. The result, together with the $K_{e 3}$ branching ratio and the low-energy end of the $K_{\mu 3}$ muon energy spectrum, is found to satisfy a constant form factor $V-A$ interaction. The muon longitudinal polarization for the $K_{\mu 3}$ events was found to be $+0.8_{-0.8}^{+0.2}$ which is two standard deviations from the calculated value -0.81 using the constant form factors which best fit the spectrum. A lifetime analysis of the decay modes gave $T(K_{\mu 3}) = (1.06 \pm 0.16) \times 10^{-8}$ sec, $T(K_{\pi 2}) = (1.197 \pm 0.032) \times 10^{-8}$ sec, and $T(K_{\mu 2}) = (1.259 \pm 0.018) \times 10^{-8}$ sec which are consistent with a single total lifetime, as expected. The total lifetime was found to be $T(K^+) = (1.231 \pm 0.011) \times 10^{-8}$ sec in agreement with previous values. The relative $K_{\mu 2}/K_{\pi 2}$ branching ratio was measured as 2.98 ± 0.25 . A search for the occurrence of an intermediate-mass lepton in the decay $K \rightarrow \chi + \nu$ established 3×10^{-4} as the upper limit for the branching ratio with $m_e < m_\chi < 75$ MeV.

INTRODUCTION

THE extension of the $V-A$ theory to describe strange-particle decays has been proposed by several authors.¹ In particular, Zachariasen, Gatto, and others² have independently calculated the matrix element

$$M = [f(P_K - P_\pi) + gP_K]_\lambda \bar{u}_l \gamma_\lambda (1 + \gamma_5) u_\nu$$

for the three-body leptonic decay of the K meson, $K \rightarrow l + \pi + \nu$. The virtual strong interactions involved are expressed in terms of two form factors, $f(P_\pi)$ and $g(P_\pi)$, from Lorentz invariance considerations.³ The energy dependence of f and g is discussed in terms of a $K\pi S$ -wave scattering model by MacDowell⁴ and in terms of a $K\pi$ resonance model by Bernstein and Weinberg.⁵ A plot of the muon energy spectra, with various assumptions made as to form factors, is shown in Fig. 1.

We have measured the high-energy end of the muon

energy spectrum in $K^+ \rightarrow \mu^+ + \pi^0 + \nu$ decay, the lifetime of the particles which decayed via this mode, and with poor statistics the longitudinal polarization of the μ^+ . The ratio of the $K_{\mu 2}$ and $K_{\pi 2}$ branching ratios was also measured. We also established an upper limit for the existence of the decay mode $K \rightarrow \chi + \nu$, where the mass of the χ lepton lies between m_e and 74 MeV.

DESCRIPTION OF APPARATUS

The 500-MeV/ c K^+ beam obtained from the Cosmotron was similar to the one described by Fitch and

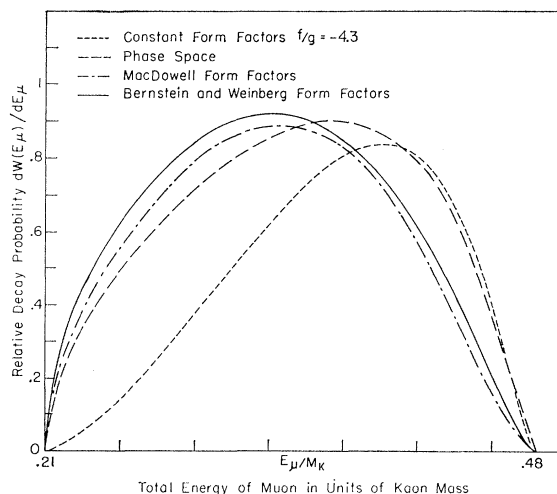


FIG. 1. Muon energy spectra in $K_{\mu 3}$ decay calculated for various form factors.

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¹ R. Dalitz, *Revs. Modern Phys.* **31**, 823 (1959); R. Dalitz, *Rend. Scuola Intern. Fis. "Enrico Fermi"* **11**, 299 (1959); V. Shekhter, *Soviet Phys.—JETP* **8**, 316 (1959).

