Method to Measure the Lifetime of the Σ^0 Hyperon*

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A method, suggested by Wattenberg, to measure the lifetime of the Σ^0 hyperon, is analyzed theoretically in this paper. The suggested method consists of measuring the distribution of angles between the Dalitz pairs emitted in the process $\Sigma^0 \rightarrow \Lambda^0+e^++e^-$, in which the Σ^0 was previously produced in a nucleus. The distribution function contains a part that depends both on the charge Z of the nucleus and on the lifetime of Σ^0. This part comes from the interference between the Z-dependent part of the matrix element with the Z-independent part. This interference owes its existence to two facts: that the target nucleon is in a bound state, and that the Σ^0 has a finite mass width. The effects of nuclear structure can be eliminated from the problem. A family of curves for the distribution functions corresponding to various values of the lifetime is obtained through the use of Born approximation for the Coulomb wave functions of the Dalitz pair. If the lifetime is in the neighborhood of 5×10^{-20} sec, then measurements with an accuracy of 10% would determine the lifetime to within a factor of 10.

1. INTRODUCTION

THE purpose of this paper is to formulate a theory for, and estimate the sensitivity of, a method suggested sometime ago by Wattenberg[1] to measure the lifetime of the Σ^0 hyperon.

The Σ^0 hyperon can be produced in reactions such as $K+N\rightarrow\Sigma^0+\pi$, where N stands for a nucleon, usually bound in a nucleus. Once created, the Σ^0 may interact with other nucleons in the nucleus, producing reaction products other than itself, or it may decay, either within the nucleus or outside. Its principal mode of decay is $\Sigma^0\rightarrow\Lambda^0+\gamma$. The secondary mode $\Sigma^0\rightarrow\Lambda^0+e^++e^-$ occurs in free space with a relative probability of 1/184 if the $\Sigma-\Lambda$ parity is even, and 1/165 is odd.[2] The total energy of the Dalitz pair is approximately equal to the $\Sigma-\Lambda$ mass difference, which is 70 MeV.

If, in an actual experiment, one observes the total rate of γ rays and Dalitz pairs originating from the decay of a Σ^0, one would obtain the rate of production of the Σ^0, but not the lifetime. The decay products in this case merely register the fact that a Σ^0 once was. To obtain the lifetime of a particle, one customarily measures either the distance traveled by the particle during its existence, or the width of the distribution of total invariant mass of the decay products. Both these methods are impracticable in the case of the Σ^0, as witnessed by the fact that present experimental knowledge of the lifetime of the Σ^0 lies between wide limits.[3] Theoretical estimates,[4] essentially based on dimensional arguments, place it at about 5×10^{-20} sec. Hence, it is desirable to find new methods to measure the Σ^0 lifetime.[5]

The method suggested by Wattenberg consists of measuring appropriate quantities, if any, associated with the Dalitz pair emitted in the decay mode $\Sigma^0\rightarrow\Lambda^0+e^++e^-$, as a function of the atomic number Z of the nucleus in which the decaying Σ^0 was orginally produced. The motivation behind this proposal is as follows. The lifetime of the Σ^0 determines the average distance from the nucleus at which Dalitz pairs originate. This distance, in turn, determines the Coulomb wave function of the pair. Therefore, appropriate observations on the Z dependence of certain distributions of the Dalitz pair may enable one to deduce the lifetime of the Σ^0. If the theoretical estimate for the lifetime, namely, 5×10^{-20} sec, turns out to be correct, then the Σ^0 will decay within the Bohr radius of the atom in which it was produced. These ideas made it promising to investigate this method in more detail.

In the present investigation, the differential rate of the decay $\Sigma^0\rightarrow\Lambda^0+e^++e^-$ is calculated with the use of Born approximation for the Coulomb wave functions of the Dalitz pair. The lifetime of the Σ^0 enters the problem through the fact that the inverse lifetime is the half-width of the mass distribution of an unstable particle. The matrix element for the decay contains a term independent of Z, and a term proportional to $Z/137$. The interference between these two yields a contribution to the differential decay rate which is proportional to $Z/137$, and is a function of the lifetime of the Σ^0.

If τ denotes the mean lifetime of the Σ^0, and $\Gamma=1/\tau$ the width of the mass distribution of the Σ^0, then the dimensionless parameter that enters into the final formulas turns out to be[6] $\lambda=M\Gamma/\gamma^2$, where M is the mass of the Σ^0 and γ is the invariant mass of the Dalitz pair. This quantity is a sensitive function of the angle

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[1] A. Wattenberg (private communication).
[2] G. Feinberg, Phys. Rev. **109**, 1019 (1958).
[3] They are "roughly analogous to a determination that the moon is closer than the sun and farther than the ceiling," as summarized by M. Gell-Mann and A. Rosenfeld, Ann. Rev. Nuclear Sci. **7**, 418 (1957).
[4] See M. Gell-Mann and A. Rosenfeld, reference 3.
[5] One of the new methods, distinct from the one discussed here, has been described by J. Dreitlein and H. Primakoff, Phys. Rev. **125**, 1671 (1962).
[6] Throughout this paper we used units in which $\hbar=c=1$.

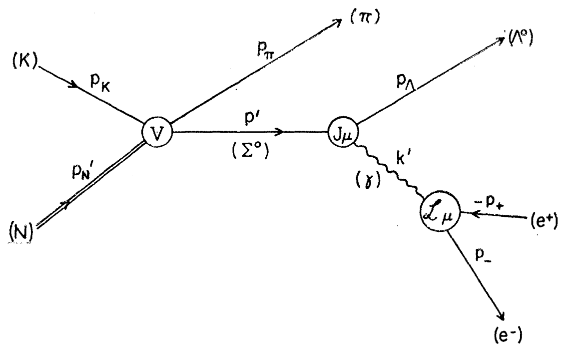

FIG. 1. Feynman diagram for the reaction discussed in this paper. For explanation of symbols see Sec. 2A.

between the Dalitz pair. Formulas suitable for direct experimental comparison are contained in Sec. 4. If the lifetime of the Σ^0 lies between 10^{-19} and 10^{-20} sec, then the sensitivity of this method is such that, for a slow Σ^0, an accuracy of 10% in the measurements would yield the lifetime to within a factor of 10.[7]

2. FORMULATION OF THE PROBLEM

A. S-Matrix Element

For definiteness, we consider a Σ^0 hyperon produced in the reaction $K+N\rightarrow\Sigma^0+\pi$. It subsequently decays through the mode $\Sigma^0\rightarrow\Lambda^0+e^++e^-$. The entire sequence of events makes up the scattering process

$$K+N\rightarrow\pi+\Sigma^0\rightarrow\pi+\Lambda^0+e^++e^-, \tag{1}$$

with such choice of final kinematics that the intermediate state given above is the only important one. Experimentally, this means that one collects only such events that the invariant mass of $\Lambda^0+e^++e^-$ lies in the neighborhood of the Σ^0 mass M.

The S-matrix element for the scattering process (1) is represented by the Feynman diagram in Fig. 1, where we use a prime to distinguish a momentum variable to be integrated over. A double line in the diagram denotes a nucleon bound in a nucleus. There are, of course, many other diagrams that will contribute to (1); but they are neglected for the reason stated previously.

If the initial and final stationary wave functions of the nucleus are, respectively, denoted by $\Psi_i(q_1,\cdots,q_A)$ and $\Psi_f(q_1,\cdots,q_{A-1})$, then the effective wave function of the single nucleon is

$$\psi_N(q,t)=e^{-iE_Nt}\int dq_1\cdots dq_{A-1}\,\Psi_f^*(q_1,\cdots,q_{A-1})\times\Psi_i(q,q_1,\cdots,q_{A-1}), \tag{2}$$

where q_i denotes all the coordinates, including spin and isotropic spin, of a nucleon in the nucleus, and $\int dq$ stands for an integration and sum over these coordinates. We shall write

$$\psi_N(q,t)=e^{-iE_Nt}\int d^3p_N'e^{i\mathbf{p}_N'\cdot\mathbf{x}}f(\mathbf{p}_N')N, \tag{3}$$

where E_N is a constant, $f(\mathbf{p}_N')$ is the Fourier transform of the wave function, and N is the nonrelativistic spinor of the nucleon. Thus, N is independent of $\mathbf{p}_N'$. The wave functions for the Dalitz pair are Coulomb wave functions about the aforementioned nucleus, taken, for this particular purpose, to be a fixed point charge of charge Ze. These Coulomb wave functions are labeled, respectively, by the quantum numbers $\mathbf{p}_+$, $\mathbf{p}_-$, which are the asymptotic momenta of the pair. The stationary energies of these Coulomb wave functions are

$$E_\pm=[|\mathbf{p}_\pm|^2+m^2]^{1/2}, \tag{4}$$

where m is the mass of the electron.

For all wave functions, we employ the convention of continuum normalization. The wave function for a spinor particle shall be so normalized that there are E/m particles per units volume, where E is the absolute magnitude of the energy and m is the mass of the particle. If $\mathbf{p}$ is the momentum of the spinor particle, then the volume element of phase space is $(2\pi)^{-3}(m/E)d^3p$. The spinor for a particle shall be denoted by the symbol representing the particle itself. Thus, Λ shall be the spinor of the Λ^0 hyperon, $\Lambda^\dagger$ its Hermitian conjugate, and $\bar{\Lambda}=\Lambda^\dagger\gamma_4$.

The wave function for a boson shall be so normalized that there are $(2E)^{-1}$ particles per unit volume. The volume element of phase space in this case is $(2\pi)^{-3}(2E)^{-1}d^3p$.

To define the symbols V, J_μ, and $\mathcal{L}_\mu$ in Fig. 1 precisely, let $\{A+B\rightarrow C+D\}$ stand for the S-matrix element of the reaction enclosed within the brackets. Then

$$\{K+N\rightarrow\pi+\Sigma^0\}_{\text{nucleus}}\equiv(2\pi)^4\delta^4(p_f-p_i)(\bar{\Sigma}VN), \tag{5}$$

$$\{\Sigma^0\rightarrow\Lambda^0+\gamma\}\equiv(2\pi)^4\delta^4(p_f-p_i)(\bar{\Lambda}J_\mu\Sigma)\epsilon_\mu, \tag{6}$$

$$\{\gamma\rightarrow e^++e^-\}_{\text{Coulomb field}}\equiv(2\pi)^4\delta(E_f-E_i)\mathcal{L}_\mu\epsilon_\mu, \tag{7}$$

where p_f and p_i denote, respectively, the total four-momentum of the final and initial state, E_f and E_i the final and initial energies, and ϵ_μ denotes the polarization of the photon. The quantities V, J_μ, and $\mathcal{L}_\mu$ are functions of all the momenta in the corresponding reaction. The function J_μ is specified in Appendix IV. It may be considered to be proportional to $\Gamma^{1/2}$. We note that the interaction (7) does not conserve momentum; but it conserves energy by virtue of the fixed-source assumption.

It is shown in Appendix I that the propagator for the Σ^0, whose four-momentum is denoted by p', may be taken to be[8]

$$S(p')=-\frac{1}{\gamma\cdot p'-i\mathfrak{M}},\qquad \mathfrak{M}=M-i\Gamma/2, \tag{8}$$

[7] Readers not concerned with theory may, at this point, make a transition to Sec. 4.

