Vector Mesons and the KN, $\bar{K}N$ Interactions*

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The effect of the exchanges of ρ and ω on the KN and $\bar{K}N$ scattering amplitudes is examined with special attention to the question of whether the ρ and ω exchanges can provide a dynamical mechanism for binding an s-wave $\bar{K}N$ system in the $T=0$ state.

THE effect of the exchanges of ρ and ω on the KN and $\bar{K}N$ scattering amplitudes is examined, using N/D techniques with special attention to the question of whether the ρ and ω exchanges can provide a dynamical mechanism for binding (in the sense of Dalitz and Tuan[1]) an s-wave $\bar{K}N$ system in the $T=0$ state. As pointed out some time ago,[2] in a theory in which the ρ and the ω are coupled, respectively, to the isospin and the hypercharge current, the KN and $\bar{K}N$ "potentials" are expected to be of the form

$$V=\pm V_\omega+\boldsymbol{\tau}_K\cdot\boldsymbol{\tau}_N V_\rho, \tag{1}$$

$$\boldsymbol{\tau}_K\cdot\boldsymbol{\tau}_N=1 \text{ for } T=1$$
$$=-3 \text{ for } T=0,$$

with

$$V_\omega>0,\quad V_\rho>0, \tag{2}$$

where the upper sign (lower sign) in Eq. (1) is applicable to the KN ($\bar{K}N$) interaction.[3] Such potentials are of particular interest because they fit nicely into the following observed facts.

(i) The KN interaction is on the average repulsive; the $\bar{K}N$ interaction is on the average attractive.

(ii) The $T=1$ KN s-wave interaction is most strongly repulsive.

(iii) The 1405-MeV Y_0^*, but not the 1385-MeV Y_1^*, is most likely to be an s-wave $\bar{K}N$ bound state resonance of the Dalitz-Tuan type, which implies that the $T=0$ $\bar{K}N$ s-wave interaction is most attractive.

In this paper we calculate the scattering lengths for the various isospin states of the KN, $\bar{K}N$ systems, using the N/D method[4] under the assumption that the ρ and ω contributions dominate the s-wave amplitudes. In particular, we demonstrate that for reasonable values of the coupling constants $f_\omega^2/4\pi$ and $f_\rho^2/4\pi$, the ω and ρ exchanges can provide the necessary attractive force to bind an s-wave $\bar{K}N$ system in the $T=0$ state.

As usual, we write the s-wave amplitude for KN, $\bar{K}N$ scattering,

$$T=\exp(i\delta)\sin\delta/k,$$

as

$$T=N(\nu)/D(\nu), \tag{3}$$
$$\nu\equiv k^2,$$

where N and D are, respectively, free of right-hand and left-hand singularities. With the assumption that the ω and ρ exchanges dominate, N is taken to be the s-wave projection of the relativistc Born amiplitude:

$$N(\nu)=-\frac{f^2}{4\pi}\frac{(W-m_N)[(W+m_N)^2-m_K^2]}{8W^2\nu}\times\ln\left(1+\frac{4\nu}{m_V^2}\right), \tag{4}$$

where

$$\frac{f^2}{4\pi}=\pm\frac{f_\omega^2}{4\pi}+\frac{\boldsymbol{\tau}_N\cdot\boldsymbol{\tau}_K}{4}\frac{f_\rho^2}{4\pi},\quad \text{for}\begin{cases}KN\\ \bar{K}N\end{cases} \tag{5}$$

and

$$W=(\nu+m_K^2)^{1/2}+(\nu+m_N^2)^{1/2}.$$

Since our calculations are expected to be crude in any case, we have set both the ω and ρ masses equal to a common value $m_V=750$ MeV. Because of the universality principle of reference 2, there are only two coupling constants,[5] $f_{\rho KK}=f_{\rho NN}=f_\rho$ and $f_{\omega KK}=f_{\omega NN}=f_\omega$. We require that T coincide with N at $\nu=\nu_0$. The normalization point ν_0 is chosen to be $\nu_0=-m_V^2/4$ where $N(\nu)$ becomes infinite. Numerically, the center-of-mass energy corresponding to ν_0 turns out to be 1170 MeV, which is to be compared to the total center-of-mass energy corresponding to the KN threshold, 1430 MeV.

We can now write down a dispersion relation for D, which yields

$$N(\nu)\nu^{1/2}\cot\delta=1-\frac{\nu-\nu_0}{\pi}\,\mathrm{Pr}\int_0^\infty\frac{N(\nu')(\nu')^{1/2}d\nu'}{(\nu'-\nu_0)(\nu'-\nu)}, \tag{6}$$

where we have used

$$\mathrm{Re}(T^{-1})=\nu^{1/2}\cot\delta,\quad \mathrm{Im}(T^{-1})=-\nu^{1/2}.$$

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[1] R. H. Dalitz and S. F. Tuan, Phys. Rev. Letters **5**, 425 (1959). See also R. H. Dalitz, *ibid.* **6**, 239 (1961).

[2] J. J. Sakurai, Ann. Phys. (New York) **11**, 1 (1960). See also Sec. 4 of R. H. Dalitz, Revs. Modern Phys. **33**, 471 (1961); J. Franklin, R. C. King, and S. F. Tuan, Phys. Rev. **124**, 1995 (1961).

[3] Equation (1) is a consequence of any theory whereas Eq. (2) is a specific consequence of the vector theory discussed in reference 2.

[4] M. Baker, Ann. Phys. (New York) **4**, 271 (1958). We closely follow the treatment of G. Feldman, P. T. Matthews, and A. Salam, Nuovo cimento **16**, 549 (1960).

