Spontaneous Breakdown of Elementary Particle Symmetries*

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Following Nambu, Jona-Lasinio, and Goldstone, we seek nonperturbative solutions of quantum field theory. We first consider a model which possesses the higher symmetry $SU(3)$. We find that without the introduction into the Lagrangian of any symmetry-breaking terms, solutions exist which have only the lower symmetries of isotopic spin and hyperchange. We also show that the usual electrodynamic interaction of the muons and the electrons allows the possibility of generating a mass splitting between them. Finally we consider a Lagrangian in which the bare coupling constants are set equal to zero, and self-generated renormalized coupling constants are found.

INTRODUCTION

THE present quantum field theory of elementary particles is beset by difficulties of many sorts: the appearances of divergences, possible inconsistencies, and the inadequacy of methods of calculations. But perhaps most discouraging is the fact that it seems necessary to employ an extremely complicated Lagrangian if there is to be any hope of describing the variety of elementary particle phenomena. It seems necessary to introduce a hierarchy of interactions with various elaborate symmetry properties: strong interactions, medium strong interactions, electromagnetic interactions, and weak interactions. The symmetries of one interaction always conflict with symmetries of others. The electron and muon are alike except for their mass difference; the β-decay interactions maximally violate conservation of parity, except for the axial-vector renormalization due to strong interactions; and possible higher symmetries, like unitary symmetry, are broken by the medium strong interactions. Thus, it seems that we must take for the Lagrangian a sum of terms with unrelated properties. None of the coupling constants and masses appearing in the world Lagrangian may be calculated and no simple principle explains why the Lagrangian has the form that it does.

This situation may be contrasted with the state of affairs in atomic, molecular, and solid-state physics where a single law describes the great variety of experimental data. Complexity arises from the complicated nature of the solutions to a simple fundamental equation involving only electromagnetic forces. Should not the complexities of the phenomena of elementary particle physics likewise arise from a "simple" fundamental theory?

Such a possibility was discussed by Heisenberg[1,2] and co-workers. They pointed out that the equations of quantum field theory are nonlinear operator equations. Since nonperturbative solutions to nonlinear equations do not in general possess the symmetry of the equations themselves, it is conceivable that the field equations may be highly symmetric expressions, while their solutions may reflect the asymmetries of nature. This is the philosophy we adopt in this paper.

This idea has been developed along different lines in the work of Nambu and Jona-Lasinio[3] and Goldstone[4] upon which this work is based. Nambu and Jona-Lasinio have shown that solutions to a field theory may exist which lack the symmetry of the Lagrangian: in particular, they show how theories with a γ_5 symmetry may admit Fermion states of finite mass. The present work is a straightforward application of these ideas to other kinds of invariances of the Lagrangian[5]: to so-called "internal symmetries" like isotopic spin and its generalizations; to a conjectured muon-electron symmetry; and to space reflection.

We base our analyses upon the coupled nonlinear equations for the one-particle Green's function and the vertex function (the Dyson equations). These were originally deduced by Dyson[6] for quantum electrodynamics by formally summing perturbation theory. Since the perturbation solution always possesses the symmetry of the Lagrangian, it is essential that these equations have meaning independent of the perturbation expansion. This was shown to be so by Schwinger[7] when he derived these equations from field theory using his action principle. Thus, they form an adequate starting point for our search for nonperturbative symmetry-violating solutions to a symmetric field theory.

We propose that a nonperturbative behavior characterizes all the interactions to which elementary particles are subject. Mass is completely dynamical; mass splittings and "approximate symmetries" result

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[3] G. Jona-Lasinio and Y. Nambu, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).
[4] J. Goldstone, Nuovo cimento **19**, 154 (1961).
[5] J. Goldstone, reference 4 also discusses other types of invariances.
[6] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).
[7] J. Schwinger, Proc. Nat'l. Acad. Sci. U. S. **37**, 452 (1951).

from nonsymmetric solutions to a fully symmetric Lagrangian theory.

Our approach has all the divergence difficulties of quantum field theories.

To carry out a sufficiently detailed analysis *proving* that the nonperturbative solutions are the ones that are actually realized in nature would require the use of reasonable approximations and realistic models. This is not the purpose of the present work. Rather, we demonstrate that several models, described by Lagrangians with extensive symmetries, do admit solutions with lesser symmetry. It is thus made plausible that the intricacies of the physical world are not reflected in an equally intricate fundamental theory.

To illustrate how the nonsymmetric solutions arise, let us consider n Fermions symmetrically coupled to m Bosons by means of unspecified Yukawa interactions. An arbitrary S-matrix element may be expressed as a power series in the exact Fermion Green's functions $G_{ij}(p)$, the exact Boson Green's function $D_{ab}(k)$, and the complete vertex operator $\Gamma_{ija}(p,k)$. The power series extends over all irreducible diagrams which contribute to the element of the S matrix being considered. The functions G, D, and Γ are themselves determined by solving the Dyson equations. The usual solutions to these equations correspond to a perturbative solution in powers of the renormalized charges, g_{ija}, and the zero-order solutions, G_{ij}^0, D_{ab}^0, and Γ_{ija}^0, which are the usual noninteracting functions referring to the physical masses. For a renormalizable theory, this is the way the renormalized perturbation theory is usually developed.

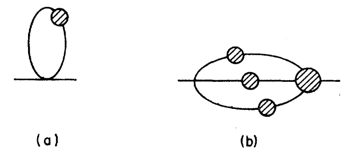
These perturbative solutions clearly have the same symmetries as the Lagrangian, and the resulting S matrix also has these symmetries.

We follow this prescription for the determination of the S matrix *with one exception*. We observe that the nonlinear equations for D, G, and Γ may possess other, nonperturbative, solutions which possess less symmetry than the Lagrangian. At this stage, we know of no way to tell which of the solutions to the Dyson equations are the physically realized ones. On the other hand, no obvious physical principle indicates that the symmetry-preserving perturbative solution should be preferred to the symmetry-breaking solutions. Perhaps a kind of stability criterion can be found that determines which are the physical solutions.

In the next sections we show explicitly that symmetry-breaking solutions do exist for G, D, and Γ in a variety of models. The S matrix constructed from these solutions necessarily violates the symmetry as well.

In Sec. II we illustrate this approach by obtaining nonsymmetric solutions in a theory possessing isotopic-spin symmetry. In Sec. III we consider a theory of physical interest which possesses the higher symmetry $SU(3)$. We find that without the introduction into the Lagrangian of any symmetry-breaking terms, solutions exist which have only the lower symmetries of isotopic spin and hypercharge. In Sec. IV we show that the usual electrodynamic interaction of muons and the electrons allows the possibility of generating a mass splitting between them. In Sec. V we consider the vertex equations for the vector and axial-vector couplings of a vector boson with Fermions. We show that the requirement that the bare coupling constants vanish imposes self-consistency conditions upon the renormalized coupling constants. In a simple approximation these conditions imply that the renormalized interactions either conserve parity or violate it maximally.