[8] $\gamma\cdot p\equiv\gamma_1p_1+\gamma_2p_2+\gamma_3p_3+\gamma_4p_4$, where γ_μ are Hermitian Dirac matrices, and p_4 is pure imaginary.

where M is the center of the Σ^0 mass distribution and Γ is the width of that distribution. As shown in Appendix I, Γ is also the total rate of decay of Σ^0. Hence, it is related to the τ by

$$\Gamma=1/\tau. \tag{9}$$

Both M and Γ are experimental constants, of which M may be assumed known.

The S-matrix element for the reaction (1) is

$$S=i(2\pi)^4\delta(E_f-E_i)T, \tag{10}$$

where T is a function of p_+, p_-, p_Λ, p_K, p_π, given by

$$T\equiv\int d^3p_N'\; f(\mathbf{p}_N')\mathcal{L}_\mu\frac{1}{(k')^2}\left(\bar{\Lambda}J_\mu\frac{1}{\gamma\cdot p'-i\mathfrak{M}}VN\right), \tag{11}$$

and

$$\begin{aligned} p'&\equiv p_K+p_N'-p_\pi,\\ p_N'&\equiv(\mathbf{p}_N',E_N),\\ k'&\equiv p_K+p_N'-p_\Lambda-p_\pi. \end{aligned} \tag{12}$$

The rate of the transition (1), denoted by $\mathcal{R}$, may be obtained from T through the formula

$$\mathcal{R}=(2\pi)^7\delta(E_f-E_i)\sum_{\text{spin}}|T|^2, \tag{13}$$

where $\sum_{\text{spin}}$ denotes a sum over the spin states of the particles in $|T|^2$.

Let

$$x'\equiv[-(p')^2]^{1/2}. \tag{14}$$

Then the following identity holds:

$$\frac{1}{\gamma\cdot p'-i\mathfrak{M}}=\frac{1}{2x'}\left[\frac{p'\cdot\gamma+ix'}{\mathfrak{M}-x'}-\frac{p'\cdot\gamma-ix'}{\mathfrak{M}+x'}\right]. \tag{15}$$

The first term represents the propagation of the Σ^0 as a particle, and the second term, as an antiparticle. When this is substituted into (11), the second term can be neglected, because in the kinematic region of interest, i.e., in the neighborhood of $x'=M$, it is at most of the order of Γ/M as compared to the first term. The first term may be rewritten in a convenient form by noting that $(\gamma\cdot p'+ix')/2ix'$ is the projection operator into positive energy states for a spinor particle of mass x'. Thus we take

$$\frac{1}{\gamma\cdot p'-i\mathfrak{M}}\approx\frac{i}{M-x'}\sum_u u\bar{u}, \tag{16}$$

where the sum extends over the two spinors u of positive energy. When (16) is substituted into (11), we have in the integrand of (11) the factor $\sum_u(\bar{\Lambda}J_\mu u)(\bar{u}VN)$. This factor is clearly invariant under a linear transformation of the u's. By choosing the appropriate transformation, we can write

$$\textstyle\sum_u(\bar{\Lambda}J_\mu u)(\bar{u}VN)=(\bar{\Lambda}J_\mu\Sigma)(\bar{\Sigma}VN), \tag{17}$$

where the spinor Σ is a linear combination of the u's, so defined that if Σ' is the spinor with spin opposite to that of Σ, then

$$(\bar{\Sigma}'VN)=0. \tag{18}$$

In fact, Σ is the spinor of the Σ^0 with the correct polarization as determined by the production process. With (16) and (17), (11) becomes

$$T=-i\int d^3p_N'\; f(\mathbf{p}_N')\frac{\mathcal{L}_\mu(\bar{\Lambda}J_\mu\Sigma)(\bar{\Sigma}VN)}{(k')^2(x'-M+i\Gamma/2)}. \tag{19}$$

If the nucleon were free, so that $f(\mathbf{p}_n')$ is a δ function, then $|T|^2$ would be of the form

$$|T|^2=\frac{|T'|^2}{(x-M)^2+\Gamma^2/4},$$

where x depends on the momenta of the initial or final particles. As $\Gamma\to 0$, this would become

$$|T|^2=\delta(x-M)(2\pi/\Gamma)|T'|^2.$$

The factor $1/\Gamma$ cancels one that is contained in $|T'|^2$, through $|(\bar{\Lambda}J_\mu\Sigma)|^2$. We would then have a complete separation between production and decay; but we would also lose the dependence on Γ. On the other hand, if Γ has been kept finite, it would appear in the combination Γ/M, which is too small to be significant, and which has been neglected in the derivation of (19).

In order that (19) depends on Γ significantly, it is necessary that the nucleon be bound. This is physically reasonable, since a bound nucleon provides a fixed point in space against which a detector, if available, can judge how far the Σ^0 has traveled during its lifetime. A detector is available. It is contained in the interaction $\mathcal{L}_\mu$.

B. Separation of Σ^0 Production from Σ^0 Decay

In the integral (19) it is convenient to change the variable of integration to $\boldsymbol{\Delta}$, which is the momentum transferred to the external Coulomb field. We put

$$\begin{aligned} \Delta&=(\boldsymbol{\Delta},0),\\ p_N'&=p_N-\Delta,\\ p'&=p-\Delta,\\ k'&=k-\Delta, \end{aligned} \tag{20}$$

where

$$\begin{aligned} p_N&\equiv(\mathbf{p}_N,E_N),\\ \mathbf{p}_N&\equiv\mathbf{p}_++\mathbf{p}_-+\mathbf{p}_\Lambda+\mathbf{p}_\pi-\mathbf{p}_K,\\ p&\equiv p_++p_-+p_\Lambda,\\ k&\equiv p_++p_-. \end{aligned} \tag{21}$$

We further define

$$\begin{aligned} x&\equiv(-p^2)^{1/2},\\ \gamma&\equiv-k^2=(E_++E_-)^2-|\mathbf{p}_++\mathbf{p}_-|^2. \end{aligned} \tag{22}$$

It is clear that $\gamma^2 \geq 2m$. The independent momenta of the problem are $\mathbf{p}_K$, $\mathbf{p}_\pi$, $\mathbf{p}_\Lambda$, $\mathbf{p}_+$, $\mathbf{p}_-$.

The decay of Σ^0 into Dalitz pairs can occur in free space, in the absence of a Coulomb field. Therefore, $\mathcal{L}_\mu$ must contain a term proportional to $\delta^3(\mathbf{\Delta})$. It is easily established that

$$\mathcal{L}_\mu = e(\bar{e}_- \gamma_\mu e_+)\delta^3(\mathbf{\Delta}) + \mathcal{L}_\mu', \tag{23}$$

where e is the charge of the electron, $e_\pm$ are free particle spinors, and $\mathcal{L}_\mu'$ is a function of $\mathbf{\Delta}$ and $\mathbf{p}_\pm$. Thus, (19) becomes

$$T = \frac{if(\mathbf{p}_N)e(\bar{e}_-\gamma_\mu e_+)[(\bar{\Lambda}J_\mu\Sigma)(\bar{\Sigma}VN)]_{\Delta=0}}{\gamma^2(x-M+i\Gamma/2)} - i\int d^3\Delta \frac{f(\mathbf{p}_N-\mathbf{\Delta})\mathcal{L}_\mu'(\bar{\Lambda}J_\mu\Sigma)(\bar{\Sigma}VN)}{[(k-\Delta)^2-i\eta](x'-M+i\Gamma/2)}. \tag{24}$$

In the second term above, the factor $(\bar{\Lambda}J_\mu\Sigma)(\bar{\Sigma}VN)$ depends on $\mathbf{\Delta}$ through invariant scalar products involving p'. It is reasonable to assume that it is not sensitive to $\mathbf{\Delta}$ if $\Delta/M \ll 1$. Since $\mathbf{\Delta}$ is the momentum transferred to the Coulomb field, we anticipate this condition to be fulfilled. Hence we shall put

$$(\bar{\Lambda}J_\mu\Sigma)(\bar{\Sigma}VN) = [(\bar{\Lambda}J_\mu\Sigma)(\bar{\Sigma}VN)]_{\Delta=0}. \tag{25}$$

Thus, in these spinor products, the Σ^0 is considered to have the fixed momentum $\mathbf{p}$ and mass M, where $\mathbf{p}$ is defined in (21).

With this, we can write

$$T = if(\mathbf{p}_N)(\bar{\Lambda}X\Sigma)(\bar{\Sigma}VN), \tag{26}$$

where

$$X \equiv J_\mu \left\{ \frac{e(\bar{e}_-\gamma_\mu e_+)}{\gamma^2(x-M+i\Gamma/2)} - \int d^3\Delta \frac{f(\mathbf{p}_N-\mathbf{\Delta})\mathcal{L}_\mu'}{f(\mathbf{p}_N)[(k-\Delta)^2-i\eta](x'-M+i\Gamma/2)} \right\}. \tag{27}$$

The "nuclear structure factor" $[f(\mathbf{p}_N-\Delta)/f(\mathbf{p}_N)]$ is not well known. Therefore, the proposed experiment must be so designed as to avoid the necessity to determine it.

Our knowledge of nuclear structure indicates that for very light nuclei, $f(\mathbf{p}_N)$ is approximately a Gaussian in $|\mathbf{p}_N|^2$, with a width of the order of 150 MeV/c. For heavier nuclei $f(\mathbf{p}_N)$ has the approximate shape of a Fermi-Dirac distributon function corresponding to a Fermi momentum of 150 MeV/c. In any case, we may put

$$f(\mathbf{p}_N-\mathbf{\Delta})/f(\mathbf{p}_N) \approx 1, \tag{28}$$

if the range of $\mathbf{\Delta}$ important for the integral in (28) is such that $\Delta \ll 150$ MeV/c. For heavier nuclei, this condition may be replaced by a weaker one, say, $\Delta < 150$ MeV/c. It will turn out that these conditions are satisfied for a reasonable range of the Σ^0 lifetime; this is a highly desirable feature, for then X becomes independent of nuclear structure.

We now discuss the disentanglement of the spin of Σ^0 from the spin of the nucleon. Let N' denote a nucleon spinor orthogonal to N, Then

$$\sum_{N \text{ spin}} |T|^2 = |f(\mathbf{p}_N)|^2\{|(\bar{\Lambda}X\Sigma_1)(\bar{\Sigma}_1VN)|^2 + |(\bar{\Lambda}X\Sigma_2)(\bar{\Sigma}_2VN')|^2\}, \tag{29}$$

where Σ_1 is identical with the Σ defined by (18), and Σ_2 is the Σ defined by (18) with N replaced by N'. In general, Σ_1 and Σ_2 are not orthogonal to each other. To assume otherwise would be to assume that the Σ^0 produced from unpolarized nucleons are unpolarized—which is untrue.