² R. Gatto, *Phys. Rev.* **111**, 1426 (1958); F. Zachariasen, *ibid.* **110**, 1481 (1958); S. G. Matinyan and L. B. Okun, *Soviet Phys.—JETP*, **9**, 933 (1959); J. Nilsson, *Nuclear Phys.* **14**, 639 (1960); N. Brene, L. Egdardt, and B. Qvist, *ibid.* **22**, 553 (1961).

³ M. Goldberger and S. Treiman, *Phys. Rev.* **111**, 354 (1958).

⁴ S. MacDowell, *Phys. Rev.* **116**, 1047 (1959).

⁵ J. Bernstein and S. Weinberg, *Phys. Rev. Letters*, **5**, 1481 (1960).

Motley.⁶ A total of 200 K mesons per 10^{11} circulating protons was stopped in a 4-in.-thick Lucite block placed 16 ft away from the point of production. The K telescope (see Fig. 2) consisted of three dE/dx scintillation counters, S_0 , S_1 , and S_2 , which differentiated K mesons in the beam from π mesons by pulse height, and a threshold Čerenkov counter, C , which detected π mesons. Scintillation counters S_5 and S_6 detected any charged particles that emanated from the lucite block and entered the μ - e hodoscope. A coincidence between S_5 and S_6 , gated by a coincidence of $S_0S_1S_2$ and simultaneously an anticoincidence of C , was defined as a K signal. The π contamination of the K signal was found to be less than 0.1%. The following set of apparatus was used to detect the K -meson decay properties:

The μ - e hodoscope was used to detect the range and polarization of the charged particle resulting from K decay. It consisted of 120 scintillation counters arranged in 12 layers as shown in Fig. 2. Each scintillator (Nuclear Enterprises Limited NE-102 20 in. \times 1½ in. \times ½ in.) was covered with aluminum foil. One end of the scintillator was mounted with an air gap against an RCA 6199 photomultiplier tube; the other end was covered by a front surfaced mirror. The pulse-height variation from one end of the scintillator to the other was less than 40%. A ¼-in.-thick aluminum sheet was inserted between layers of counters, and the counters were biased to detect minimum ionization particles with the aid of a Co^{60} source. With the given geometry and size of the stopping block, the monoenergetic π and μ mesons from the $K_{\pi 2}$ and $K_{\mu 2}$ modes of decay appeared in the μ - e hodoscope as normal distributions with $\sigma = 2.21$ g/cm² and $\sigma = 1.67$ g/cm², respectively, where σ is the half-width at half-maximum of the distribution of stopping particles. The difference in σ was due entirely to straggling. The entire μ - e hodoscope was magnetically shielded by a 3/8-in.-thick iron box and a 3/64-in. mu-metal box to minimize the interaction between the stopping muons and the magnetic field (~ 20 G) generated by the Cosmotron. In order to calibrate the hodoscope as a polarization analyzer, 2434 μ mesons from