FIG. 1. Contributions to $\Sigma_{ij}(\gamma\cdot p)$.



We emphasize that these results are all obtained within the context of the conventional Green's function equations of field theory once it is realized that these nonlinear equations, in general, do possess solutions without the symmetry of the equations.

II. THE TWO-FERMION PROBLEM

For our first model, we consider two self-coupled Fermion fields with bare mass zero and with an interaction invariant under isotopic rotations,

$$\psi\to\exp(i\boldsymbol{\alpha}\cdot\boldsymbol{\tau})\psi,\quad \bar\psi\to\bar\psi\exp(-i\boldsymbol{\alpha}\cdot\boldsymbol{\tau}),\tag{2.1}$$

and under the discrete γ_5 transformation,[8]

$$\psi\to\gamma_5\psi,\quad \bar\psi\to\bar\psi\gamma_5.\tag{2.2}$$

We choose the Lagrangian used by Banerjee[9] in his discussion of the work of Nambu and Jona-Lasinio,

$$\mathcal{L}=+i\bar\psi\gamma^\mu\partial_\mu\psi+g(\bar\psi\psi)^2+f(\bar\psi\gamma_5\psi)^2 +g'(\bar\psi\boldsymbol{\tau}\gamma_5\psi)^2+f'(\bar\psi\boldsymbol{\tau}\psi)^2.\tag{2.3}$$

If

$$f=g\quad\text{and}\quad f'=g',\tag{2.4}$$

then $\mathcal{L}$ is invariant under the continuous γ_5 transformation,

$$\psi\to e^{\alpha\gamma_5}\psi,\quad \bar\psi\to\bar\psi e^{\alpha\gamma_5},\tag{2.5}$$

while if $f'=g$ and $f=g'$, $\mathcal{L}$ is invariant under a continuous $\boldsymbol{\tau}\gamma_5$ rotation

$$\psi\to\exp[(\boldsymbol{\alpha}\cdot\boldsymbol{\tau})\gamma_5]\psi,\quad \bar\psi\to\bar\psi\exp[(\boldsymbol{\alpha}\cdot\boldsymbol{\tau})\gamma_5].\tag{2.6}$$

We wish to express the integral equations for the self-energy operator $\Sigma(p)$. $\Sigma(p)$ is obtained by summing all the diagrams of Figs. 1(a) and 1(b). Denote the contributions of Fig. 1(a) to the self-energy matrix $\Sigma_{ij}(\gamma\cdot p)$ by $\Sigma_{ij}{}^{(1)}(\gamma\cdot p)$:

$$\Sigma_{ij}{}^{(1)}=-i\sum_k h_kO_{ij}{}^{(k)}\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}[O^{(k)}G(\gamma\cdot p)],\tag{2.7}$$

[8] We use anti-Hermitian γ_μ's satisfying $\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu=-2g_{\mu\nu}$, $g_{00}=-1$, $g_{ii}=1$, $\gamma_5=\gamma^0\gamma^1\gamma^2\gamma^3$.

[9] H. Banerjee, Nuovo cimento 23, 597 (1962).

where

$$O^{(k)}=1,\ \gamma_\mu,\ \gamma_\mu\gamma_5,\ \sigma_{\mu\nu},\ \gamma_5,\ \boldsymbol{\tau},\ \boldsymbol{\tau}\gamma_\mu,\ \boldsymbol{\tau}\gamma_\mu\gamma_5,\ \boldsymbol{\tau}\sigma_{\mu\nu},\ \boldsymbol{\tau}\gamma_5,$$

and h_k are definite linear combinations of f, g, f', and g'. In terms of the self-energy operator, the Green's function $G_{ij}(p)$ is determined by

$$(G^{-1}(\gamma\cdot p))_{ij}=(\gamma\cdot p\delta_{ij}+\Sigma_{ij}(\gamma\cdot p)). \tag{2.8}$$

Contributions to $\Sigma_{ij}(p)$ from Fig. 1(b) involve the four-point vertex operator Γ, which itself can be expressed in terms of G and Γ by summing all irreducible vertex diagrams. This yields a set of coupled integral equations (not in closed form) for Σ and Γ. We wish to determine whether there are solutions to these equations which break the continuous isotopic spin symmetry. We must emphasize that the Lagrangian (2.3) is used only to illustrate the possibility that there may be solutions to the Dyson equations which lack this symmetry. It is not intended to describe real neutrons and protons.

The theory described by (2.3) is not renormalizable. Even after the Dyson equations and the S matrix are re-expressed in terms of physically measured coupling constants and masses, cutoffs must still be used in order to obtain finite results. The procedure of replacing bare parameters by physical parameters should nevertheless be carried out in order to develop a consistent approximation scheme in terms of physical quantities.

In the usual fashion, we may obtain equations which determine the bare masses and charges in terms of physical masses and charges (and in terms of the cutoff). Like Nambu, we take the view that all masses are dynamical, and can be determined from these equations self-consistently by setting the bare masses equal to zero.

The Lehmann representation requires $G_{ij}(\gamma p)$ to have poles at the physical masses of the particles. The renormalized Green's function $\bar{G}_{ij}$ is defined so that

$$\bar{G}_{11}\sim\frac{1}{\gamma\cdot p+m_1}\qquad \bar{G}_{22}\sim\frac{1}{\gamma\cdot p+m_2}, \tag{2.9}$$

while $\bar{G}_{12}$, $\bar{G}_{21}$ have no pole singularities. The wave-function renormalization (here, a two-by-two matrix) bringing G_{ij} to the form (2.9) is undefined to within an arbitrary rotation in isospace because of the symmetry of the Lagrangian. We may thereby write, with no loss of generality,

$$\bar{G}_{ij}(\gamma\cdot p)=(Z_iZ_j)^{-\frac{1}{2}}G_{ij}(\gamma\cdot p). \tag{2.10}$$

We shall use only a lowest approximation in this work, so that we need not carry out the renormalization procedure in any detail—we only emphasize its importance for a more refined calculation.

Our approximation to the Dyson equations consists in taking

$$\Sigma_{ij}(\gamma\cdot p)\cong\Sigma_{ij}^{(1)},$$

as given by (2.7). In this approximation an expression for the vertex operator is not required. Moreover, $\Sigma_{ij}^{(1)}$ is independent of $\gamma\cdot p$, so we may write (after a suitable isotopic rotation of the fields),

$$\Sigma_{ij}(\gamma\cdot p)=m_V\delta_{ij}+m_S(\tau_3)_{ij}\equiv M_{ij}. \tag{2.11}$$

Because $\Sigma_{ij}(\gamma\cdot p)$ is a constant, the Dyson integral equations reduce simply to a set of algebraic equations for m_S and m_V:

$$(\bar{G}^{-1}(\gamma\cdot p))_{ij}=(\gamma p+M_{ij})$$

and

$$M_{ij}=-i\sum_k h_kO_{ij}^{(k)}\int\frac{d^4p}{(2\pi)^4}\operatorname{tr}(O^{(k)}\bar{G}). \tag{2.12}$$

We can interpret the h_k as renormalized coupling constants. $G_{ij}=\bar{G}_{ij}$, since the Z_i of (2.10) are one in this approximation.