Let

$$\mathcal{P} \equiv |f(\mathbf{p}_N)|^2\{|(\bar{\Sigma}_1VN)|^2 + |(\bar{\Sigma}_2VN')|^2\}, \tag{30}$$

which is the probability for producing a Σ^0 from an unpolarized nucleon, and is a function of $\mathbf{p}_K$, $\mathbf{p}_\pi$, and $\mathbf{p}$. Then

$$\sum_{N \text{ spin}} |T|^2 = \mathcal{P}\{\tfrac{1}{2}(1+s')|(\bar{\Lambda}X\Sigma_1)|^2 + \tfrac{1}{2}(1-s')|(\bar{\Lambda}X\Sigma_2)|^2\}, \tag{31}$$

where

$$s' \equiv \frac{|(\bar{\Sigma}_1VN)|^2 - |(\bar{\Sigma}_2VN)|^2}{|(\bar{\Sigma}_1VN)|^2 + |(\bar{\Sigma}_2VN)|^2}. \tag{32}$$

This quantity is not what is conventionally called the "degree of polarization," because Σ_1 and Σ_2 are not orthogonal to each other. However, it is shown in Appendix II that there exist two mutually orthogonal Σ^0 spinors, denoted by $\Sigma_\uparrow$ and $\Sigma_\downarrow$, such that

$$\sum_{N \text{ spin}} |T|^2 = \mathcal{P}\{\tfrac{1}{2}(1+s)|(\bar{\Lambda}X\Sigma_\uparrow)|^2 + \tfrac{1}{2}(1-s)|(\bar{\Lambda}X\Sigma_\downarrow)|^2\}, \tag{33}$$

where s is now the degree of polarization, and is given in Appendix II in terms of s' and $(\bar{\Sigma}_1\Sigma_2)$. It is also shown there that a 4×4 matrix ρ may be introduced for conciseness, so that we can rewrite (33), after summing over the spin of the Λ^0, in the form

$$\sum_{\text{spin}} |T|^2 = \mathcal{P} \sum_{\text{spin}} |(\bar{\Lambda}X\rho\Sigma)|^2. \tag{34}$$

The spinor Σ may now be taken to be a member of an arbitrary set of mutually orthogonal spinors for the Σ^0. The matrix ρ should be regarded as an experimental quantity. For example, if experimentally we find that the Σ^0 is unpolarized, then $\rho = 1/\sqrt{2}$. If one expresses (34) as a spin trace in the usual manner, one would see that ρ appears only in the form ρ^2, which is the density matrix for the Σ^0 polarization.

We now complete the separation of Σ^0 production from Σ^0 decay by separating the kinematics of these two

processes. From (27) and (28) we have

$$|(\bar{\Lambda}X\rho\Sigma)|^2 = \left|\frac{B}{x-M+i\Gamma/2} - C\right|^2 = \frac{|B|^2}{(x-M)^2+\Gamma^2/4} - 2\,\mathrm{Re}\left[\frac{B^*C}{x-M-i\Gamma/2}\right] + |C|^2, \quad (35)$$

where

$$B \equiv (e/\gamma^2)(\bar{e}_-\gamma_\mu e_+)(\bar{\Lambda}J_\mu\rho\Sigma), \quad (36)$$

$$C \equiv (\bar{\Lambda}J_\mu\rho\Sigma)\int d^3\Delta \frac{\mathcal{L}_\mu'}{[(k-\Delta)^2-i\eta](x'-M+i\Gamma/2)}. \quad (37)$$

The kinematic region of interest lies in the neighborhood of $x=M$. It is seen that $|C|^2$ does not vary rapidly with x, because it depends on x only through the combination

$$x'-M = (x^2-\Delta^2+2\mathbf{p}\cdot\mathbf{\Delta})^{1/2} - M, \quad (38)$$

which involves the integration variable $\mathbf{\Delta}$. At $x=M$ it is of order $(\Gamma/M)^2$ smaller than the first term in (35). Hence, we shall neglect it. On the other hand, the interference term is expected to be significant, as we shall see.

Neglecting $|C|^2$, we rewrite (35) in the form

$$|(\bar{\Lambda}X\rho\Sigma)|^2 = \frac{|B|^2 - 2\,\mathrm{Re}[(x-M+i\Gamma/2)B^*C]}{(x-M)^2+\Gamma^2/4}. \quad (39)$$

Let the probability for the decay of the Σ^0 into Dalitz pairs in free space be

$$\mathfrak{D}_0 \equiv (e^2/\gamma^4)\sum_{\text{spin}}|(\bar{\Lambda}J_\mu\rho\Sigma)(\bar{e}_-\gamma_\mu e_+)|^2, \quad (40)$$

which is not only a function of $\mathbf{p}_\Lambda$, $\mathbf{p}_+$, $\mathbf{p}_-$, but also depends on $\mathbf{p}_N$ and $\mathbf{p}_\pi$ through ρ. We can now write

$$\sum_{\text{spin}}|(\bar{\Lambda}X\rho\Sigma)|^2 = \frac{\mathfrak{D}}{(x-M)^2+\Gamma^2/4}, \quad (41)$$

where

$$\mathfrak{D} = \mathfrak{D}_0(1+F), \quad (42)$$

$$F \equiv -\frac{2e}{\gamma^2\mathfrak{D}_0}\sum_{\text{spin}}\mathrm{Re}\left\{[(\bar{\Lambda}J_\mu\rho\Sigma)(\bar{e}_-\gamma_\mu e_+)]^*(\bar{\Lambda}J_\nu\rho\Sigma) \times \int d^3\Delta \frac{\mathcal{L}_\nu'}{(k-\Delta)^2-i\eta}\left[\frac{x-M+i\Gamma/2}{x'-M+i\Gamma/2}\right]\right\}. \quad (43)$$

Again, F is not a rapidly varying function of x. Its x dependence, like that of $|C|^2$, gives rise to small corrections to the shape of the Σ^0 mass distribution, in which we are not interested. Hence we set $x=M$ in F, and obtain, after explicitly writing out $(k-\Delta)^2$,

$$F = \frac{e\Gamma}{\gamma^2\mathfrak{D}_0}\sum_{\text{spin}}\mathrm{Im}\left\{[(\bar{\Lambda}J_\mu\rho\Sigma)(\bar{e}_-\gamma_\mu e_+)]^*(\bar{\Lambda}J_\nu\rho\Sigma) \times \int d^3\Delta \frac{\mathcal{L}_\nu'}{(\Delta^2-2\mathbf{k}\cdot\mathbf{\Delta}-\gamma^2-i\eta)[y(\mathbf{\Delta})+i\Gamma/2]}\right\}, \quad (44)$$

where

$$y(\mathbf{\Delta}) \equiv (M^2+2\mathbf{p}\cdot\mathbf{\Delta}-\Delta^2)^{1/2} - M \approx (1/M)(\mathbf{p}\cdot\mathbf{\Delta}-\tfrac{1}{2}\Delta^2). \quad (45)$$

The last approximation is based on the neglect of $(\Delta/M)^2$ and $(p\Delta/M)$ as compared to unity.

It is consistent to let $\Gamma \to 0$ in the denominator of (41), but keep Γ finite in F. The reason is that in the denominator, Γ stands in comparison with M, whereas in F such is not the case. As mentioned earlier, it will turn out that Γ appears in F in the combination $M\Gamma/\gamma^2$, which can be of the order of unity. Accordingly, we reduce (41) to

$$\sum_{\text{spin}}|(\bar{\Lambda}X\rho\Sigma)|^2 = \delta(x-M)(2\pi/\Gamma)\mathfrak{D}, \quad (46)$$

where the factor $1/\Gamma$ will cancel one contained in $\mathfrak{D}$ through J_μ. Substituting this into (34), we obtain

$$\sum_{\text{spin}}|T|^2 = \mathcal{P}\delta(x-M)(2\pi/\Gamma)\mathfrak{D}. \quad (47)$$

The factor $\delta(x-M)$ separates the kinematics of production from that of decay, for, *as far as kinematics is concerned*, the mass of Σ^0 now has the definite value M.

The kinematics of the problem may be summarized as follows. The independent momenta are $\mathbf{p}_K$, $\mathbf{p}_\pi$, $\mathbf{p}_\Lambda$, $\mathbf{p}_+$, $\mathbf{p}_-$. A beam of K mesons, of fixed momentum $\mathbf{p}_K$, interacts with a bound nucleon of variable momentum $\mathbf{p}_N'$. It produces a π meson of fixed momentum $\mathbf{p}_\pi$, and a Σ^0 of variable momentum $\mathbf{p}'$ but of definite mass M. The Σ^0 decays into a Λ^0 of momentum $\mathbf{p}_\Lambda$ and a Dalitz pair of momenta $\mathbf{p}_\pm$, all of which are fixed. In the absence of a Coulomb field, $\mathbf{p}'$ would have the definite value $\mathbf{p}$, and $\mathbf{p}_N'$ the definite value $\mathbf{p}_N$, as required by momentum conservation at every step. With the Coulomb field, neither $\mathbf{p}'$ nor $\mathbf{p}_N'$ has a definite value. In fact, $\mathbf{p}'=\mathbf{p}-\mathbf{\Delta}$, $\mathbf{p}_N'-\mathbf{p}_N-\mathbf{\Delta}$, where $\mathbf{\Delta}$, the momentum absorbed by the Coulomb field, does not have a definite value. We are interested in the interference between the non-Coulombic part of the matrix element and the Coulombic part of the matrix element. While the initial state and final state must be, respectively, identical for these two parts of the matrix element, the nucleon and Σ^0 momentum in one part *must* be different from those in other part, (i.e., $\mathbf{\Delta}\neq 0$). This is possible only because the nucleon is bound.

We can now substitute (47) into (13) and obtain a cross section for the reaction (1) after appropriate integrations over final states. This calculation is contained

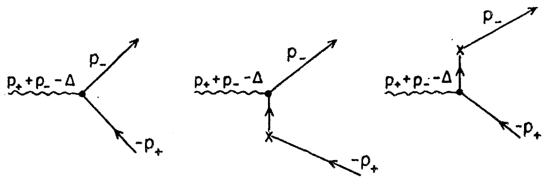

FIG. 2. Feynman diagrams whose sum is the Born approximation for the matrix element $\mathfrak{L}_\mu$, defined by (7). A cross denotes an interaction with the nuclear Coulomb field.

in Appendix III, which also contains formulas suitable for use in an experiment to measure the function F of (43). These formulas will be taken up in Sec. 4. We proceed to our immediate task, the actual calculation of F.

3. BORN APPROXIMATION

The object of this section is to outline the calculation of the function F of (43). We consider the case $Z/137 \ll 1$. Hence, the Coulomb wave functions for the Dalitz pair can be treated in Born approximation. This approximation is equivalent to the statement that

$$(2\pi)^4\delta(E_f-E_i)\mathfrak{L}_\mu = \text{sum of Feynman diagrams of Fig. 2.} \quad (48)$$

In Fig. 2, a cross on the electron or positron line denotes an interaction with an unscreened Coulomb field whose four-potential is given by

$$A_\mu(x)=(0,0,0,i\phi(x)), \quad \phi(x)=Ze/4\pi|\mathbf{x}|, \quad e^2/4\pi=1/137. \quad (49)$$

As no confusion can arise, we denote the Fourier transform of $A_\mu(x)$ by $A_\mu(\Delta)$. Thus,

$$\gamma\cdot A(\Delta)=i\gamma_4\int d^4x\, e^{-ix\cdot\Delta}\frac{Ze}{4\pi|\mathbf{x}|}=\left(\frac{2\pi Ze}{\Delta^2}\right)\delta(\Delta_0)i\gamma_4. \quad (50)$$

The four-vector Δ, with fourth component $i\Delta_0$, is the momentum transferred to the Coulomb field.