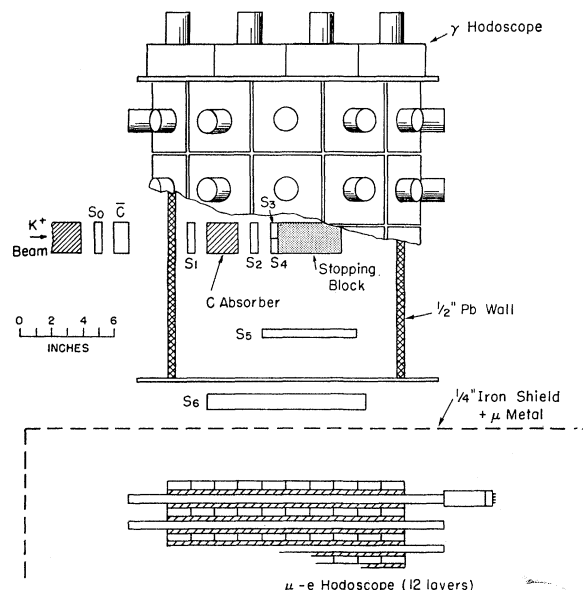


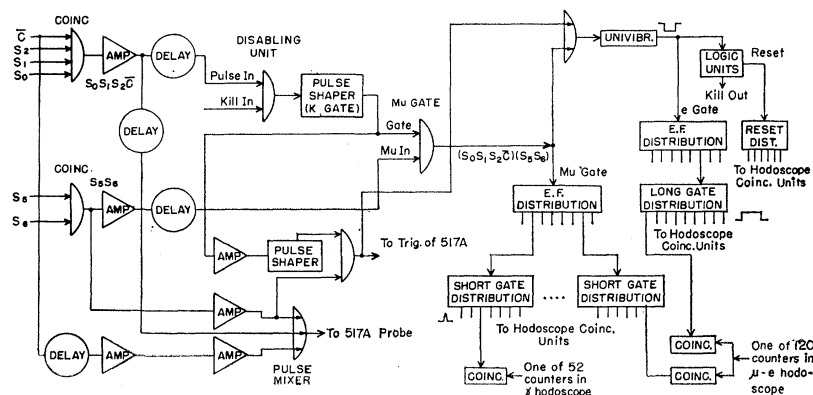
FIG. 2. Diagram of apparatus.

the $K_{\mu 2}$ decay mode and 231 π mesons from the $K_{\pi 2}$ decay mode were stopped in the hodoscope. The polarizations as determined from the up-down asymmetry of these particles were found to be $-1(+0.15, -0)$ and 0.52 ± 0.45 , respectively.

The gammas derived from the π^0 mesons from $K_{\pi 2}$, $K_{\mu 3}$, and K_{e3} decay modes were detected by the γ hodoscope, an array of 52 scintillation counters, each 4¼ in. \times 4¼ in. \times 1 in. Each scintillator was mounted in a box covered with aluminized styrene and viewed by an RCA 6655 A photomultiplier tube 3 in. above the plastic. A ½-in. lead converter was placed in front of each counter.

The time interval between the K stopping and the subsequent decay was recorded by displaying and photographing the K trigger pulse $S_0S_1S_2\bar{C}$, the coincidence pulse S_5S_6 , and \bar{C} simultaneously on a Tektronix 517A oscilloscope (See Fig. 3). The scope was

FIG. 3. Block diagram of electronics.



⁶ V. Fitch, R. Motley, Phys. Rev. **101**, 496 (1956).

calibrated periodically by a 175.00-Mc/sec sine wave generated by a GR 1209-BL Oscillator which was calibrated in turn against a TS-174/U frequency standard. The block diagram of the electronics is shown in Fig. 3.

The electronics, with the exception of Wentzel-type coincidence circuits, were all transistorized. The signals from each μ - e hodoscope counter were fed into two transistorized coincidence units. One unit was gated by a 30-nsec pulse generated by the K signal to interrogate the counter for μ mesons; the other unit was gated by a pulse 2 μ sec long delayed 0.2 μ sec after the μ pulse (S_5S_6) to detect the electrons resulting from μ - e decay. The γ hodoscope units were gated by the 30-nsec pulse. The output of each coincidence unit was used to flash on a 6977 visual indicator tube for 200 μ sec. A modified oscillograph camera with a $f/1.4$ lens was used to record the array of 6977 lights on Agfa Record film. The dead time of the recording system was 2 msec.

SCANNING

The range of the charged particle from each K decay where stopped in the μ - e hodoscope was determined from the coordinate of its end point. Photographs which contained double tracks were not considered, and particles which scattered more than 30° from the point of entry in the μ - e hodoscope were also discarded. The probability of a μ track being falsely extended one layer (2.8 g/cm^2) by background was found to be $(0.053 \pm 0.005)\%$. The average efficiency of μ - e hodoscope counters was found to be $98.6 \pm 0.2\%$.