This is the starting point of the work of Nambu and Jona-Lasinio. We have presented a detailed discussion only to illustrate what approximations are required and what must be done in order to find better approximation solutions. For a discussion of the axial-vector currents and of the vertex operator, it is imperative to carry out the renormalization procedure. This will be seen in Sec. V with reference to a different model.

Unlike Nambu and Jona-Lasinio, we allow M_{ij} to be a matrix (2.11) and we anticipate solutions to (2.12) for which a mass difference between the two particles appears. This is the essential point of the present work.

With the Lagrangian of (2.3), (2.12) becomes

$$\begin{aligned}M_{ij}=-\frac{i}{8}\int\frac{d^4p}{(2\pi)^4}\{&(7g+f+3g'-3f')\\&\times\delta_{ij}\operatorname{tr}[(M+\gamma\cdot p)^{-1}]+(f-g-g'+9f')\\&\times(\tau_3)_{ij}\operatorname{tr}[\tau_3(M+\gamma\cdot p)^{-1}]\}.\end{aligned} \tag{2.13}$$

This equation for the two masses,

$$m_1=m_V+m_S\quad\text{and}\quad m_2=m_S-m_V,$$

can be written in the form:

$$\begin{aligned}m_1&=\lambda_1m_1F(m_1^2)+\lambda_2m_2F(m_2^2),\\m_2&=\lambda_2m_1F(m_1^2)+\lambda_1m_2F(m_2^2),\end{aligned} \tag{2.14}$$

where

$$\lambda_1=3g+g'+3f'+f, \tag{2.15}$$

$$\lambda_2=4g+2g'-6f', \tag{2.16}$$

and

$$F(m^2)=-i\int\frac{d^4p}{(2\pi)^4}(p^2+m^2)^{-1}G(p^2/\Lambda^2). \tag{2.17}$$

With the same covariant cutoff factor $G(p^2/\Lambda^2)$ used in reference 3, we obtain

$$F(m^2)=\frac{\Lambda^2}{16\pi^2}\left[1-\frac{m^2}{\Lambda^2}\ln\left(1+\frac{\Lambda^2}{m^2}\right)\right]. \tag{2.18}$$

We define the more convenient parameters

$$x=m_1/\Lambda, \quad y=m_2/\Lambda, \tag{2.19}$$

$$\alpha=\Lambda^2\lambda_1/16\pi^2, \tag{2.20}$$

$$\gamma=\Lambda^2\lambda_2/16\pi^2, \tag{2.21}$$

$$f(x)=1-x^2\ln(1+1/x^2), \tag{2.22}$$

and rewrite (2.14) in the form

$$x=\alpha x f(x)+\gamma y f(y), \quad y=\gamma x f(x)+\alpha y f(y). \tag{2.23}$$

Note that the continuous symmetry of $\mathcal{L}$ is reflected in the permutation symmetry of (2.23).

There are several kinds of solutions to (2.23). Certainly, $x=y=0$ is one solution. This solution is consistent with all the symmetries of the Lagrangian and is the usual perturbation theory solution. There are also solutions for which $x=y>0$. Such solutions require

$$(\alpha+\gamma)f(x)=1, \tag{2.24}$$

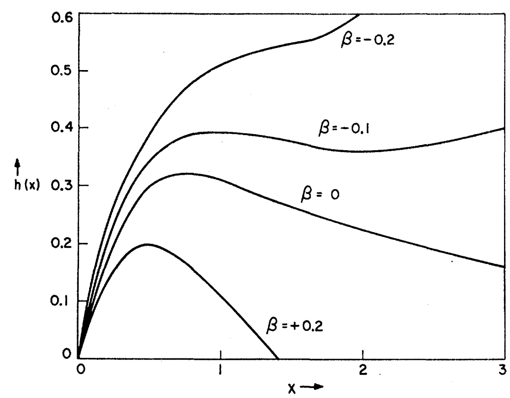


FIG. 2. $h(x)=f(x)-\beta x$ for several representative values of β.

and exist so long as

$$\alpha+\gamma>1. \tag{2.25}$$

These are the nonperturbative solutions discussed by Nambu and Jona-Lasinio. They are compatible with isospin invariance (2.1), but they break γ_5 symmetry (2.2).

There are also solutions for which $x\neq y$. They break both the symmetries (2.1) and (2.2).

To see that such solutions do appear for certain ranges of the coupling constants, it is convenient to rewrite (2.23) in the form

$$x+y=G[xf(x)-\beta x], \quad x+y=G[yf(y)-\beta y], \tag{2.26}$$

where

$$G=\gamma^{-1}(\gamma^2-\alpha^2), \tag{2.27}$$

$$\beta=(\alpha-\gamma)^{-1}. \tag{2.28}$$

For two solutions to occur with $x_1\neq x_2$, it is necessary for the function

$$h(x)=xf(x)-\beta x \tag{2.29}$$

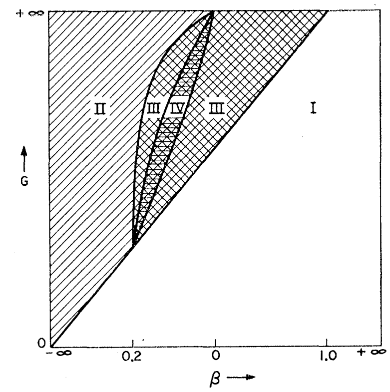


FIG. 3. Schematic diagram indicating values of G and β for which exceptional solutions occur in the three-field model. In I the three masses must vanish; in II there are nontrivial solutions with all masses equal; in III there are solutions with two masses equal; in IV nondegenerate solutions exist.

to assume the same value c, for at least two distinct positive arguments, say a and b. Then either

$$x=a, \quad y=b$$

or

$$x=b, \quad y=a \tag{2.30}$$

satisfies (2.26) providing that

$$G=(a+b)/c. \tag{2.31}$$

It is evident from Fig. 2 that $h(x)$ has the required property for

$$\sim -0.2<\beta<+1. \tag{2.32}$$

For β fixed and within this range, there will be a range of values of G [determined by (2.31)] for which asymmetric solutions exist. This is illustrated in Fig. 3. In Fig. 4 the possible values of x and y are shown for several representative values of β. Note that the situation is most complicated for $\sim -0.2<\beta<0$, when

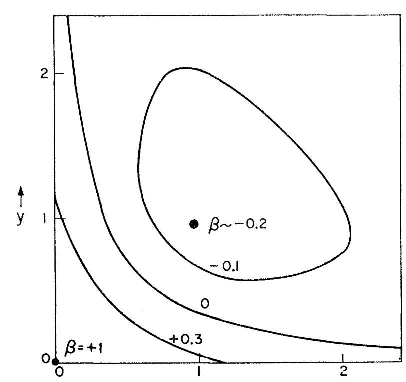


FIG. 4. Mass spectrum (x,y) is shown for representative values of β. Each curve is parametrized with respect to G. This figure applies both to the two-field solutions and to the partly degenerate three-field solutions (but with different parametrizations in the two situations). For each curve, the minimum value of G corresponds to the point of closest approach to the origin.

the function inverse to h is three valued (all values positive), and (2.30) have a variety of solutions.