Using (50) with (48), we obtain

$$\mathfrak{L}_\mu=e(\bar{e}_-\gamma_\mu e_+)\delta^3(\mathbf{\Delta})+\mathfrak{L}_\mu', \quad \mathfrak{L}_\mu'=(Ze/137)(\bar{e}_-L_\mu e_+), \quad (51)$$

where

$$L_\mu=-\frac{i}{2\pi^2\Delta^2}\left[\gamma_4\frac{1}{\gamma\cdot(p_--\Delta)-i(m-i\epsilon)}\gamma_\mu +\gamma_\mu\frac{1}{\gamma\cdot(\Delta-p_+)-i(m-i\epsilon)}\gamma_4\right]. \quad (52)$$

Since L_μ appears only in $(\bar{e}_-L_\mu e_+)$, and since $(\gamma\cdot p_\pm \pm im)e_\pm=0$, (52) may be effectively reduced to the following:

$$L_\mu=\frac{1}{\pi^2\Delta^2}\left[\frac{(E_-+\frac{1}{2}\boldsymbol{\alpha}\cdot\mathbf{\Delta})\gamma_\mu}{\Delta^2-2\mathbf{p}_-\cdot\mathbf{\Delta}-i\epsilon}-\frac{\gamma_\mu(E_+-\frac{1}{2}\boldsymbol{\alpha}\cdot\mathbf{\Delta})}{\Delta^2-2\mathbf{p}_+\cdot\mathbf{\Delta}-i\epsilon}\right], \quad (53)$$

where $\boldsymbol{\alpha}\equiv i\gamma_4\boldsymbol{\gamma}$.

Substituting (51) and (53) into (44), and using (40), we obtain

$$F=(Z/137)(G+G'), \quad (54)$$

where

$$G=(E_-I_--E_+I_+), \quad (55)$$

$$G'=\frac{\frac{1}{2}\sum_{\text{spin}}\{[(\bar{\Lambda}J_\mu\rho\Sigma)(\bar{e}_-\gamma_\mu e_+)]^*(\bar{\Lambda}J_\nu\rho\Sigma)(\bar{e}_-[\mathbf{Q}_-\cdot\boldsymbol{\alpha}\gamma_\nu+\gamma_\nu\boldsymbol{\alpha}\cdot\mathbf{Q}_+]e_+)\}}{\sum_{\text{spin}}|(\bar{\Lambda}J_\mu\rho\Sigma)(\bar{e}_-\gamma_\mu e_+)|^2}, \quad (56)$$

in which

$$(I_\pm,\mathbf{Q}_\pm)\equiv-\left(\frac{2\gamma^2M\Gamma}{\pi^2}\right)\text{Im}\int d^3\Delta\frac{(1,\mathbf{\Delta})}{\Delta^2(\Delta^2-2\mathbf{p}\cdot\mathbf{\Delta}-iM\Gamma)(\Delta^2-2\mathbf{k}\cdot\mathbf{\Delta}-\gamma^2-i\eta)(\Delta^2-2\mathbf{p}_\pm\cdot\mathbf{\Delta}-i\epsilon)}. \quad (57)$$

The approximation (45) has been used.

To proceed further, the following assumptions are introduced:

(a) The Σ^0 and Λ^0 are nonrelativistic. Hence

$$E_++E_-\approx\Delta M=70 \text{ MeV}. \quad (58)$$

(b) The Dalitz pair is extremely relativistic. Hence

$$|\mathbf{p}_\pm|\approx E_\pm. \quad (59)$$

(c) The angle θ between the Dalitz pairs is neither too large nor too small, say,

$$10° < \theta < 120°. \quad (60)$$

(d) The energies of $e^\pm$ are unequal, say,

$$\frac{1}{5}<\frac{|E_+-E_-|}{E_++E_-}<1. \quad (61)$$

As a consequence we can write

$$\gamma^2\approx 2E_+E_-(1-\cos\theta). \quad (62)$$

It is shown in Appendix V [Eq. (V25)] that the average momentum transferred to the Coulomb field is of the order of

$$\bar{\Delta}=M\Gamma/\Delta M=17\Gamma. \quad (63)$$

Values of $\bar{\Delta}$ corresponding to various values of Γ are given in Table I. It is seen that the neglect of nuclear

TABLE I. A list of some parameters in the problem corresponding to various values of the Σ^0 lifetime τ.

τ (sec)	$\Gamma=1/\tau$ (MeV)	$\bar{\Delta}=M\Gamma/(\Delta M)$ (MeV/c)	$\zeta=M\Gamma/(\Delta M)^2$
(a) 0.8×10^{-18}	0.82×10^{-3}	0.014	$\frac{1}{5}\times10^{-3}$
(b) 0.8×10^{-19}	0.82×10^{-2}	0.14	$\frac{1}{5}\times10^{-2}$
(c) 0.8×10^{-20}	0.82×10^{-1}	1.4	$\frac{1}{5}\times10^{-1}$
(d) 0.8×10^{-21}	0.82	14	$\frac{1}{5}$

structure, as expressed by (28), is a good approximation.

It is also shown in Appendix V [Eq. (V28)] that

$$\left|\frac{G'}{G}\right|\lesssim\left|\frac{E_++E_-}{E_+-E_-}\right|\zeta, \tag{64}$$

where

$$\zeta\equiv M\Gamma/(\Delta M)^2. \tag{65}$$

Table I also lists the values of ζ corresponding to various values of Γ. Near the theoretical estimate $\tau=5\times10^{-20}$ sec, the neglect of G' introduces an error of a few percent, as estimated in Appendix V.C. Therefore, we shall take

$$F=(Z/137)G. \tag{66}$$

In the calculation of G, the approximation is made that the momentum transferred to the Coulomb field, $\mathbf{\Delta}$, is mainly orthogonal to the momenta of the Dalitz pair. [See Appendix V.A and V.B.] The result can be written in the following form:

$$G=\frac{\pi(E_--E_+)}{|\mathbf{p}_-+\mathbf{p}_+|}\Phi(\lambda,\mathbf{K},\mathbf{P}), \tag{67}$$

where Φ is given in (V11). Its value lies between 0 and 1, and its arguments are

$$\begin{aligned}\lambda&\equiv M\Gamma/\gamma^2,\\ \mathbf{K}&\equiv(\mathbf{p}_++\mathbf{p}_-)/\gamma,\\ \mathbf{P}&\equiv\mathbf{p}/\gamma,\end{aligned} \tag{68}$$

where $\mathbf{p}$ is the Σ^0 momentum. Thus Γ enters only in the combination λ, which is a sensitive function of the angle between the Dalitz pair. The limiting values of Φ are as follows:

$$\begin{aligned}&\text{(a)}\quad \Phi\underset{P\to\infty}{\longrightarrow}0,\\ &\text{(b)}\quad \Phi\underset{\lambda\to0}{\longrightarrow}0,\\ &\text{(c)}\quad \Phi\underset{\lambda\to\infty}{\longrightarrow}1,\\ &\text{(d)}\quad \Phi\underset{P\to0}{\longrightarrow}\Phi_0(\lambda,K),\end{aligned} \tag{69}$$

where $\Phi_0(\lambda,K)$ is a function to be displayed later. The cases (a) and (b) confirm one's intuitive feeling that the effect described here should vanish, either when the Σ^0 is moving too fast, or when the lifetime is too long. Thinking classically, one might expect the effect to vanish also in cases (c) and (d), for in these limiting cases the Σ^0 always decays at the same point in space. But our results contradict this expectation. The reason is that the effect discussed here is not classical but quantum mechanical. Specific quantum effects enter the problem in two essential ways: in the existence of a mass width for the Σ^0, and in the interference of probability amplitudes.

4. RESULTS

In the proposed experiment to measure the lifetime of the Σ^0 hyperon, one is to produce the Σ^0 in various nuclei by the reaction $K+N\to\pi+\Sigma^0$, or its equivalent, and observe the decay $\Sigma^0\to\Lambda^0+e^-+e^+$ for different atomic numbers Z. The dominant decay mode $\Sigma^0\to\Lambda^0+\gamma$ must also be observed in order to eliminate the production cross section, which depends on Z. Experimentally it is easy to distinguish between these two modes of decay. For the dominant mode, the γ ray is observed by allowing it to materialize in the target material. Most of the pairs from the materialization have an opening angle less than 1°. On the other hand, an appreciable number of the Dalitz pairs will have an opening angle greater than 10°.

The following quantities are to be measured: $d\sigma_{\text{pair}}$= differential cross section for the over-all process $K+N\to\pi+\Sigma^0\to\pi+\Lambda^0+e^++e^-$, with given Σ^0 momentum $\mathbf{p}$, given solid angles of the pair $\Omega_\pm$, and given energy difference of the pair $\nu=\frac{1}{2}(E_+-E_-)$; σ_{photon} = differential cross section for the over-all process $K+N\to\pi+\Sigma^0\to\pi+\Lambda^0+\gamma$, with given Σ^0 momentum $\mathbf{p}$. Experimentally, of course, a given value of the Σ^0 momentum is chosen by choosing the momentum of the Λ^0. The kinematics should be so chosen that the assumptions (58)–(61) are satisfied.

The theory developed in this paper shows that, for light nuclei, [Eqs. (III25), (66), and (67)]

$$\frac{d\sigma_{\text{pair}}}{\sigma_{\text{photon}}}=\mathcal{C}\left[1-\left(\frac{Z}{137}\right)\frac{2\pi\nu}{|\mathbf{p}_++\mathbf{p}_-|}\Phi\right]d\Omega_+d\Omega_-d\nu, \tag{70}$$

where the only Z dependence is contained in the factor $Z/137$. The goal of the experiment is to obtain the function Φ, from which one can obtain the lifetime of Σ^0 by comparing it with theoretical curves.

The quantity $\mathcal{C}$ depends on the $\Sigma-\Lambda$ parity and on the Σ^0 polarization, and hence must be determined experimentally before one can obtain Φ. If the Σ^0 is unpolarized, then a calculation based on certain assumptions concerning the decay interaction of the Σ^0 gives [Eq. (IV8)]

$$\mathcal{C}=\frac{e^2E_+E_-}{32\pi^4(\Delta M)^3}\left[\frac{E_+^2+E_-^2}{E_+E_-(1-\cos\theta)}\mp1\right], \tag{71}$$

where the $-(+)$ sign is to be taken with even (odd) $\Sigma-\Lambda$ parity, $\Delta M = E_+ + E_- = 70$ MeV ($\Sigma-\Lambda$ mass difference), and θ is the angle between the Dalitz pair.

The quantity Φ, given in (V11), is a function of λ, $\mathbf{K}$, and $\mathbf{P}$, defined in (68). Of these, only λ contains the Σ^0 lifetime. Φ is independent of the $\Sigma-\Lambda$ parity, the Σ^0 polarization, and the structure of the nucleus from which the Σ^0 is produced. Some of its general properties are listed in (69).

Because Φ depends on so many variables, it would be helpful to discuss a simple special case. Let us assume that the momentum $\mathbf{p}$ of the Σ^0 is so small that $P \ll \lambda$, or

$$p \ll M\Gamma/(E_+E_-)^{1/2}\sin(\theta/2). \tag{72}$$

As a numerical illustration, let $\tau = 5\times10^{-20}$ sec and $E_+ - E_- = 20$ MeV. Then the above requires that

$$p \ll [1.5/\sin(\theta/2)]\ \text{MeV}. \tag{73}$$

Under such circumstances, Φ may be replaced by its limiting form for $\mathbf{p}=0$. The experiment is hereby greatly simplified, because the experimenter need not ascertain the precise value of the Σ^0 momentum. The function Φ can now be replaced by

$$\Phi_0(\lambda,K) \equiv \Phi(\lambda,\mathbf{K},0) = (2/\pi)(\beta_1+\beta_2-\beta_3), \tag{74}$$

where

$$\beta_1 = \tan^{-1}[1+(1/K)(\lambda/2)^{1/2}], \tag{75}$$

$$\beta_2 = \tan^{-1}[1/\lambda h(K)],$$
$$h(K) = 1+2K[(1+K^2)^{1/2}+1], \tag{76}$$

$$\beta_3 = \tan^{-1}\left[\frac{(2\lambda)^{-1/2}-K}{(\lambda/2)^{1/2}+K}\right], \tag{77}$$

where $\tan^{-1}x$ denotes the angle between $-\pi/2$ and $+\pi/2$ whose tangent is x. Some general properties of Φ_0 are as follows:

$$\begin{aligned} &\tfrac{1}{2} \leqslant \Phi_0 \leqslant 1, \\ &\Phi_0 \xrightarrow[\lambda\to 0]{} \tfrac{1}{2}, \\ &\Phi_0 \xrightarrow[\lambda\to\infty]{} 1. \end{aligned} \tag{78}$$

The second of these properties is different from that of Φ, as given in (69), because the limit $\mathbf{p}=0$ has already been taken.