The corresponding decay time of each particle stopped in the μ - e hodoscope was determined from the distance between half-maxima of the coincidence pulse $S_0S_1S_2\bar{C}$ and the pulse S_5S_6 which were recorded on film. Measurements of the sine wave positions on the calibration traces served to calibrate the time base.

RESULTS

A. Muon Energy Spectrum and Polarization at the High-Energy End in $K_{\mu 3}$ Decay

Absorbers were placed between S_5 and S_6 so that the π^+ mesons from the $K_{\pi 2}$ mode were stopped in the third layer of the μ - e hodoscope. The muons resulting from the $K_{\mu 3}$ mode were selected in the following way:

(1) *Range criteria.* We selected those events whose energy lay between that of the π mesons from $K_{\pi 2}$ decay and the muons from $K_{\mu 2}$ decay, i.e., $115 \text{ MeV} \pm 3 \text{ MeV} \leq E \leq 134 \text{ MeV}$. Within the accepted energy region, the $K_{\pi 2}$ contamination was estimated to be less than 9% .

TABLE I. The number of events in the high-energy end of the muon energy spectrum in $K_{\mu 3}$ decay is tabulated. This corresponds to a total of 27 000 $K_{\mu 2}$ events.

| Energy interval in MeV | 115-120 | 121-125 | 126-131 | 132-134 |
|------------------------------|-------------|-------------|-------------|-----------|
| Number of $K_{\mu 3}$ events | 45 ± 15 | 55 ± 15 | 21 ± 11 | 1 ± 1 |

This was deduced from the width and shape of the $K_{\pi 2}$ line spectrum.

(2) *Lifetime criteria.* Out of the sample selected by range, we accepted only those events whose decay time was greater than 4.6 nsec and less than 25.6 nsec. This eliminated mesons which triggered the K gate and scattered into the μ - e hodoscope.

(3) *Decay electron requirement.* We selected events which exhibited μ - e decays. K_{e3} events, π mesons from $K_{\pi 2}$ mode decaying within the fast gate, and other spurious events such as potons from π stars and electrons from π^0 conversion are possible sources of contamination. They were largely subtracted out from the above selection of events by a μ - e decay requirement. The efficiency of the hodoscope in detecting μ - e decays was determined from π and μ mesons which stopped in the hodoscope.

(4) The γ hodoscope information was used to determine the number of $K_{\mu 3}$ events near the $K_{\mu 2}$ peak.

The high-energy end of the muon energy spectrum, corrected for all the biases mentioned above, is listed in Table I. The ratio of $K_{\mu 3}$ to $K_{\mu 2}$ for this part of the spectrum is $(0.72 \pm 0.17)\%$ which gives $(0.42 \pm 0.10)\%$ as the $K_{\mu 3}$ branching ratio for $(115 \pm 3) \text{ MeV} \leq E_\mu \leq 134 \text{ MeV}$. The low-energy end of the muon energy spectrum has been measured by many authors.⁷ The combined data of Taylor *et al.*⁸ and Alexander *et al.*⁹ (see Fig. 4) yield $(1.3 \pm 0.17)\%$ as the relative $K_{\mu 3}$ branching ratio for $E_\mu \leq 63 \text{ MeV}$. A combined plot of the high- and low-energy data normalized to each other is shown in Fig. 4. A comparison between the experimentally obtained data and the spectra shown in Fig. 1 is listed in Table II.

The constant form factor hypothesis with $-14 \leq f/g \leq -4.3$ gives a spectrum which agrees with the measured data. The computed spectrum for $f/g = -4.3$ is shown in Fig. 4. Using $f/g = -4.3$ and the experimental $K_{\mu 3}$ data, one predicts $(4.5 \pm 2)\%$ for the $K_{\mu 3}$ branching ratio and $(5.7 \pm 3)\%$ for the K_{e3} branching ratio. Both are consistent with recent results given by Roe *et al.*¹⁰

Our muon spectrum is in agreement with the muon spectrum measurements of Mann *et al.*¹¹ Mann finds that $\zeta = -9$ fits their data best where $\zeta = 2(f/g) + 1$, i.e., $f/g = -5$ fits their data best.