If G and β (or equivalently, the initial coupling constants, f, g, f' and g') are chosen so that solutions with $x \neq y$ exist, then there are also solutions for these same values of G and β for which $x=y=0$, or $x=y>0$. Evidently, some other principle must be invoked in order to determine which kind of solution is actually realized in nature. Perhaps the symmetric solutions may be eliminated by some kind of stability criterion, but we have not answered this question.

We used an extreme approximation for Σ in which it is a constant. If more involved contributions are kept, we are forced to solve integral equations to determine G. Let us suppose that these equations, like our simple algebraic equations, have as a solution a Green's function lacking the isotopic symmetry. This cannot be the solution corresponding to the usual perturbation theory based upon a symmetric zero-order solution. However, we might conjecture that this asymmetric solution can be developed in a perturbation expansion about the symmetry-breaking solution we have just determined. If this conjecture is true (and we have no firm reason to believe that it is) then the predictions of this theory may be expressed in a perturbation series in terms of the physical (nonsymmetric) masses and physical (also nonsymmetric) coupling constants.

This formal solution to the exact equation always exists, just as the fully symmetric solution in terms of a perturbation theory compatible with the symmetries of $\mathcal{L}$ always exists. The question is whether or not there appear *still other* solutions to the Dyson equations which cannot even be expressed as a perturbation theory of this kind. We might assume that there are no such solutions, and moreover, that the stability criterion which is used to exclude the symmetric solutions does not also exclude the symmetry-breaking solutions. A more detailed consideration of this point is necessary.

Under the above assumptions, we can attach physical significance to the nonsymmetric lowest order solutions of the Dyson equations.

Consider the behavior of the physical masses as functions of one of the coupling constants, $m_i(g)$. In general, there is a critical coupling constant g_c so that for $g<g_c$ this particular solution does not appear. Having excluded the symmetric solution by a stability criterion, we are left with no physically realizable solutions for $g<g_c$. Thus $G_{ij}\equiv 0$ in this approximation. This suggests the behavior of an unstable particle for which the Green's function indeed has no pole. One might look for complex solutions to (2.26) if there are no real solutions of the desired type, and determine the lifetime from the imaginary part of the mass. However, it is really necessary to carry out a better approximation for the Green's function in such a situation.

If this is the correct origin of unstable particles, an important corollary appears: either a solution exists in lowest order and all the particles are stable; or there is no solution in lowest order, and all the particles are unstable.

In the next section, we discuss a more realistic model with $SU(3)$ as the starting symmetry. In such a theory, all eight baryons comprise a unitary octet as do the eight pseudoscalar mesons. There also appears to be an octet of vector bosons. It is a remarkable fact that all eight baryons are stable with respect to strong interactions, as are all the pseudoscalar mesons. This requires many inequalities among their masses, $m_\Sigma - m_\Lambda < m_\pi$, $m_\Xi - m_\Lambda < \mu_K$, $\mu_\chi < 4\mu_\pi$, etc. For the vector particles, all the inequalities run the other way, $\mu_{K^*} > \mu_K + \mu_\pi$, $\mu_\omega > 3\mu_\pi$, etc; all there particles are unstable. This behavior is understood from the point of view of the preceding paragraph.

There is a problem with this approach which has been discussed by Nambu[3] and Banerjee.[9] The continuous symmetry of the Lagrangian implies certain conservation laws. These laws put restrictions on the form factors determined by the matrix elements of the conserved current between single particle states. If there are one-particle states which violate the original symmetry of the Lagrangian, the resulting conditions on the form factors indicate the existence of zero-mass bosons. At this stage, we do not know under what circumstances such particles actually do arise. A detailed discussion of this problem is essential.

To summarize this section, we have shown how a spontaneous breakdown of isotopic spin symmetry can occur in a theory described by a Lagrangian with this symmetry. This is not the situation of physical interest. In the next section we will see the possibility of the spontaneous breakdown of unitary symmetry leaving only the observed isotopic spin and hypercharge symmetries. Thus the concept of a higher symmetry is given an exact meaning even in a world where only a lower symmetry appears.

III. THE THREE-FERMION PROBLEM

We consider three self-coupled Fermion fields with zero bare mass and with interactions that are invariant under the unitary symmetry group $SU(3)$,

$$\psi \to \exp(i\boldsymbol{\alpha}\cdot\boldsymbol{\lambda})\psi, \quad \bar{\psi} \to \exp(-i\boldsymbol{\alpha}\cdot\boldsymbol{\lambda})\bar{\psi}, \tag{3.1}$$

where the $\boldsymbol{\lambda}$ are eight properly normalized traceless three-by-three matrices,[10] and under the discrete γ_5 transformation (2.2). We choose for the Lagrangian

$$\mathcal{L} = +i\bar{\psi}\gamma^\mu\partial_\mu\psi + g(\bar{\psi}\psi)^2 + f(\bar{\psi}\gamma_5\psi)^2 + g'(\bar{\psi}\boldsymbol{\lambda}\gamma_5\psi)^2 + f'(\bar{\psi}\boldsymbol{\lambda}\psi)^2. \tag{3.2}$$

If $f=g$ and $f'=g'$, $\mathcal{L}$ is also invariant under the continuous γ_5 rotations of (2.5), while invariance under continuous $\boldsymbol{\lambda}\gamma_5$ rotations requires[11] $f=g=f'=g'$. Proceeding exactly as in Sec. II, we deduce the following

[10] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[11] S. Coleman and S. L. Glashow, Ann. Phys. (New York) **17**, 41 (1962).