We introduce the following parameters:

$$\begin{aligned} &\zeta = M\Gamma/(\Delta M)^2, \\ &r = \nu/\Delta M = (E_+-E_-)/2(E_+-E_-), \\ &\chi = (1-r^2)\sin^2(\theta/2). \end{aligned} \tag{79}$$

The constant ζ is the object of the experiment, and r and χ are variables under the control of the experimenter. Some of the values of ζ corresponding to various values of the Σ^0 lifetime are listed in Table I. Its physical meaning is that $\zeta(\Delta M)$ is the average momentum transfer between the Dalitz pair and the nuclear Coulomb field. In terms of these parameters, we have

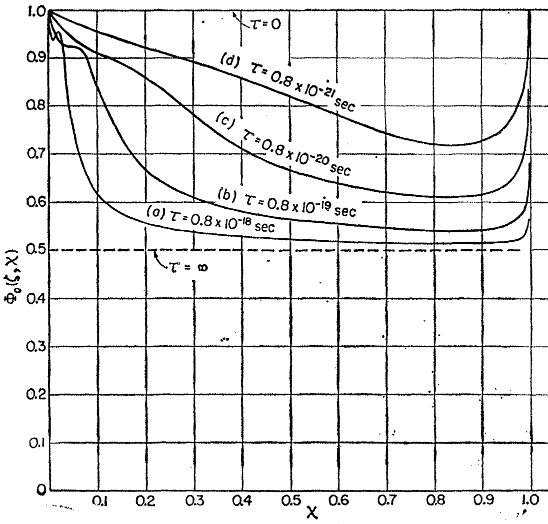

FIG. 3. The function $\Phi_0(\zeta,\chi)$ which appears in (85). The curves are labeled with the values of τ corresponding to the values of ζ listed in Table I.

$$\begin{aligned} &\lambda = \zeta/\chi, \\ &\kappa = [(1-\chi)/\chi]^{1/2}. \end{aligned} \tag{80}$$

Let $\Phi_0(\lambda,K)$ be expressed in terms of ζ and χ. As no confusion can arise, we denote the resulting function by $\Phi_0(\zeta,\chi)$:

$$\Phi_0(\zeta,\chi) = (2/\pi)(\beta_1+\beta_2-\beta_3), \tag{81}$$

$$\beta_1 = \tan^{-1}\left[1+\left(\frac{\zeta/2}{1-\chi}\right)^{1/2}\right], \tag{82}$$

$$\beta_2 = \tan^{-1}\left[\frac{1}{\zeta h(\chi)}\right],$$
$$h(\chi) = \frac{1}{\chi}\left[1+2\left(\frac{1-\chi}{\chi}\right)^{1/2}(1+\chi^{-1/2})\right], \tag{83}$$

$$\beta_3 = \tan^{-1}\left\{\frac{\chi-[2\zeta(1-\chi)]^{1/2}}{(2\zeta)^{1/2}[(1-\chi)^{1/2}+(\zeta/2)^{1/2}]}\right\}. \tag{84}$$

Rewriting (70) in terms of ζ, χ, and r, and integrating over the solid angle Ω_+ and the azimuthal angle of Ω_-, we obtain

$$\frac{d\sigma_{\text{pair}}}{\sigma_{\text{photon}}} = \frac{e^2}{(4\pi)^2}\left(\frac{1+r^2}{\chi}\mp 1\right)\left(\frac{1-r^2}{2}\right) \times\left[1-\left(\frac{Z}{137}\right)\frac{\pi r}{(1-\chi)^{1/2}}\Phi_0(\zeta,\chi)\right]d(\cos\theta)d\nu, \tag{85}$$

where we have used (71). A family of curves for the

TABLE II. The function $H(\zeta,\theta)/(1-\cos\theta)^2$.

$\theta \backslash \zeta$	Even Σ-Λ parity $\frac{1}{5}\times10^{-3}$	$\frac{1}{5}\times10^{-2}$	$\frac{1}{5}\times10^{-1}$	$\frac{1}{5}$	Odd Σ-Λ parity $\frac{1}{5}\times10^{-3}$	$\frac{1}{5}\times10^{-2}$	$\frac{1}{5}\times10^{-1}$	$\frac{1}{5}$
10°	448	458	462	464	450	460	465	467
20°	111	112	115	116	113	114	117	118
30°	45.3	49.3	50.5	51.7	47.2	51.5	52.7	54.0
40°	23.3	27.3	28.2	29.2	25.0	29.4	30.5	31.5
50°	14.0	16.8	18.0	18.8	15.5	18.8	20.3	21.2
60°	9.33	11.3	12.5	13.1	10.9	13.1	14.8	15.6
70°	6.74	8.06	9.20	9.78	8.25	9.87	11.4	12.2
80°	5.15	6.09	7.06	7.61	6.69	7.88	9.27	10.11
90°	4.11	4.82	5.61	6.14	5.69	6.62	7.82	8.70
100°	3.40	3.95	4.62	5.11	5.04	5.97	6.84	7.74
110°	2.89	3.34	3.91	4.36	4.62	5.26	6.19	7.07
120°	2.52	2.91	3.40	3.82	4.36	4.92	5.77	6.63
130°	2.26	2.60	3.04	3.42	4.23	4.73	5.53	6.38
140°	2.07	2.38	2.77	3.13	4.20	4.67	5.44	6.29
150°	1.94	2.22	2.59	2.93	4.27	4.72	5.48	6.36
160°	1.85	2.12	2.47	2.79	4.44	4.89	5.68	6.61
170°	1.80	2.06	2.40	2.72	4.79	5.29	6.17	7.19

function $\Phi_0(\zeta,\chi)$ is plotted in Fig. 3, for the values of ζ listed in Table I.

If one integrates over the energy spectrum of the Dalitz pair by integrating (85) over all values of r between ±1, the Z-dependent term vanishes because it is an odd function of r. This is a consequence of the approximatons used in the derivation of (85). It means that for light nuclei, the completely integrated spectrum has a negligible Z dependence as compared to, say, the half-integrated spectrum. The present calculation says nothing about heavy nuclei.

We shall integrate over the portion of the energy spectrum in which $E_+>E_-$. In doing so, we ignore the restriction (61), which has a negligible effect on (85). The resulting distribution function shall be denoted by

$$\frac{dN}{d(\cos\theta)}\equiv\int_{E_+>E_-}\frac{1}{\sigma_{\text{photon}}}\frac{d\sigma_{\text{pair}}}{d(\cos\theta)}. \tag{86}$$

From (85) one easily obtains

$$\frac{dN}{d(\cos\theta)}=\frac{e^2}{3(4\pi)^2}\left\{\left[\frac{4}{1-\cos\theta}-(\pm1)\right]-\left(\frac{Z}{137}\right)\frac{H(\zeta,\theta)}{(1-\cos\theta)^2}\right\}, \tag{87}$$

where

$$H(\zeta,\theta)\equiv3\pi\int_0^{\frac{1}{2}(1-\cos\theta)}d\chi\left[2-\left(\frac{1}{1-\cos\theta}\pm1\right)\chi\right]\frac{\Phi_0(\zeta,\chi)}{(1-\chi)^{1/2}}, \tag{88}$$

with the $+$ $(-)$ sign taken for even (odd) $\Sigma-\Lambda$ parity. If instead of (86) one wishes to consider the case $E_+<E_-$, then the corresponding distribution function is obtained from (87) by formally replacing Z by $-Z$. Tables for $H(\zeta,\theta)/(1-\cos\theta)^2$ are given in Table II.

One can deduce from more general formula for Φ, given in (V11), that for large Σ^0 momentum the function Φ is very sensitive to the lifetime of the Σ^0 near $\lambda=0$. However, the magnitude of Φ becomes quite small in that region, and the experiment may, in fact, turn out to be more difficult.

The theoretical formulas given above are not exact. The error amounts to about 5%.

We conclude that, in general, the sensitivity of the proposed experiment is such that measurements of 10% accuracy would yield the Σ^0 lifetime to within a factor of 10. The ease with which the experimenter can achieve the desired degree of accuracy, of course, depends on what he does.

Note added in proof. The results of some numerical computations carried out by Dr. J. Eisenberg (unpublished) show that, for heavy nuclei ($Z>20$) and for large opening angles ($\theta>20°$), the sensitivity of the proposed experiment becomes much smaller than that estimated here on the basis of the Born approximation.

ACKNOWLEDGMENTS

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APPENDIX I. THE PROPAGATOR OF AN UNSTABLE PARTICLE

Consider an unstable spinor particle whose state is denoted by $|i\rangle$. It can decay into a state $|f\rangle$, which is different from $|i\rangle$, through the interaction represented by the interaction Lagrangian density $J(x)$. For what we shall discuss, it is necessary that $J(x)$ be considered a small perturbation, for otherwise the production of

the particle cannot be separated from its decay. The decay process is described by the following quantities:

$$\{i\to f\}=\left\langle f\left|\int d^4x\, J(x)\right|i\right\rangle$$
$$=(2\pi)^4\delta^4(p_f-p_i)\langle f|J(0)|i\rangle, \quad \text{(I1)}$$

(Transition probability/sec-cm^3)

$$=(2\pi)^4\delta^4(p_f-p_i)|\langle f|J(0)|i\rangle|^2, \quad \text{(I2)}$$

$$1/\tau=\sum_f(2\pi)^4\delta^4(p_f-p_i)|\langle f|J(0)|i\rangle|^2, \quad \text{(I3)}$$

where τ, an invariant, is defined to be the mean lifetime in the rest frame.

If $J(x)$ were zero, the propagator of the particle would be

$$S(p)=-\frac{1}{p\cdot\gamma-iM_0}, \quad \text{(I4)}$$

where M_0 is a real number. Since $J(x)$ is not zero, its propagator is $S'(p)$, which is obtained by summing all Feynman graphs in which the initial and final states are $|i\rangle$.[9] Let $W(p)$ be the proper self-energy part for the particle. Then

$$[S'(p)]^{-1}=[S(p)]^{-1}-W(p). \quad \text{(I5)}$$

As we shall see, the fact that $|i\rangle$ can make a real transition to a state $|f\rangle\neq|i\rangle$, with energy-momentum conservation, implies that $W(p)$ has a nonvanishing imaginary part.