[5] f_ω and f_ρ are, respectively, f_Y and f_T of reference 2. Our definitions are related to those of M. Gell-Mann and F. Zachariasen [Phys. Rev. **124**, 953 (1961)] by $f_\rho=2\gamma_\rho$, $f_\omega=\sqrt{3}\gamma_\omega$.

We calculate the scattering length,

$$a=\lim_{\nu\to 0+}(\nu^{1/2}\cot\delta)^{-1},$$

as a function of the coupling constant $f^2/4\pi$ which, because of (5), may take both positive (repulsive interaction) and negative (attractive interaction) values. Numerical results are shown in Fig. 1. For large positive values of $f^2/4\pi$, the scattering length becomes rather insensitive of $f^2/4\pi$, which is, of course, characteristic of any strong short-ranged repulsive interaction. The Born approximation appears to be valid only in a small region $|f^2/4\pi|\lesssim 1$. For $f^2/4\pi<-6.7$ we expect an s-wave bound state.[6]

Current experiments, taken together with an argument given by Akiba and Capps,[7] strongly suggest that the 1405-MeV Y_0^* is a bound-state resonance of the Dalitz-Tuan type and that there are no other s-wave $\bar{K}N$ bound states. This leads to the following conditions on the coupling constants

$$f_\omega^2/4\pi+\tfrac{3}{4}f_\rho^2/4\pi>6.7,$$
$$f_\omega^2/4\pi-\tfrac{1}{4}f_\rho^2/4\pi<6.7.$$

From the decay width for the process $\rho\to 2\pi$ and s-wave πN scattering,[8] $f_\rho^2/4\pi$ has been determined to be ≈ 2.2. So if we take our calculations seriously, we obtain

$$f_\omega^2/4\pi\sim 5-7,$$

which is not unreasonable. It might be worth mentioning in this connection that the "eightfold way" of Gell-Mann and Ne'eman[9] leads to

$$f_\omega^2/4\pi=\tfrac{3}{4}f_\rho^2/4\pi\approx 1.6.$$

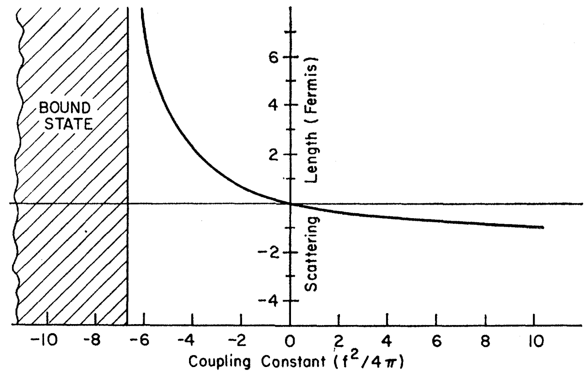

FIG. 1. Scattering length as a function of the coupling constant $f^2/4\pi=\pm f_\omega^2/4\pi+\frac{1}{4}\boldsymbol{\tau}_K\cdot\boldsymbol{\tau}_N f_\rho^2/4\pi$, where the upper (lower) sign is for the s-wave KN ($\bar{K}N$) interaction. The vector meson masses are taken to be $m_V=m_\omega=m_\rho=750$ MeV.

As for the KN interaction, the K^+p scattering length is predicted to be of the order of -0.7 F if $f_\omega^2/4\pi\sim 6$ and $f_\rho^2/4\pi\sim 2$. Experimentally σ_{tot} for K^+p scattering is about 15 mb, which corresponds to $a=-0.36$ F.

To conclude, the aesthetically simple idea that the low-energy KN, $\bar{K}N$ interactions are dominated by the exchanges of the ρ and the ω meson which are, respectively, coupled universally to the conserved isospin current and the hypercharge current leads to a unified picture for the main qualitative features of KN, $\bar{K}N$ processes. In particular, there definitely exists a possibility for a $T=0$ bound $\bar{K}N$ state (but no other bound states) for reasonable values of coupling constants. This is seen to be quite an achievement, especially if we recall that the K^-p "potential" based on pseudoscalar meson theory ($KN\Lambda$ and $KN\Sigma$ couplings) turns out to be repulsive. Quantitative details of our calculations, however, should not be taken literally since we have completely ignored:

(i) reactive effects due to the inelastic channels $\bar{K}N\to\pi\Lambda$, $\pi\Sigma$ (which may be brought about by K^* exchanges?);

(ii) conventional Yukawa-type singularities which are fortunately far away for pseudoscalar $KN\Lambda$ and $KN\Sigma$ couplings.

It is a pleasure to thank Professor R. H. Dalitz for illuminating discussions on the present status of pion-hyperon resonances.

[6] It is interesting to compare this result with a calculation based on the Schrödinger theory with an equivalent Yukawa potential. If we assume that the $\bar{K}N$ interaction is due to the Yukawa potential $V=(f^2/4\pi)\exp(-m_V r)/r$, the condition for the existence of a bound state turns out to be $-f^2/4\pi>1.683m_V(m_K+m_N)/2m_K m_N\approx 2.0$. See, e.g., R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Co., Cambridge, 1953), p. 74.

[7] T. Akiba and R. H. Capps, Phys. Rev. Letters **8**, 457 (1962). For a list of references on the experimental aspects of Y_0^* and other hyperon isobars, see, e.g., G. Alexander, G. R. Kalbfleish, D. H. Miller, and G. A. Smith, *ibid.* **8**, 447 (1962).

[8] J. J. Sakurai, "Proceedings of the 1962 International Conference on High-Energy Physics" (to be published).

[9] M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report, CSTL-20, 1961 (unpublished) [see also M. Gell-Mann, Phys. Rev. **125**, 1067 (1962)]. Y. Ne'eman, Nuclear Phys. **26**, 222 (1961).