equations for the physical masses by setting the bare masses equal to zero,

$$\begin{aligned} m_1&=\lambda_1 m_1 F(m_1^2)+\lambda_2\{m_2F(m_2^2)+m_3F(m_3^2)\},\\ m_2&=\lambda_1 m_2 F(m_2^2)+\lambda_2\{m_3F(m_3^2)+m_1F(m_1^2)\},\\ m_3&=\lambda_1 m_3 F(m_3^2)+\lambda_2\{m_1F(m_1^2)+m_2F(m_2^2)\},\end{aligned} \tag{3.3}$$

where $F(m^2)$ is given by (2.18), and λ_1 and λ_2 are linear combinations of f, g, f', and g' different from (2.15) and (2.16). The continuous symmetry group of $\mathcal{L}$ shows itself in the symmetry of (3.3) under any permutation of the three masses. This permutation group is a discrete subgroup of $SU(3)$. Introducing the dimensionless masses,

$$x=m_1/\Lambda,\quad y=m_2/\Lambda,\quad z=m_3/\Lambda, \tag{3.4}$$

we find

$$\begin{aligned} x+y+z&=G[xf(x)-\beta x],\\ x+y+z&=G[yf(y)-\beta y],\\ x+y+z&=G[zf(z)-\beta z],\end{aligned} \tag{3.5}$$

where G and β are related to λ_1 and λ_2 through (2.20), (2.21), (2.27), and (2.28). Again, there are several kinds of solutions to (3.5). The perturbative result is $x=y=z=0$. There is also a completely degenerate solution with $x=y=z>0$ providing that

$$(1-\beta)G>3.$$

Nondegenerate solutions occur only when the right-hand side (rhs) of (3.5) can take the same value for several distinct arguments.

For $0<\beta<1$ there are at most two abscissas corresponding to each value of the left-hand side (lhs) of (3.6) (see Fig. 2). For this range of β, there is no solution with three distinct masses. If x denotes the repeated mass and y the unequal mass, (3.5) reduces to

$$2x+y=G[xf(x)-\beta x],\quad 2x+y=G[yf(y)-\beta y], \tag{3.6}$$

and may be solved in the same way as (2.26). The curve of Fig. 4 for $0<\beta<1$ gives the two masses for β fixed. Each point of the curve represents two solutions for two different values of G: one solution with the degenerate mass greater than the third mass ($x>y$) and one with $x<y$. Evidently, the same curves are pertinent for the three-field case as for the two-field case (except for their parametrization in G) because they merely specify the pairs of arguments (x,y) for which the rhs of (3.6) is the same.

The situation is more complicated in the narrow range $\sim -0.2<\beta<0$. Again, there are many ways to satisfy (3.6), for the rhs is now the inverse of a three-valued positive function. The discussion of the preceding paragraph applies with no change for the solutions with $x=y\neq z>0$.

There are now additional solutions for which no two masses are equal. From Fig. 2 it is clear that x, y, and z may be chosen all distinct, yet satisfying (3.6) with an appropriate value of G. In practice, it is difficult to find these solutions since the ranges of β and especially of G that give rise to such solutions are very small. This happens because, with $f(x)$ given by (2.22), there is very little variation of the lhs of (3.5) as x, y, z run through their permitted values. This region in G and β is shown schematically in Fig. 3.

Observe that there is a hierarchy of solutions with decreasing symmetries:

(1) $x=y=z=0$; symmetry of $U(3)$ and γ_5

(2) $x=y=z>0$; symmetry of $U(3)$

(3) $x=y\neq z$; symmetry of $U(2)\times U(1)$

(4) $x\neq y\neq z\neq x$; symmetry of $U(1)\times U(1)\times U(1)$.

As the symmetry is reduced, the constraints upon the coupling constants become more severe. Whenever a given kind of solution does exist, solutions with lesser symmetries also exist.

In order to relate these results to strong interactions, we must assume three things:

(1) The "stablest" kind of solution is the one with least symmetry, thus we eliminate solutions with too much symmetry.

(2) The strong interactions are *exactly* invariant under $SU(3)$, and f, g, f', and g' are such that solutions of the third variety, but not of the fourth, occur, thus we eliminate solutions with too little symmetry.

(3) There is a convergent perturbative expansion based on zero-order Green's functions with the nonsymmetric physical masses.

With these assumptions, we have the makings of a theory of strong interactions:

Strong interactions are exactly invariant under $SU(3)$. There are no "medium strong symmetry-breaking interactions." Because of dynamic instability of the fully symmetric solution, the physically realized solutions partially break the symmetry of the Lagrangian. However, some symmetry remains: two of the underlying fields remain degenerate. Of the original eight-parameter group $SU(3)$, only a four-parameter subgroup survives. We are left with isospin invariance (unitary transformations among the two degenerate fields), and with conservation of hypercharge [the remaining kind of rotation in $SU(3)$ not mixing fields of unequal mass]. Dynamic instabilities provide a likely explanation of why just this subgroup of $SU(3)$ is an exact symmetry of nature.

We now digress. Departures from exact unitary symmetry appear to be of a simple sort. The Gell-Mann mass formulas

$$\tfrac{1}{2}(m_N+m_\Xi)=\tfrac{1}{4}(m_\Sigma+3m_\Lambda)$$

and

$$\mu_K^2=\tfrac{1}{4}(3\mu_\chi^2+\mu_\pi^2),$$

are satisfied to within several MeV. In other words, the effective mass Lagrangian has the transformation

property under $SU(3)$ of a superposition of a unitary singlet and a member of an octet. These formulas also describe the imaginary parts of the masses—all sixteen particles are stable with respect to strong interactions. Perhaps the reason for this is the very simple way the exact unitary symmetry of the Lagrangian is violated in our model.

One may ask why a theory with three basic Fermions should describe a world in which octets, not triplets, of elementary particles appear. We have little to add to the remarks of Gell-Mann[10] about this. But three is the only number of fields that can give a successful theory along the lines we pursue. Certainly, three fields are a minimum number with sufficient symmetries to describe isospin and hypercharge.

With more than three fields, the residual symmetry group is always too great. From (3.5) and Fig. 2 it is clear that no more than three masses can appear—independent of the number of participating fields. This result depends only on very general properties, like the monotonicity of $f(x)$ in (2.23). Four fields, with interactions invariant under $SU(4)$, yield the following possibilities for exact strong interaction symmetries:

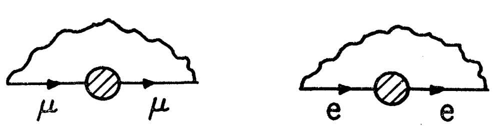
$$\begin{gathered} SU(4), \\ SU(3)\times U(1), \\ SU(2)\times SU(2)\times U(1), \\ SU(2)\times U(1)\times U(1), \end{gathered}$$

all of which contain more than the known exact symmetries of strong interactions.

IV. THE MUON-ELECTRON PROBLEM

Recent experiments[12] demonstrate that the interactions of the electron and of the muon are identical (i.e., purely electromagnetic) aside from weak interactions. Conventional theory accounts for their mass difference by introducing different bare mass terms into the Lagrangian. We point out the possibility that their mass difference can result from a self-consistent solution to the Green's function equations with a Lagrangian containing no bare mass terms. Thus, the explanation of the electron-muon puzzle may lie entirely within the realm of ordinary quantum electrodynamics.