The proper self-energy part $W(p)$ is a relativistic invariant. Therefore, its most general form is

$$W(p)=A(p^2)+(p\cdot\gamma-iM_0)B(p^2). \quad \text{(I6)}$$

Substituting this and (I4) into (I5), we obtain

$$[S'(p)]^{-1}=[1-B(p^2)](p\cdot\gamma-M_0)-A(p^2).$$

Let

$$L(p^2)=1-B(p^2),$$
$$A'(p^2)=A(p^2)/L(p^2). \quad \text{(I7)}$$

Then

$$S'(p)=-\frac{1}{L(p^2)[p\cdot\gamma-iM_0-A'(p^2)]}. \quad \text{(I8)}$$

If $S'(p)$ is inserted into a graph in which the particle propagates between its production and its decay, the factor $L(p^2)$ merely introduces a coupling constant renormalization, in which the original production and decay vertices are both multiplied by $[L(p^2)]^{-1/2}$. Therefore, $L(p^2)$ can be left as understood. Similarly, $A'(p^2)$ may be replaced by $A(p^2)$ if the Formier transform of $J(x)$ is understood to have absorbed a factor $[L(p^2)]^{-1/2}$. Thus we take

$$S'(p)=-\frac{1}{p\cdot\gamma-iM_0-A(p^2)}. \quad \text{(I9)}$$

[9] F. E. Low, Phys. Rev. **88**, 53 (1952).

We now calculate $A(p^2)$. From (I6) we have

$$A(-M_0^2)=\langle i|W(p)|i\rangle. \quad \text{(I10)}$$

Treating $J(x)$ as a perturbation, we calculate $W(p)$ by taking only the lowest order self-energy graph:

$$\langle i|W(p)|i\rangle=\tfrac{1}{2}(-i)^2\int d^4(x-y)\,e^{-ip\cdot(x-y)}$$
$$\times\langle i|T[J(x)J(y)]|i\rangle, \quad \text{(I11)}$$

where T denotes the time-ordered product. Consider first the matrix element $\langle i|J(y)J(x)|i\rangle$. By inserting a complete set of intermediate states $|n\rangle$, of momentum p_n, we have

$$\langle i|J(y)J(x)|i\rangle=\sum_n\langle i|J(y)|n\rangle\langle n|J(x)|i\rangle$$
$$=\sum_n e^{i(p-p_n)\cdot(y-x)}|\langle i|J(0)|n\rangle|^2, \quad \text{(I12)}$$

where the last step follows from translational invariance:

$$\partial J(x)/\partial x_\mu=i[P_\mu,J(x)], \quad \text{(I13)}$$

where P_μ is the momentum operator. Substitution of (I12) into (I11) yields

$$\langle i|W|i\rangle=\tfrac{1}{2}(-i)^2\int d^4z\,e^{-ip\cdot z}f_n(z)|\langle i|J(0)|n\rangle|^2, \quad \text{(I14)}$$

where

$$f_n(z)=e^{-i(p-p_n)\cdot z}\quad\text{if}\quad z_0>0,$$
$$=e^{i(p-p_n)\cdot z}\quad\text{if}\quad z_0<0, \quad \text{(I15)}$$

and

$$\int d^4z\,f_n(z)e^{-ipz}=2i(2\pi)^3\delta^3(\mathbf{p}-\mathbf{p}_n)$$
$$\times\left[\frac{P}{E-E_n}+i\pi\delta(E-E_n)\right], \quad \text{(I16)}$$

where iE and iE_n are, respectively, the fourth components of p and p_n, and P denotes the principal part. Substitution of (I16) into (I14) yields

$$A(-M_0^2)=\langle i|W(p)|i\rangle=-iM_1+\tfrac{1}{2}\Gamma, \quad \text{(I17)}$$

where

$$M_1=(2\pi)^3\sum_n\delta^3(\mathbf{p}-\mathbf{p}_n)$$
$$\times[P/(E-E_n)]|\langle i|J(0)|n\rangle|^2, \quad \text{(I18)}$$

$$\Gamma=(2\pi)^4\sum_n\delta^4(p-p_n)|\langle i|J(0)|n\rangle|^2. \quad \text{(I19)}$$

Both M_1 and Γ are real numbers, and are to be regarded as small quantities of second order in $J(x)$. By comparison with (I3) it is seen that

$$\Gamma=1/\tau. \quad \text{(I20)}$$

In general, we write

$$A(p^2)=-iM_1(p^2)+\tfrac{1}{2}\Gamma(p^2), \quad \text{(I21)}$$

where $M_1(-M_0^2)=M_1$, $\Gamma(-M_0^2)=\Gamma$. Thus

$$S'(p)=-\frac{1}{p\cdot\gamma-i[M(p^2)-i\Gamma(p^2)/2]}$$
$$=-\frac{p\cdot\gamma+i[M(p^2)-i\Gamma(p^2)/2]}{p^2+[M(p^2)-i\Gamma(p^2)/2]^2}, \quad \text{(I22)}$$

where $M(p^2)\equiv M_0-M_1(p^2)$. $S'(p)$ has a pole at $p=p_0$, where

$$p_0^2+[M(p_0^2)-\tfrac{1}{2}i\Gamma(p_0^2)]^2=0. \quad \text{(I23)}$$

We assume that either this is the only pole, or other poles are sufficiently far away to be ignored. Then it is straightforward to show that to second order in the decay interaction $J(x)$, p_0 is given by

$$p_0^2+M^2=iM\Gamma, \quad \text{(I24)}$$

the error being of fourth order in $J(x)$. The constant M, formally given by

$$M=M_0-M_1,$$

is an experimental constant, the renormalized mass of the particle. Therefore, in the neighborhood of the pole $p=p_0$, we can take the propagator to be

$$S'(p)=-\frac{1}{p\cdot\gamma-i(M-i\Gamma/2)}. \quad \text{(I25)}$$

APPENDIX II. POLARIZATION OF THE Σ^0

In the notation of (31), let

$$\mathcal{Q}\equiv\tfrac{1}{2}(1+s')|(\bar{\Lambda}X\Sigma_1)|^2+\tfrac{1}{2}(1-s')|(\bar{\Lambda}X\Sigma_2)|^2, \quad \text{(II1)}$$

where, for the present purpose, s', is any given number between 0 and 1. It is required to find a set of mutually orthogonal spinors $\Sigma_\uparrow$ and $\Sigma_\downarrow$, which are linear combinations of Σ_1 and Σ_2, such that

$$\mathcal{Q}=\tfrac{1}{2}(1+s)|(\bar{\Lambda}X\Sigma_\uparrow)|^2+\tfrac{1}{2}(1-s)|(\bar{\Lambda}X\Sigma_\downarrow)|^2, \quad \text{(II2)}$$

where s is a number between 0 and 1.

The quantity $\mathcal{Q}$ is independent of the relative phase between Σ_1 and Σ_2. Hence, we can choose the phase such that $(\bar{\Sigma}_1\Sigma_2)$ is real. This number, being smaller than unity, can be represented by

$$(\bar{\Sigma}_1\Sigma_2)=\sin(\delta/2). \quad \text{(II3)}$$

In fact δ is the smallest angle through which, if Σ_2 were rotated, the resulting spinor would be orthogonal to Σ_1. The axis of the rotation is unique. Let Σ_1 be rotated through an angle θ about this axis. Let the resulting spinor be $\Sigma_\uparrow$, and the spinor orthogonal to $\Sigma_\uparrow$, $\Sigma_\downarrow$. Then

$$\begin{aligned}\Sigma_1&=\Sigma_\uparrow\cos(\theta/2)+\Sigma_\downarrow\sin(\theta/2),\\ \Sigma_2&=-\Sigma_\uparrow\sin[(\theta-\delta)/2]+\Sigma_\downarrow\cos[(\theta-\delta)/2].\end{aligned} \quad \text{(II4)}$$

We substitute these into (II1) and determine θ by requiring that (II1) reduces to (II2). The result is

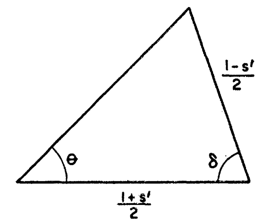

FIG. 4. Graphical construction to obtain the angle θ of (II5).

$$\sin\theta=\frac{(1-s')\sin\delta}{2[\sin^2(\delta/2)+s'^2\cos^2(\delta/2)]^{1/2}}. \quad \text{(II5)}$$

Clearly θ always exists, and may be obtained from the geometrical construction shown in Fig. 4. The number s in (II2) is then given by

$$s=[(1+s')/2]\cos\theta-[(1-s')/2]\cos(\theta-\delta). \quad \text{(II6)}$$

Let a 4×4 matrix ρ be defined by

$$\begin{aligned}(\bar{\Sigma}_\uparrow\rho\Sigma_\uparrow)&=[(1+s)/2]^{1/2},\\ (\bar{\Sigma}_\downarrow\rho\Sigma_\downarrow)&=[(1-s)/2]^{1/2},\\ (\bar{\Sigma}_\uparrow\rho\Sigma_\downarrow)&=(\bar{\Sigma}_\downarrow\rho\Sigma_\uparrow)=0.\end{aligned} \quad \text{(II7)}$$

Then (II2) is equivalent to

$$\mathcal{Q}=|(\bar{\Lambda}X\rho\Sigma_\uparrow)|^2+|(\bar{\Lambda}X\rho\Sigma_\downarrow)|.^2 \quad \text{(II8)}$$

This is invariant under a linear transformation of $\Sigma_\uparrow$ and $\Sigma_\downarrow$. Therefore,

$$\mathcal{Q}=\sum|(\bar{\Lambda}X\rho\Sigma)|^2, \quad \text{(II9)}$$

where the sum extends over any set of two mutually orthogonal spinors for the Σ^0. By comparison with (II2), we see that $\mathcal{Q}$ depends only on ρ^2, which is the density matrix for the Σ^0 polarization. This fact becomes manifest if one writes (II9) in the form of a spin trace in the usual manner.

APPENDIX III. CROSS SECTIONS

We shall derive the cross section for the reaction (1). Substitution of (47) into (13) yields the rate for the reaction (1):

$$\mathcal{R}=(2\pi)^8\delta(E_f-E_i)\delta(x-M)\mathcal{P}(1/\Gamma)\mathfrak{D}. \quad \text{(III1)}$$

The following notation will be used: For a particle of momentum $\mathbf{p}_A$ and mass m_A,

$$\begin{aligned}p_A&\equiv|\mathbf{p}_A|,\\ E_A&\equiv(p_A^2+m_A^2)^{1/2}.\end{aligned} \quad \text{(III2)}$$

The only exception to the last definition is E_N, which is a given fixed number. In this notation

$$\begin{aligned}E_i&=E_K+E_\pi+E_N,\\ E_f&=E_\Lambda+E_++E_-,\\ x&=[(E_\Lambda+E_++E_-)^2-|\mathbf{p}_\Lambda+\mathbf{p}_++\mathbf{p}_-|^2]^{1/2}.\end{aligned} \quad \text{(III3)}$$

By virtue of the factor $\delta(E_f-E_i)$ in (IV1), we can also write

$$x=[(E_K+E_\pi+E_N)^2-|\mathbf{p}_\Lambda+\mathbf{p}_++\mathbf{p}_-|^2]^{1/2}. \quad \text{(III4)}$$

To obtain a cross section for the reaction (1), we multiply (III1) by the factor

$$\frac{m_\Lambda m^2}{(2\pi)^{12}2E_\pi E_\Lambda E_+E_-I}d^3p_\pi d^3p_\Lambda d^3p_+d^3p_-, \quad \text{(III5)}$$

and integrate over any region of phase space as we see fit. Here, I is the incident flux of K mesons.