The possibility of $K_{\mu 3}$ decaying via a vector boson¹² was investigated by assuming constant form factors f' and g' with $f'/g' \approx -4$. Taking the boson mass to be

⁷ The data are summarized by R. W. Birge, D. H. Perkins, J. R. Peterson, D. H. Stork, and M. N. Whitehead, *Nuovo cimento* 4, 834 (1956) and C. O'Cealleigh, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957).

⁸ S. Taylor, G. Harris, J. Orear, J. Lee, and P. Baumel, *Phys. Rev.* 114, 359 (1959).

⁹ G. Alexander, R. H. Johnston, and C. O'Cealleigh, *Nuovo cimento* 6, 478 (1957).

¹⁰ B. P. Roe, D. Sinclair, J. L. Brown, D. Glaser, J. A. Kadyk, and G. H. Trilling, *Phys. Rev. Letters*, 7, 346 (1961).

¹¹ A. K. Mann, K. Reibel, F. J. Sciulli, H. Uto, D. H. White, and K. K. Young, *Phys. Rev. Letters*, 8, 295 (1962).

¹² J. Pati, thesis, University of Maryland (unpublished).

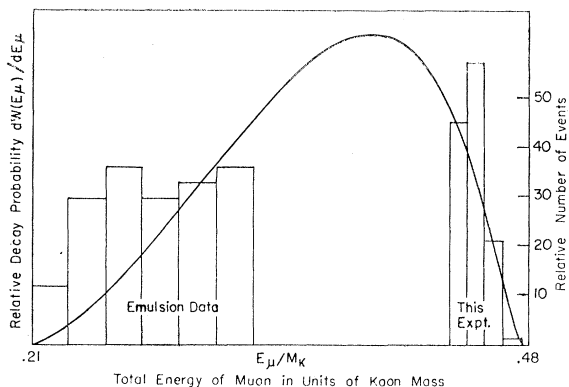


FIG. 4. The measured muon spectrum from $K_{\mu 3}^+$ decay is shown together with the low-energy data from Taylor *et al.* (reference 8). The solid curve is the theoretical spectrum calculated for constant form factors with $f/g = -4.3$. Both the low-energy data and the theoretical spectrum have been normalized to our data.

$M_{\pi} + M_K$, the $K_{e 3}$ branching ratio then comes out to be about $\frac{1}{2}\%$, which is an order of magnitude less than the measured value. The form factors calculated with the assumption that the $K\pi$ system has a strong S -wave resonance near 800 MeV yields a muon spectrum which exhibits a peak near 70 MeV. Chew¹³ points out that a P -wave $K\pi$ resonance at 880 MeV yields a similar spectrum. The experimental spectrum does not support either the boson or resonance theories.

The longitudinal polarization of the muon was measured for muons near the high-energy end. Fifty events had acceptable electron tracks from which the muon polarization was found to be $+0.8_{-0.8}^{+0.2}$ where the plus sign denotes the spin of the muon parallel to the muon momentum. The geometry of the hodoscope and depolarization effects of the plastic scintillator were considered in deriving the above number.

The f/g value of -4.3 obtained from the muon spectrum predicts an average polarization of -0.81 for $115 \text{ MeV} < E_{\mu} < 134 \text{ MeV}$, which is two standard deviations from our measured value. This appears to be a contradiction to the constant form factor theory with the f/g value determined from the spectrum. However, the large statistical error in the polarization measurements makes it difficult to conclude whether or not this is a true disagreement.

B. Lifetime

The $K_{\mu 3}$ events used for lifetime analysis were selected from the sample mentioned in the previous section by requiring a μ - e decay so as to minimize any contamination from other backgrounds. Events whose decay time ranged from 4.6 to 25.3 nsec were included in the analysis.