With very crude approximations, we have been unable to determine the electron-muon mass ratio. We



(a) (b)

FIG. 5. (a) Approximation for $\Sigma_\mu(\gamma\cdot p)$. (b) Approximation for $\Sigma_e(\gamma\cdot p)$.

[12] G. E. Masek, L. D. Heggie, Y. B. Kim, and R. R. Williams, Phys. Rev. **122**, 937 (1961). G. Charpak, F. J. M. Farley, R. L. Garwin, T. Muller, J. C. Sens, V. L. Telegdi, and A. Zichichi, Phys. Rev. Letters **6**, 128 (1961).

are hopeful that more refined calculations, perhaps along the lines we suggest, will give the correct result. The electron-muon problem appears a good proving ground for the kind of spontaneous breakdown of elementary particle symmetries we discuss.

We start with only the electrodynamic interaction and no bare mass terms,

$$\mathcal{L}=+i\bar{\psi}_e\gamma^\lambda\partial_\lambda\bar{\psi}_e+i\bar{\psi}_\mu\gamma^\lambda\partial_\lambda\psi_\mu+e_0A_\lambda(\bar{\psi}_e\gamma^\lambda\bar{\psi}_e+\bar{\psi}_\mu\gamma^\lambda\psi_\mu),$$

where ψ_e, ψ_μ, and A_λ are the electron, muon, and photon fields, respectively, and e_0 is the bare electric charge.

We seek solutions to the renormalized Dyson equations such that the renormalized electron Green's function G_e has a pole at the physical electron mass m_e, and the renormalized muon Green's function G_μ has a pole at the physical muon mass m_μ:

$$G_e\sim(\gamma\cdot p+m_e)^{-1} \quad\text{and}\quad G_\mu\sim(\gamma\cdot p+m_\mu)^{-1}. \tag{4.2}$$

As a first approximation to the electron and muon self-energy operators, $\Sigma_e(\gamma\cdot p)$ and $\Sigma_\mu(\gamma\cdot p)$, we replace the exact vertex operator Γ_μ by γ_μ and the exact photon propagator $D(k^2)$ by $1/k^2$ (Fig. 5).

Because we calculate self-energies, we encounter divergences, and a cutoff energy Λ must be introduced. From the two equations determining m_μ and m_e, it should be possible to deduce an unambiguous (cutoff independent) value for their ratio m_μ/m_e.

In our study of direct four-Fermion couplings, the approximation for $\Sigma(\gamma\cdot p)$ was independent of $\gamma\cdot p$, so that the solution to the approximate equations was given exactly by the pole terms. The integral equations then reduced simply to algebraic equations for m_1 and m_2. In the electromagnetic problem, the approximation described by Fig. 5 does not yield a constant $\Sigma(\gamma\cdot p)$, so that the approximate Green's functions contain spectral functions as well as pole terms.

To carry through this approximation, it is necessary to solve integral equations in terms of m_e, m_μ and the spectral functions. We have not done this. Suppose the spectral functions do not play a vital role in determining the mass ratio. If just the pole parts of the Green's function are put into the integrals for $\Sigma_e(\gamma\cdot p)$ and $\Sigma_\mu(\gamma\cdot p)$, we find

$$m_e=(3\alpha/4\pi)m_e\ln(\Lambda^2/m_e^2), \tag{4.3}$$

$$m_\mu=(3\alpha/4\pi)m_\mu\ln(\Lambda^2/m_\mu^2), \tag{4.4}$$

so long as $\Lambda\gg m_e$ and $\Lambda\gg m_\mu$. There are four solutions to these equations:

$$\begin{gathered} m_\mu=m_e=0; \\ m_\mu=0,\quad m_e=\Lambda e^{-2\pi/3\alpha}; \\ m_e=0,\quad m_\mu=\Lambda e^{-2\pi/3\alpha}; \\ m_\mu=m_e=\Lambda e^{-2\pi/3\alpha}. \end{gathered}$$

Besides the fact that these solutions disagree with experiment, they are suspect for two reasons:

(i) Neglect of the spectral functions of G_e and G_μ is questionable.

(ii) In the approximation of Fig. 5 the muon and electron Green's functions are not coupled to each other. This is unlike the model discussed in Sec. II where m_1 entered into the equation determining m_2.

We can attempt to deal with the second objection by considering the diagrams of Fig. 6 as well as those of Fig. 5. Replacing (4.3) and (4.4), we now find

$$m_e = (3\alpha/4\pi)m_e \ln(\Lambda^2/m_e^2) + \alpha^2 m_e F(m_e, m_\mu, \Lambda), \quad (4.5)$$

$$m_\mu = (3\alpha/4\pi)m_\mu \ln(\Lambda^2/m_\mu^2) + \alpha^2 m_\mu F(m_\mu, m_e, \Lambda). \quad (4.6)$$

These equations also admit uninteresting solutions like those of (4.3) and (4.4). We have not determined whether they have solutions for which the two masses are unequal, and neither mass vanishes.

It may be over optimistic to believe that Eqs. (4.5) and (4.6) will give the correct lepton mass ratio, for the expansion parameter is $\alpha \ln(\Lambda^2/m^2)$ and not α. It may be necessary to include the most divergent parts of a wide class of diagrams in order to find reliable algebraic equations for the masses.

Once the masses have been self-consistently determined, a perturbation theory may be developed about the physical (nonsymmetric) masses. The predictions of such a theory are just those of ordinary renormalized perturbation theory, which agrees so well with experiment. The absence of $\mu \to e+\gamma$ remains true to any (electrodynamic) order because the vertex operator remains diagonal.

V. VERTEX EQUATION

In this work, we have been exploiting the fact that the Dyson equations can admit solutions other than the usual perturbative one. Thus far in our study of these solutions, we have limited ourselves to the consideration of the equation for the self-energy operator. $\Sigma(p)$ was decoupled from the vertex operator Γ by approximating the complete vertex by the zero-order vertex γ in the expression for $\Sigma(p)$. In this section we reverse this procedure. We now study the equation for the vertex operator which we decouple from the self-energy operator by replacing the complete Green's function by its zero-order value in the experssion for Γ.

We consider a single complex vector Boson field B_μ of mass μ coupled to two massless Fermion fields ψ_1 and ψ_2 with the interaction Lagrangian $\mathfrak{L}_I$:

$$\mathfrak{L}_I = B_\lambda\{f_0\bar{\psi}_1\gamma^\lambda\psi_2 + g_0\bar{\psi}_1 i\gamma^\lambda\gamma_5\psi_2\} + \text{H.c.}, \quad (5.1)$$

where f_0 and g_0 are the unrenormalized vector and axial-vector coupling constants. The complete vertex function is a sum of a "vector" part Γ_μ and an "axial-vector" part $\Gamma_{\mu5}$. Γ_μ includes γ_μ and all radiative corrections to the γ_μ vertex, $\Lambda(\Gamma_\mu)$, while $\Gamma_{\mu5}$ includes $i\gamma_\mu\gamma_5$ and all radiative corrections to the $i\gamma_\mu\gamma_5$ vertex, $\Lambda(\Gamma_{\mu5})$.