First we shall integrate over $\mathbf{p}_\pi$. We write

$$d^3p_\pi=d\Omega_\pi\, p_\pi E_\pi dE_\pi. \quad \text{(III6)}$$

For the integration over E_π, we note that if $f(E_\pi)$ is a function of E_π, then

$$\int dE_\pi\,\delta(x-M)f(E_\pi) =\left[\frac{\partial E_\pi}{\partial x}f(E_\pi)\right]_{x=M}=\frac{M}{E_\Sigma}f(E_\pi), \quad \text{(III7)}$$

where, in the last expression,

$$\begin{aligned}E_\Sigma&=[|\mathbf{p}_\Lambda+\mathbf{p}_++\mathbf{p}_-|^2+M^2]^{1/2},\\ E_\pi&=E_\Sigma-E_K-E_N.\end{aligned} \quad \text{(III8)}$$

Now we integrate over Ω_π. This integration affects only $\mathcal{P}$ and the Σ^0 polarization contained in $\mathfrak{D}$. It is clear that the only effect of this integration on $\mathfrak{D}$ is to replace the density matrix ρ^2 by its average value $\langle\rho^2\rangle$:

$$\langle\rho^2\rangle=\int d\Omega_\pi\,\mathcal{P}\rho^2\Big/\int d\Omega_\pi\,\mathcal{P}. \quad \text{(III9)}$$

As long as this is understood, we need not introduce a new notation. Thus

$$\int d^3p_\pi\,\mathcal{R}=\left[\left(\frac{M}{E_\Sigma}\right)E_\pi p_\pi(2\pi)^4\int d\Omega_\pi\,\mathcal{P}\right] \times\left[(2\pi)^4\delta(E_\Sigma-E_f)\frac{1}{\Gamma}\mathfrak{D}\right], \quad \text{(III10)}$$

where p_π has the fixed value consistent with (III8), and where E_Σ replaces E_i because $x=M$. The first factor is proportional to the differential cross section for the production of a Σ^0 of momentum $\mathbf{p}$:

$$d\sigma_{\text{prod}}\equiv d^3p\left[\frac{p_\pi M}{2IE_\Sigma(2\pi)^2}\right]\int d\Omega_\pi\,\mathcal{P}. \quad \text{(III11)}$$

We now choose $\mathbf{p}$ instead of $\mathbf{p}_\Lambda$ to be an independent momentum. Thus, in all subsequent formulas it shall be understood that

$$\begin{aligned}\mathbf{p}_\Lambda&=\mathbf{p}-\mathbf{p}_+-\mathbf{p}_-,\\ E_\Sigma&=(p^2+M^2)^{1/2}.\end{aligned} \quad \text{(III12)}$$

The differential cross section for the reaction (1) is

$$d\sigma_{\text{pair}}=d\sigma_{\text{prod}}d^3p_+d^3p_-\frac{m_\Lambda m^2}{(2\pi)^2E_\Lambda E_+E_-} \times\delta(E_\Sigma-E_\Lambda-E_+-E_-)\frac{\mathfrak{D}}{\Gamma}. \quad \text{(III13)}$$

Let us write

$$d^3p_+d^3p_-=d\Omega_+d\Omega_-(p_+E_+)(p_-E_-)d\omega d\nu, \quad \text{(III14)}$$

where

$$\begin{aligned}\omega&\equiv E_++E_-,\\ \nu&\equiv\tfrac{1}{2}(E_+-E_-).\end{aligned} \quad \text{(III15)}$$

Integrating (III13) over ω, we obtain

$$d\sigma_{\text{pair}}=d\sigma_{\text{prod}}(\xi/\Gamma)\mathfrak{D}d\Omega_+d\Omega_-dD, \quad \text{(III16)}$$

where

$$\xi\equiv\frac{m_\Lambda m^2}{(2\pi)^2}\int d\omega\,\delta(\omega+E_\Lambda-E_\Sigma)\frac{p_+p_-}{E_\Lambda}, \quad \text{(III17)}$$

and where $\mathbf{p}_\pm$ have the values consistent with

$$E_++E_-=(p^2+M^2)^{1/2} -[|\mathbf{p}-\mathbf{p}_+-\mathbf{p}_-|^2+m_\Lambda{}^2]^{1/2}. \quad \text{(III18)}$$

We shall assume that both Σ^0 and Λ^0 are nonrelativistic, and that $e^\pm$ are extremely relativistic. Then

$$d\sigma_{\text{pair}}=d\sigma_{\text{prod}}(m/2\pi)^2(E_+E_-/\Gamma)\mathfrak{D}d\Omega_+d\Omega_-d\nu, \quad \text{(III19)}$$

in which

$$E_++E_-=\Delta M=70\text{ MeV}. \quad \text{(III20)}$$

It is noted that (III19) depends on Z both through $\mathfrak{D}$ and $d\sigma_{\text{prod}}$. To eliminate the latter, we consider the dominant decay mode $\Sigma^0\to\Lambda^0+\gamma$, which may be observed as part of the process

$$K+N\to\pi+\Sigma^0\to\pi+\Lambda^0+\gamma. \quad \text{(III21)}$$

The differential cross section for this shall be denoted by $d\sigma_{\text{photon}}$. In analogy with (III13) we have

$$d\sigma_{\text{photon}}=d\sigma_{\text{prod}}d^3q \times(\pi m_\Lambda/qE_\Lambda)\delta(E_\Sigma-E_\Lambda-q)(\mathcal{E}_0/\Gamma), \quad \text{(III22)}$$

where $\mathbf{q}$ is the momentum of the photon,

$$E_\Lambda\equiv[|\mathbf{p}-\mathbf{q}|^2+m^2]^{1/2},$$

and

$$\mathcal{E}_0\equiv\sum_{\text{spin,pol}}|(\bar{\Lambda}J_\mu\rho\Sigma)\epsilon_\mu|^2, \quad \text{(III23)}$$

where ρ is understood to correspond to the average polarization of the Σ^0, and the sum above includes a sum over the two independent polarizations of the photon. We keep $\mathbf{p}$ fixed and let the integration in (III22) extend over all $\mathbf{q}$. The result shall be denoted

by σ_{photon}. For nonrelativistic Σ^0 and Λ^0, it is given by

$$\sigma_{\text{photon}}=d\sigma_{\text{prod}}(4\pi^2\Delta M)\,\mathcal{E}_0/\Gamma. \tag{III24}$$

Dividing (III19) by this quantity, we obtain

$$\frac{d\sigma_{\text{pair}}}{\sigma_{\text{photon}}}=\frac{m^2E_+E_-}{16\pi^4\Delta M}\left(\frac{\mathfrak{D}_0}{\mathcal{E}_0}\right)(1+F)d\Omega_+d\Omega_-dv, \tag{III25}$$

where $\mathfrak{D}_0$ is given by (40) and F by (43). The Z dependence is now solely contained in F, which is the quantity to be measured experimentally. In Appendix IV, $\mathfrak{D}_0/\mathcal{E}_0$ is calculated under certain assumptions.

APPENDIX IV. DECAY OF Σ^0 IN FREE SPACE

The S-matrix element $\{\Sigma^0\to\Lambda^0+\gamma\}$ is given by (6), which defines the decay interaction J_μ. Under the assumptions of Lorentz invariance and gauge invariance, one can immediately write down the most general form of J_μ. It would contain two unknown form factors, one of which does not contribute to the rate of the decay mode $\Sigma^0\to\Lambda^0+\gamma$. If we assume that this form factor can be neglected, and that the other form factor is a constant, then we are effectively assuming that

$$\begin{aligned} J_\mu&=-igk_\nu\sigma_{\nu\mu} &&\text{for even } \Sigma-\Lambda \text{ parity,}\\ &=-ig\gamma_5k_\nu\sigma_{\nu\mu} &&\text{for odd } \Sigma-\Lambda \text{ parity,}\end{aligned} \tag{IV1}$$

where $\sigma_{\nu\mu}\equiv(\gamma_\nu\gamma_\mu-\gamma_\mu\gamma_\nu)/2i$. Consider an unpolarized Σ^0. Then $\mathcal{E}_0$, defined by (III23), becomes a relativistic invariant:

$$\mathcal{E}_0=\tfrac{1}{2}\sum_{\text{spin,pol}}|(\bar{\Lambda}J_\mu\Sigma)\epsilon_\mu|^2=\frac{g^2(M^2-m_\Lambda{}^2)^2}{2Mm_\Lambda}. \tag{IV2}$$

The decay rate for the dominant mode in the rest frame of the Σ^0 is

$$\Gamma_0=\frac{g^2(M^2-m_\Lambda{}^2)^3}{4\pi M(M^2+m_\Lambda{}^2)}. \tag{IV3}$$

This determines the effective coupling constant g in terms of Γ_0, which, as we know, differs from the total decay rate by less than 1%.

Consider now the decay mode $\Sigma^0\to\Lambda^0+e^++e^-$ in free space for an unpolarized Σ^0. We have

$$\begin{aligned}\{\Sigma^0\to\Lambda^0+e^++e^-\}&=(2\pi)^4\delta^4(p_f-p_i)(e/\gamma^2)\\ &\quad\times(\bar{\Lambda}J_\mu\Sigma)(\bar{e}_-\gamma_\mu e_+).\end{aligned} \tag{IV4}$$

The quantity $\mathfrak{D}_0$, defined by (40), also reduces to a relativistic invariant:

$$\begin{aligned}\mathfrak{D}_0&=\frac{e^2}{2\gamma^4}\sum_{\text{spin}}|(\bar{\Lambda}J_\mu\Sigma)(\bar{e}_-\gamma_\mu e_+)|^2\\ &=\frac{g^2e^2M}{2m^2m_\Lambda}\left\{1-\left(1\pm\frac{m_\Lambda}{M}-\frac{k\cdot p}{M^2}\right)\left(1+\frac{2m^2}{\gamma^2}\right)\right.\\ &\qquad\left.+\frac{(k\cdot p)^2}{\gamma^2M^2}\left(1+\frac{4m^2}{\gamma^2}\right)+\frac{4(p\cdot q)^2}{\gamma^2M^2}\right\},\end{aligned} \tag{IV5}$$

where $k=p_++p_-$, $q=\frac{1}{2}(p_+-p_-)$, and $\gamma^2=-k^2$. The $+(-)$ sign is associated with even (odd) $\Sigma-\Lambda$ parity.

