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Λ-Nucleon Scattering and the Pion-Hyperon Coupling Constants*

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Using the prescription laid down by Brueckner and Watson, we have calculated the pion contribution to the hyperon-nucleon potentials for both even and odd Σ parity. The coupled Schrödinger equations are solved numerically taking into account the ΛΣ mass difference. Fitting the Λ-nucleon scattering lengths, as obtained from the hyperfragment data, we have determined the coupling constants. When we use the same core ($x_c=0.35\ \mu^{-1}$) in all channels and $f_{NN}^2=0.08125$, we find $f_{\Lambda\Sigma}^2=f_{\Sigma\Sigma}^2=0.0841$ for the even Σ-parity case and $f_{\Sigma\Sigma}^2=0.0225$ ($f_{\Sigma\Sigma}<0$) and $g_{\Lambda\Sigma}^2=0.578$ for the odd Σ-parity case. The rationalized pseudovector coupling constant we denote by f and the rationalized scalar coupling constant by g. For these coupling constants and cores, we have calculated the cross sections for Λ-nucleon scattering below Σ threshold. The results suggest the presence of attractive spin-orbit potentials.

1. INTRODUCTION

SOME recent attempts to understand the structure of the strong interactions have hinged on the possible values of the pion-baryon coupling constants and the relative parity. Global symmetry,[1–3] the doublet approximation or restricted symmetry,[4] unitary symmetry,[5] the superconductor model,[6] and the conserved vector current theory[7] have more or less specific predictions about the coupling strengths. For this reason, the numerical values themselves are of considerable interest. In this article we obtain fairly sharp estimates for the $\pi\Lambda\Sigma$ and $\pi\Sigma\Sigma$ coupling constants. Results are given for both odd and even Σ parity (we take the Λ parity to be even by definition). We use as input data the triplet and singlet ΛN scattering lengths (a_1 and a_0) determined from analyses of the light hyperfragments.[8–10] It will be shown that for either parity it is possible to fit these data with pion forces only.

Our general approach is in strict analogy with the nucleon-nucleon problem. In that case the pion field can be eliminated by finding the effective potential. When one uses this potential in the Schrödinger equation and tries to fit the low-energy NN scattering parameters and the binding energy and quadrupole moment of the deuteron then one reaches the conclusions that the pion should be pseudoscalar, and that the pion-nucleon coupling constant f^2 should be about 0.07 to 0.09.[11,12] This is in quite good agreement with the best accepted value $f^2=0.080\pm0.005$ obtained from much more complicated experiments and analyses.[13]

For the hyperon-nucleon case the problem is harder. There is no hyperon-nucleon bound state (like the deuteron) observed. Also, there are at present much fewer scattering data[14,15]; the Λ-nucleon scattering lengths are uncertain because they had to be "unscrambled" from the hypernuclei; practically nothing is known about angular distributions in the hyperon-nucleon scattering. In addition, there will be a greater sensitivity to the values of the K couplings than in the NN case, because a single kaon exchange is possible in the hyperon-nucleon system. In this article we will neglect the K mesons entirely.

Despite these difficulties, the problem of the pion-hyperon couplings has attracted considerable attention. The method which we shall use has already been tried by a number of other workers.[16,17] The essential difference between us and the other workers is that, although our final result is the s-wave Λ-nucleon scattering length, we solve the Schrödinger equation exactly, including the effects of the tensor force and of the closed Σ channel. That the tensor force is important is well known in the nucleon-nucleon problem, where it furnishes almost all the binding in the deuteron; the importance in the hyperon-nucleon case is also quite evident from other calculations.[10,18,19] In a complicated multichannel prob-