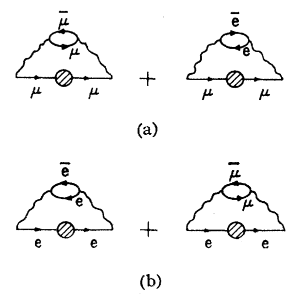


FIG. 6. (a) Further contributions to $\Sigma_\mu(\gamma\cdot p)$. (b) Further contributions to $\Sigma_e(\gamma\cdot p)$.

Since $\mathfrak{L}_I$ does not conserve parity, Γ_μ and $\Gamma_{\mu5}$ will each contain vector and axial-vector contributions.

Replacing all propagators by their zero-order values, we obtain a pair of coupled integral equations for Γ_μ and $\Gamma_{\mu5}$,

$$\Gamma_\mu = \gamma + \Lambda(\Gamma_\mu,\ f_0\Gamma + g_0\Gamma_5), \quad (5.3)$$

$$\Gamma_{\mu5} = i\gamma_\mu\gamma_5 + \Lambda(\Gamma_{\mu5},\ f_0\Gamma + g_0\Gamma_5). \quad (5.4)$$

Renormalized vector and axial-vector functions, $\bar{\Gamma}_\mu$ and $\bar{\Gamma}_{\mu5}$, are defined so that

$$\bar{\Gamma}_\mu(p_1,p_2) \to \gamma_\mu \quad (\text{as} \quad \gamma\cdot p_1 \to 0 \quad \text{and} \quad \gamma\cdot p_2 \to 0), \quad (5.5)$$

and

$$\bar{\Gamma}_{\mu5}(p_1,p_2) \to i\gamma_\mu\gamma_5 \quad (\text{as} \quad \gamma\cdot p_1 \to 0 \quad \text{and} \quad \gamma\cdot p_2 \to 0), \quad (5.6)$$

and they are related to the unrenormalized quantities through a two-by-two renormalization matrix $\mathbf{Z}$,

$$\begin{pmatrix}\bar{\Gamma}_\mu \\ \bar{\Gamma}_{\mu5}\end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22}\end{pmatrix}\begin{pmatrix}\Gamma_\mu \\ \Gamma_{\mu5}\end{pmatrix}. \quad (5.7)$$

The requirements (5.5) and (5.6) uniquely determine $\mathbf{Z}$. From (5.3), (5.4), and (5.7) we find

$$\begin{pmatrix}\bar{\Gamma}_\mu \\ \bar{\Gamma}_{\mu5}\end{pmatrix} = \mathbf{Z}\begin{pmatrix}\gamma_\mu \\ i\gamma_\mu\gamma_5\end{pmatrix} + \begin{pmatrix}\Lambda(\bar{\Gamma}_\mu,\ f_0\Gamma + g_0\Gamma_5) \\ \Lambda(\bar{\Gamma}_{\mu5},\ f_0\Gamma + g_0\Gamma_5)\end{pmatrix}. \quad (5.8)$$

We define renormalized coupling constants by the condition

$$f_0\Gamma_\nu + g_0\Gamma_{\nu5} = f\bar{\Gamma}_\nu + g\bar{\Gamma}_{\nu5}, \quad (5.9)$$

and put (5.8) into the form

$$\begin{pmatrix}\bar{\Gamma}_\mu(p_1,p_2) \\ \bar{\Gamma}_{\mu5}(p_1,p_2)\end{pmatrix} = (\mathbf{Z}+\mathbf{L})\begin{pmatrix}\gamma_\mu \\ i\gamma_{\mu5}\end{pmatrix} + \begin{pmatrix}\Lambda^R(\bar{\Gamma}_\mu,\ f\bar{\Gamma} + g\bar{\Gamma}_5,\ p_1,\ p_2) \\ \Lambda^R(\bar{\Gamma}_{\mu5},\ f\bar{\Gamma} + g\bar{\Gamma}_5,\ p_1,\ p_2)\end{pmatrix}, \quad (5.10)$$

where

$$\mathbf{L}\begin{pmatrix}\gamma_\mu \\ i\gamma_\mu\gamma_5\end{pmatrix} \equiv \begin{pmatrix}\Lambda(\bar{\Gamma}_\mu,\ f\bar{\Gamma} + g\bar{\Gamma}_5,\ 0,\ 0) \\ \Lambda(\bar{\Gamma}_{\mu5},\ f\bar{\Gamma} + g\bar{\Gamma}_5,\ 0,\ 0)\end{pmatrix} \quad (5.11)$$

and

$$\Lambda_\mu^R(p_1,p_2) = \Lambda_\mu(p_1,p_2) - \Lambda_\mu(0,0). \quad (5.12)$$

By choosing

$$\mathbf{Z}=1-\mathbf{L}, \tag{5.13}$$

we satisfy (5.5) and (5.6).

This is the usual renormalization procedure for the vertex function. Equation (5.10) is just the integral equation for the renormalized vertex functions, $\bar{\Gamma}_\mu$ and $\bar{\Gamma}_{\mu 5}$ and it involves only the renormalized coupling constants, f and g, which by (5.7) and (5.9) are related to f_0 and g_0 by the equation

$$\begin{pmatrix} f_0 \\ g_0 \end{pmatrix}=\mathbf{Z}\begin{pmatrix} f \\ g \end{pmatrix}. \tag{5.14}$$

From the solution of (5.10) for $\bar{\Gamma}_\mu$ and $\bar{\Gamma}_{\mu 5}$ in terms of f and g, we use (5.11), (5.12), and (5.13) to calculate f_0 and g_0 in terms of f and g.

In accordance with our viewpoint that the original Lagrangian should be simple, we impose the condition that the bare coupling constants vanish,

$$f_0=g_0=0,$$

just as in our study of the self-energy operator we had set all bare masses equal to zero. Thus (5.14) becomes

$$\mathbf{Z}(f,g)\begin{pmatrix} f \\ g \end{pmatrix}=0. \tag{5.15}$$

The matrix $\mathbf{Z}$ is determined from (5.11), (5.12), and (5.13) in terms of the solutions $\bar{\Gamma}_\mu$ and $\bar{\Gamma}_{\mu 5}$ to the renormalized vertex equation (5.10). Then, (5.15) yields and equation determining f and g. There is also the possibility that (5.12) is an identity in f and g.[13]

Equations (5.15) may be combined with (5.11) and (5.13) to yield

$$f\gamma_\mu+g\gamma_\mu\gamma_5=\Lambda(f\bar{\Gamma}_\mu+g\bar{\Gamma}_{\mu 5},\ f\bar{\Gamma}+g\bar{\Gamma}_5,\ 0,\ 0). \tag{5.16}$$

This has the obvious interpretation as the condition for a self-generated interaction.