The mean life and variance were obtained by the maximum likelihood method. The value obtained is

$$\tau(K_{\mu 3}) = (1.06 \pm 0.16) \times 10^{-8} \text{ sec.}$$

¹³ H. Chew, Phys. Rev. Letters, **8**, 297, (1962).

As an over-all check of the measurement procedure a sample of 1414 $K_{\mu 2}$ events was taken over the same interval. These data handled in the same way gave $\tau(K_{\mu 2}) = 1.18 \pm 0.05 \times 10^{-8} \text{ sec}$. The present result agrees with the $K_{\mu 3}$ lifetime measured by Taylor *et al.*⁸ $\tau(K_{\mu 3}) = (1.20_{-0.4}^{+1.0}) \times 10^{-8} \text{ sec}$ and Hoang¹⁴ $\tau(K_{\mu 3}) = (0.88 \pm 0.23) \times 10^{-8} \text{ sec}$.

Lifetime determinations for the more abundant $K_{\pi 2}$, $K_{\mu 2}$, and total K^+ decays were made also. These modes were selected from the range data of the decay particle entering the μ - e hodoscope. Lifetime measurements were made from approximately 100 000 oscilloscope traces with the time scale extending from zero to $8\tau_K$. From the range information, 5000 decays were identified as $K_{\pi 2}$ and 16 800 as $K_{\mu 2}$. The time dispersion (± 2.2 nsec for electronic and scanning jitter) in the lifetime frequency distribution required discarding measurements which were less than $\sim 0.6\tau_K$ or greater than $7\tau_K$. This left 2900 $K_{\pi 2}$ events, 11 000 $K_{\mu 2}$ events, and 49 000 K^+ events for the lifetime determinations.

The separate frequency distributions of decay times for $K_{\pi 2}$, and $K_{\mu 2}$, and K decays were each fitted with curves of the form $Ae^{-\lambda t} + B$ by the method of least squares in which the parameters A , B , and λ were varied. A maximum likelihood analysis gave almost identical results for these parameters. The resultant lifetime estimates from this analysis, with allowance for a systematic error of 0.5% are

$$\tau(K_{\pi 2}) = (1.197 \pm 0.032) \times 10^{-8} \text{ sec,}$$

$$\tau(K_{\mu 2}) = (1.259 \pm 0.018) \times 10^{-8} \text{ sec.}$$

The ratio of these two mode lifetimes is 1.7 standard deviations from unity. We conclude that the $K_{\pi 2}$ and $K_{\mu 2}$ lifetimes do not differ significantly in this measurement. As expected, all the mode lifetimes measured here are consistent with a single value.

The total K^+ mean lifetime estimate from this measurement is

$$\tau(K^+) = (1.231 \pm 0.011) \times 10^{-8} \text{ sec,}$$

where the error includes a 0.5% systematic error with the 0.75% statistical error.

TABLE II. A comparison of the various models for $K_{\mu 3}$ decay and the experimental muon energy spectrum is made by calculating R , the ratio of the decay rate for low- to high-energy muons in $K_{\mu 3}$ decay. In calculating R ,^a the resolution of the apparatus was taken into consideration.

| | R |
|--|---------------|
| Bernstein form factors | 17 ± 4 |
| MacDowell form factors, Brene <i>et al.</i> ^b | 15 ± 3 |
| Constant form factors $f/g \leq -4$ | 3.5 ± 0.5 |
| Phase-space density | 4.5 ± 1 |
| Experimental value | 3.0 ± 0.9 |

^a R is defined as $\int_{E_{\mu}=0}^{E_{\mu}=63 \text{ MeV}} dW(E_{\mu}) / \int_{E_{\mu}=115 \text{ MeV}}^{E_{\mu}=134 \text{ MeV}} dW(E_{\mu})$

^b See reference 2.

¹⁴ T. F. Hoang, M. F. Kaplon, and G. Yekutieli, Phys. Rev. **105**, 278 (1957).