Let us neglect the $\Sigma-\Lambda$ mass difference ΔM as compared to M, assume that the Σ^0 is nonrelativistic, and that the $e^\pm$ are extremely relativistic. Further assume that the angle between the Dalitz pair is sufficiently large so that $m^2/\gamma^2\ll1$. Then

$$\begin{aligned}\mathcal{E}_0&=2g^2(\Delta M)^2,\\ \mathfrak{D}_0&=\frac{g^2e^2}{m^2}\left[\frac{2(E_+{}^2+E_-{}^2)}{\gamma^2}\mp1\right],\end{aligned} \tag{IV6}$$

where the $-(+)$ sign corresponds to even (odd) $\Sigma-\Lambda$ parity. In the same approximation we may take

$$\gamma^2=2E_+E_-(1-\cos\theta), \tag{IV7}$$

in which θ is the angle between the Dalitz pair. Therefore,

$$\frac{\mathfrak{D}_0}{\mathcal{E}_0}=\frac{e^2}{2m^2(\Delta M)^2}\left[\frac{E_+{}^2+E_-{}^2}{E_+E_-(1-\cos\theta)}\mp1\right]. \tag{IV8}$$

APPENDIX V. DETAILS OF CALCULATIONS

A. The Function G

We shall calculate the function G defined by (55). From (55) and (57), we have

$$G=-\left(\frac{2\gamma^2M\Gamma}{\pi^2}\right)\mathrm{Im}\int d^3\Delta\times\frac{W(\mathbf{\Delta})}{\Delta^2(\Delta^2-2\mathbf{p}\cdot\mathbf{\Delta}-iM\Gamma)(\Delta^2-2\mathbf{k}\cdot\mathbf{\Delta}-\gamma^2-i\eta)}, \tag{V1}$$

where

$$\begin{aligned}W(\mathbf{\Delta})&=\frac{E_-}{\Delta^2-2\mathbf{p}_-\cdot\mathbf{\Delta}-i\epsilon}-\frac{E_+}{\Delta^2-2\mathbf{p}_+\cdot\mathbf{\Delta}-i\epsilon}\\ &=\frac{(E_--E_+)\Delta^2+2E_+E_-(\mathbf{v}_--\mathbf{v}_+)\cdot\mathbf{\Delta}}{\Delta^2(\Delta^2-2\mathbf{k}\cdot\mathbf{\Delta})+4(\mathbf{p}_+\cdot\mathbf{\Delta})(\mathbf{p}_-\cdot\mathbf{\Delta})-i\epsilon},\end{aligned} \tag{V2}$$

in which $\mathbf{v}_\pm$ are the respective velocities of $e^\pm$. We may

rewrite the above in the form

$$W(\boldsymbol{\Delta})=\frac{1}{\Delta^2-2\mathbf{k}\cdot\boldsymbol{\Delta}-i\epsilon}\left[(E_--E_+)-(4E_+E_-)\frac{(\mathbf{v}_+\cdot\boldsymbol{\Delta})^2E_+-(\mathbf{v}_-\cdot\boldsymbol{\Delta})^2E_--\Delta^2(\mathbf{v}_--\mathbf{v}_+)\cdot\boldsymbol{\Delta}}{\Delta^2(\Delta^2-2\mathbf{k}\cdot\boldsymbol{\Delta})+4E_+E_-(\mathbf{v}_+\cdot\boldsymbol{\Delta})(\mathbf{v}_-\cdot\boldsymbol{\Delta})-i\epsilon}\right]. \tag{V3}$$

The second term will be neglected. It is small because the important contributions to (V1) comes from the region in which $\boldsymbol{\Delta}\cdot\mathbf{p}_+\approx\boldsymbol{\Delta}\cdot\mathbf{p}_-\approx 0$. Accordingly, we take

$$W(\boldsymbol{\Delta})\approx\frac{E_--E_+}{\Delta^2-2\mathbf{k}\cdot\boldsymbol{\Delta}-i\epsilon}. \tag{V4}$$

The error incurred by doing this is probably a few percent in the final formula (87).

Using the well-known identity

$$\frac{1}{ABC}=2\int_0^1 dt\,t\int_0^1 ds\frac{1}{[A(1-t)+Bts+Ct(1-s)]^3}, \tag{V5}$$

we obtain from (V1) and (V4)

$$G=-\left(\frac{4\lambda}{\pi^2}\right)\left(\frac{E_--E_+}{\gamma}\right)\mathrm{Im}\int_0^1 dt\,t\int_0^1 ds\int_0^\infty d\Delta\int d\Omega \times\frac{1}{(\Delta^2-2\mathbf{V}\cdot\boldsymbol{\Delta}-N-i\epsilon)^3}, \tag{V6}$$

where $d\Omega$ is an element of solid angle of the vector $\boldsymbol{\Delta}$, and

$$\mathbf{V}=(1-t)\mathbf{P}+t\mathbf{K},\quad N=ts+i\lambda(1-t), \tag{V7}$$

with λ, $\mathbf{P}$, $\mathbf{K}$ defined by (63). The integration over Ω leads to

$$G=-\frac{4\lambda(E_--E_+)}{\pi^2\gamma}\mathrm{Im}\int_0^1 dt\,t\int_0^1 ds\,4\pi\int_0^\infty d\Delta \times\frac{\Delta^2-N}{(\Delta^2-N-2V\Delta-i\epsilon)^2(\Delta^2-N+2V\Delta-i\epsilon)} \tag{V8}$$

where $V\equiv|\mathbf{V}|$. The integration over Δ leads to

$$G=-\frac{2\lambda(E_--E_+)}{\gamma}\mathrm{Re}\int_0^1 dt\,t\int_0^1 ds\frac{V^2+\frac{3}{2}N}{N^2(V^2+N)^{3/2}}. \tag{V9}$$

One easily recognizes that the integrand above is a perfect differential in s. Thus the s integration is elementary. For the t integration, the integrand has a number of branch points; but one can verify that the path of integration remains on a single Riemann sheet. Then the t integration too is elementary. The final result is

$$G=\frac{\pi(E_--E_+)}{|\mathbf{p}_-+\mathbf{p}_+|}\Phi(\lambda,\mathbf{K},\mathbf{P}), \tag{V10}$$

where

$$\Phi(\lambda,\mathbf{K},\mathbf{P})=(2/\pi)(\phi_1+\phi_2-\phi_3), \tag{V11}$$

$$\phi_1=\tan^{-1}\left\{\frac{(\lambda/2K)+(\lambda/2)^{1/2}\mathfrak{F}_2}{(\mathbf{K}\cdot\mathbf{P}/K)+(\lambda/2)^{1/2}\mathfrak{F}_1}\right\}, \tag{V12}$$

$$\phi_2=\tan^{-1}\left\{\frac{(1-2\mathbf{P}\cdot\mathbf{K})\mathcal{G}_1-[1+2(P/\lambda)^2-2(K^2/\lambda)(\mathcal{G}_1^2+\mathcal{G}_2^2)]\mathcal{G}_2}{\lambda(1-2\mathbf{P}\cdot\mathbf{K})\mathcal{G}_2+\lambda[1+2(P/\lambda)^2]\mathcal{G}_1+2\lambda K[(1+K^2)^{1/2}+K\mathcal{G}_1](\mathcal{G}_1^2+\mathcal{G}_2^2)}\right\}, \tag{V13}$$

$$\phi_3=\tan^{-1}\left\{\frac{(1-2\mathbf{P}\cdot\mathbf{K})\mathcal{G}_1-\lambda[1+2(P/\lambda)^2]\mathcal{G}_2-(2\lambda)^{1/2}K(\mathcal{G}_1^2+\mathcal{G}_2^2)\mathfrak{F}_1}{(1-2\mathbf{P}\cdot\mathbf{K})\mathcal{G}_2+\lambda[1+2(P/\lambda)^2]\mathcal{G}_1+(2\lambda)^{1/2}K(\mathcal{G}_1^2+\mathcal{G}_2^2)\mathfrak{F}_2}\right\}, \tag{V14}$$

$$\mathfrak{F}_1=\left\{\left[1+\left(\frac{P^2}{\lambda}\right)^2\right]^{1/2}+\frac{P^2}{\lambda}\right\}^{1/2}, \tag{V15}$$

$$\mathfrak{F}_2=\left\{\left[1+\left(\frac{P^2}{\lambda}\right)^2\right]^{1/2}-\frac{P^2}{\lambda}\right\}^{1/2}, \tag{V16}$$

$$\mathcal{G}_1=\frac{1}{\sqrt{2}}\left\{\left[\left(1+\frac{P^2}{\lambda^2K^2}\right)^2+\left(\frac{2\mathbf{P}\cdot\mathbf{K}}{\lambda K}\right)^2\right]^{1/2}+\left[1+\frac{P^2}{\lambda^2K^2}\right]^{1/2}\right\}^{1/2}, \tag{V17}$$

$$\mathcal{G}_2=\frac{1}{\sqrt{2}}\frac{\mathbf{K}\cdot\mathbf{P}}{KP}\left\{\left[\left(1+\frac{P^2}{\lambda^2K^2}\right)^2+\left(\frac{2\mathbf{P}\cdot\mathbf{K}}{\lambda K}\right)^2\right]^{1/2}-\left[1+\frac{P^2}{\lambda^2K^2}\right]^{1/2}\right\}^{1/2}. \tag{V18}$$

B. The Average Momentum Transfer

We shall give a rough estimate of the average momentum transferred by the Dalitz pair to the Coulomb field. This is γ times the value of Δ that is most important for the integral (V8).

The integrand in (V8) has four second-order poles, located, respectively, at

$$(V^2+N)^{1/2}-V,\quad (V^2+N)^{1/2}+V, \tag{V19}$$

and their negatives. Of these, only $(V^2+N)^{1/2}-V$ is important, because it occurs near the origin $\Delta=0$. This may be seen as follows. From (V7) we have

$$V^2+N=(1-t)^2P^2+t^2K^2 \\ +t(1-t)\mathbf{P}\cdot\mathbf{K}+ts+i\lambda(1-t). \tag{V20}$$

By assumptions (58)–(61), we have

$$\frac{P}{K}=\frac{|\mathbf{p}|}{|\mathbf{p}_+ + \mathbf{p}_-|}\ll 1, \\ \frac{\lambda}{K^2}=\frac{M\Gamma}{|\mathbf{p}_+ + \mathbf{p}_-|^2}\ll 1, \tag{V21}$$

where the last statement holds only for $\Gamma<1$ MeV ($\tau>10^{-21}$ sec). Accordingly, we put

$$V^2\approx t^2K^2, \\ V^2+N\approx t^2K^2\left[1+\frac{i\lambda}{K^2}\left(\frac{1-t}{t^2}\right)\right]. \tag{V22}$$

The quantity $(1-t)/t^2$ may be treated as a quantity of the order of unity, because in the t integration the neighborhood of $t=0$ is suppressed by the factor t in (V8). Thus, the most important pole in (V8) is located at

$$\frac{i\lambda(1-t)}{2Kt}. \tag{V23}$$

This gives rise to a maximum in the integrand of (V8), located at $\Delta=0$, with a width of the order of

$$\delta=\lambda/K. \tag{V24}$$

Hence the average momentum transferred to the Coulomb field is of the order of $\gamma\delta=M\Gamma/|\mathbf{p}_+ + \mathbf{p}_-|$. This, except for very wide angles between the Dalitz pair, is of the order of

$$\bar{\Delta}=M\Gamma/(E_+ + E_-), \tag{V25}$$

which is a small quantity for part of the range of Γ listed in Table I.

C. The Function G'

The function G' is defined by (56). An order-of-magnitude estimate of G' may be made by replacing $\boldsymbol{\alpha}\cdot\mathbf{Q}_\pm$ in (56) by $Q_\pm\equiv|\mathbf{Q}_\pm|$, respectively. This is justified because both $e^\pm$ are extremely relativistic. What one usually calls the "large" and "small" components of the spinors are, in this case, comparable in magnitude. Hence the nonvanishing matrix elements of the matrices $\boldsymbol{\alpha}$, which connect the "large" component to the "small" component, are of the order of unity. Thus,

$$G'\lesssim\tfrac{1}{2}(Q_+ + Q_-). \tag{V26}$$

From (57), it can be seen that

$$|\mathbf{Q}_\pm|/I_\pm\lesssim\bar{\Delta}. \tag{V27}$$

In the approximation (V4), I_+ and I_- are the same. Hence

$$\left|\frac{G'}{G}\right|\lesssim\frac{\bar{\Delta}}{|E_+ - E_-|}=\left|\frac{E_+ - E_-}{E_+ + E_-}\right|\zeta. \tag{V28}$$

Under the assumption (61) this is less than unity for the range of ζ covered by Table I. Near the theoretical estimate of $\tau=5\times10^{-20}$ sec, the neglect of G' as compared to G is estimated to produce an error of less than 5% for the worse choice of energies, i.e., $|E_+ - E_-|=\frac{1}{5}(E_+ + E_-)$. The corresponding error for the integrated distribution (87) should be less than 2%.