It should be pointed out that because f_0 and g_0 no longer appear in (5.10), the Lagrangian (5.1) simply determines the structure of the renormalized equations. The requirement (5.15) imposes the condition that the allowable interactions are just those that can arise spontaneously without the introduction of the arbitrary parameters f_0 and g_0. This treatment of the vertex function is analogous to our previous discussion of the self-energy function. There, we used the condition $m_0=0$ as a requirement that all masses arise self-consistently. Here, we demand that all interactions arise self-consistently. In a complete treatment, both problems must be handled together.

We do not solve the integral equation (5.10) for $\bar{\Gamma}_\mu$ and $\bar{\Gamma}_{\mu 5}$, but as a first approximation we use their threshold values γ_μ and $i\gamma_\mu\gamma_5$ in our determination of L according to (5.11). From the structure of the nth order irreducible contribution to Λ_μ, it is easy to show that

$$\mathbf{L}=\begin{pmatrix} A(f,g,\Lambda/\mu), & B(f,g,\Lambda/\mu) \\ B(f,g,\Lambda/\mu), & A(f,g,\Lambda/\mu) \end{pmatrix}, \tag{5.17}$$

where $A(f,g,\Lambda/\mu)$ and $B(f,g,\Lambda/\mu)$ are power series in f and g. A cutoff Λ is introduced to make the integrals defining A and B finite. We also have

$$\begin{gathered} A(0,0,\Lambda/\mu)=B(0,g,\Lambda/\mu)=B(f,0,\Lambda/\mu)=0, \\ A(-f,\,g)=A(f,\,-g)=A(f,g), \\ B(-f,\,g)=B(f,\,-g)=-B(f,g). \end{gathered} \tag{5.18}$$

Equations (5.15) become

$$f(1-A)+gB=0,\quad g(1-A)+fB=0, \tag{5.19}$$

and can have the following types of solution:

(a) $f=g=0$;

(b) $f=0$, $g\neq 0$,

where g satisfies

$$A(0,g,\Lambda/\mu)=1; \tag{5.20}$$

(c) $g=0$ and $f\neq 0$,

where f satisfies

$$A(f,0,\Lambda/\mu)=1; \tag{5.21}$$

(d) $g=\pm f\neq 0$,

where f satisfies

$$1=A(f,f)-B(f,f); \tag{5.22}$$

(e) $f^2\neq g^2$, $f\neq 0$, $g\neq 0$,

where f and g satisfy the *two* relations

$$A(f,g)=1 \quad\text{and}\quad B(f,g)=0. \tag{5.23}$$

(a) gives no interaction and is the usual perturbation theory solution when $f_0=g_0=0$.

(b) and (c) give self-generated axial-vector and vector interactions, respectively.

(d) gives self-generated $\gamma_\mu(1\pm i\gamma_5)$ interactions which violate parity maximally.

(e) gives solutions which neither conserve parity nor violate it maximally. To lowest order, $B\propto fg$ and no such solutions exist. We can make no general statement about the existence of solutions to (5.23) of this kind.

Hopefully, interactions like (e) are not realized in nature, (d) corresponds to the weak interactions, (b) and (c) to the strong interactions.

To summarize this section, we have shown the possibility that the fundamental interactions can generate themselves from a "bootstrap mechanism" in a theory where the bare coupling constants vanish. We emphasize that in our discussion of the vertex equation we have neglected all modifications of the propagators, including the wave function renormalization constants

[13] See the discussion of the no-subtraction philosophy in dispersion relations. S. D. Drell and F. Zachariasen, Phys. Rev. **111**, 1727 (1958); M. Baker and F. Zachariasen, *ibid.* **119**, 438 (1960).

$Z_2^{(i)}$. In our model, there is no conserved current and we should not expect that a zero of $Z_2^{(i)}$ will occur to cancel the zero of Z.

VI. CONCLUSION

We have explored the possibility that the complex of fundamental interactions can be understood in terms of the stable self-generated solutions of the coupled Green's function equations of field theory. Since in this paper we have not tried to solve any integral equations, we have made no predictions which conclusively test the validity of this idea. Rather, we have consistently made the most naive approximations to the integral equations in order to reduce them to algebraic equations. Our purpose has been to show that there is the *possibility* of explaining fundamental interactions along these lines. There remains the more difficult practical problem of finding more reasonable approximations to the equations. There are also the basic problems dealing with stability criteria, the possible appearance of zero-mass particles,[14] and the existence of divergences, which must be understood before these ideas can become a complete theory.

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[14] J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 965 (1962).

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Classical Radiation Recoil*

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The conditions under which a material system may recoil while emitting electromagnetic or gravitational radiation are investigated. The lowest order secular effects in the electromagnetic case arise from an interference of the electric dipole radiation with the electric quadrupole or magnetic dipole radiations. In the gravitational case, the lowest order terms involve the interference of the mass quadrupole radiation with the mass octopole or the flow quadrupole radiations. The investigation of the gravitational radiation recoil is carried out in complete analogy with the more elementary electromagnetic case, so that this paper should be accessible to physicists having no previous knowledge of general relativity theory.

1. INTRODUCTION AND NOTATIONS

IT is well known that a material system can dissipate energy in the form of spherical waves radiated to infinity. In classical theory, these waves may be either electromagnetic[1] or gravitational,[2,3] and if they are emitted mostly in a preferred direction, then the emitting system will recoil in the opposite direction, like a rocket. However, while "photon rockets" have been largely publicized, the possibility of measuring gravitational radiation recoil is still far beyond our experimental techniques,[4] and even theoretical investigations have hitherto been restricted to a very special model.[5]

The main purpose of this paper is to present the general theory of gravitational radiation recoil. The calculations will be valid for any kind of motion (rotational, vibrational, or other) but with the condition that the material system remains localized within a finite volume. Only secular effects will be considered, i.e., those effects which do not average to zero over a long time interval.

In order to make these interesting questions accessible to the reader who is not a specialist of general relativity, we first present, in Sec. 2, the theory of electromagnetic radiation recoil, using the same tools as will later be needed in the gravitational case. It is found that the lowest order effects arise from the interference of the electric dipole radiation with either the electric quadrupole radiation or the magnetic dipole radiation. This could have been expected on general grounds, because the recoil force must be bilinear in the various multipoles (since the Poynting vector is), and the only way to construct a three-dimensional vector is to contract a 2^n pole with a 2^{n+1} pole. (Magnetic 2^n poles are homogeneous to electric 2^{n+1} poles.)

In gravitational theory, the analogs of the electric and magnetic 2^n poles are the mass and flow 2^n poles, which are defined in the same way, masses playing the role of charges. However, there can be neither mass nor

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