These values are in agreement with the previous measurements made by Fitch and Motley^{15,16}: $\tau(K^+) = (1.221 \pm 0.026) \times 10^{-8}$ sec, $\tau(K_{\mu 2}) = (1.21_{-0.10}^{+0.11}) \times 10^{-8}$ sec, $\tau(K_{\mu 2}) = (1.17_{-0.07}^{+0.08}) \times 10^{-8}$ sec; and by Alvarez *et al.*^{16,17}: $\tau(K^+) = (1.227 \pm 0.015) \times 10^{-8}$ sec, $\tau(K_{\pi 2}) = (1.3 \pm 0.2) \times 10^{-8}$ sec, $\tau(K_{\mu 2}) = (1.4 \pm 0.2) \times 10^{-8}$ sec.

C. $K_{\mu 2}/K_{\pi 2}$ Ratio

The ratio of the $K_{\mu 2}$ branching ratio to the $K_{\pi 2}$ branching ratio was determined from a run where the π mesons from $K_{\mu 2}$ decay mode were stopped in the hodoscope. All but 1% of the μ mesons from $K_{\mu 2}$ decay mode penetrated the hodoscope. Events from the two decay modes were identified by their ranges and the following corrections were made

(a) The difference in solid angle of the different detecting regions of the μ - e hodoscope was taken into account.

(b) The $K_{\pi 2}$ events were contaminated by $K_{\mu 3}$, K_{e3} and other spurious events due to π mesons scattered from the beam. Events which did not undergo secondary decay such as K_{e3} were estimated to be $(5 \pm 3.9)\%$. The scattered π -meson contamination was reduced to $< 0.5\%$ by selecting those events which had a decay time greater than $\frac{1}{2}$ mean life. The $K_{\mu 3}$ contribution was calculated by using the experimentally determined partial decay rate and $f/g = -4.3$.

(c) The loss of pions from $K_{\pi 2}$ resulting from large angle scattering and inelastic nuclear interactions was corrected using the cross sections reported by Stork and others.¹⁸ This correction amounted to $(31 \pm 5)\%$.

¹⁵ V. Fitch and R. Motley, Phys. Rev. **101**, 496 (1956); **105**, 265, (1957).

¹⁶ Private communications reported by G. A. Snow and M. M. Shapiro, Revs. Modern Phys. **33**, 231 (1961).

¹⁷ L. W. Alvarez, F. S. Crawford, M. L. Good, and M. L. Stevenson, Phys. Rev. **101**, 503 (1956).

¹⁸ G. Bernadini, E. T. Booth, and L. Lederman, Phys. Rev., **83**, 1075 (1951); H. Byfield, J. Kessler, and L. M. Lederman, *ibid.*

The $K_{\mu 2}/K_{\pi 2}$ ratio after corrections is 2.98 ± 0.25 . Our result is statistically not in disagreement with 3.46 ± 0.07 published by Roe *et al.*¹⁰

D. Search for an Intermediate-Mass Lepton

A search was undertaken to find a possible decay

$$K^+ \rightarrow \chi^+ + \nu,$$

where χ has a range greater than 75 g/cm² of carbon. We observed 20 events with ranges greater than 75 g/cm² in 10 209 $K_{\mu 2}$ events. The contribution of electrons to the above 20 events from the K_{e3} mode of decay was calculated from $f/g \simeq -4.3$ to be less than one event. The time distribution of this sample of events was observed to be random and inconsistent with that of the K lifetime. These events were probably pions from the 500-MeV/ c beam which were scattered into the apparatus while the K gate was opened. By allowing possibly two to three events in the given range and time interval, an upper limit to the decay mode $K \rightarrow \chi + \nu$ relative to the $K_{\mu 2}$ mode is less than 3×10^{-4} .

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86, 17 (1952); R. L. Martin, *ibid.* **87**, 1052 (1952); A. Shapiro, *ibid.* **84**, 1063 (1951); C. Chedester, P. Isaacs, A. Sachs, and J. Steinberger, *ibid.* **82**, 958 (1951); D. H. Stork, *ibid.* **93**, 868 (1954); J. F. Tracy, *ibid.* **91**, 960 (1953).