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[1] E. P. Wigner, Proc. Natl. Acad. Sci. U. S. **38**, 449 (1952).
[2] J. Schwinger, Ann. Phys. (New York) **2**, 407 (1957).
[3] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).
[4] A. Pais, Phys. Rev. **110**, 574, 1480 (1958).
[5] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).
[6] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).
[7] J. J. Sakurai, Ann. Phys. (New York) **11**, 1 (1960).
[8] R. H. Dalitz and B. W. Downs, Phys. Rev. **110**, 958 (1958); **111**, 967 (1958); **114**, 593 (1959).
[9] K. Dietrich, R. Folk and H. J. Mang, *Proceedings of the Rutherford Jubilee International Conference, Manchester, 1961*, edited by J. B. Beirks (Academic Press Inc., New York, 1961).
[10] J. J. de Swart and C. Dullemond, Ann. Phys. (New York) **19**, 458 (1962).
[11] J. Iwadare, S. Otsuki, and W. Watari, Progr. Theoret. Phys. (Kyoto) **15**, 86 (1956).
[12] J. Iwadare, S. Otsuki, R. Tamagaki and W. Watari, Progr. Theoret. Phys. (Kyoto) **16**, 455 (1956); **16**, 472 (1956).
[13] T. Spearman, Nuclear Phys. **16**, 402 (1960). G. Salzman and H. Schnitzer, Phys. Rev. **113**, 1153 (1959).
[14] F. S. Crawford, Marcello Cresti, Myron L. Good, Frank T. Solmitz, M. Lynn Stevenson, and Harold K. Ticho, Phys. Rev. Letters **2**, 174 (1959).
[15] G. Alexander, J. Anderson, F. Crawford, W. Laskar, and L. Lloyd, Phys. Rev. Letters **7**, 348 (1961).
[16] D. B. Lichtenberg and M. H. Ross, Phys. Rev. **107**, 1714 (1957).
[17] F. Ferrari and L. Fonda, Nuovo cimento **9**, 842 (1958).
[18] J. J. de Swart and C. Dullemond, Ann. Phys. (New York) **16**, 263 (1961),
[19] C. Dullemond and J. J. de Swart, Nuovo cimento **25**, 1072 (1962).

lem such as the YN scattering, it is difficult to make reliable predictions using "effective" central potentials.

In this article we shall consider only the Λ-nucleon scattering below the Σ threshold. Section 2 introduces our isospin notation, that this is less trivial than it appears, is evident from the errors made on this point in the existing literature. In Sec. 3 we will give a discussion of the equivalence of the pseudovector (PV) and the pseudoscalar (PS) coupling, and of the vector (V) and the scalar (S) coupling for the pion-baryon system. This equivalence is important in the sense that it reduces the number of possibilities. It allows us to write a unique interaction Hamiltonian for either Σ parity if we are willing to neglect higher order corrections. The YN potentials are then derived in Sec. 4 using the prescription given by Brueckner and Watson.[20] The presence of a hard core is now well established in the nucleon-nucleon potential. We will assume cores to be present in the YN potentials also. Specific features of the YN potentials for even and odd Σ parity will be discussed in Secs. 5 and 6. Certain details are given in the Appendices.

In Sec. 7 we give the numerical results. These numerical results are discussed in Sec. 8. Special attention is given there to the treatment of the cores, because this appears to be the main source of uncertainty. It is shown that with the simplest assumption of equal cores of $x_c=0.35\,\mu^{-1}$ in all states and channels we obtain global pseudovector symmetry as the solution for the even Σ-parity case. Also in the odd Σ-parity case we find a solution. This implies that even with pion forces alone, it is possible to obtain a spin-dependent potential for the odd Σ-parity case. We do not have to resort to the introduction of K mesons to obtain the required spin dependence.[17] Only if $f_{\Sigma\Sigma}=0$ do we have a spin-independent potential as is shown in Appendix C. The spin dependence is due to the strong spin-dependent forces in the Σ-nucleon channel. A correct treatment of this channel is therefore necessary to obtain meaningful results. The effect of the forces in the Σ-nucleon channel is discussed in Sec. 9. Finally, in Sec. 10 we have calculated the total cross section for our "favored" solution in either Σ-parity case. These calculations are compared with the experimental data[14,15] and with earlier calculations.[18,19] It is shown that there are indications of attractive spin-orbit potentials in the Λ-nucleon and/or the Σ-nucleon channel for either Σ parity.

2. ISOSPIN NOTATION

We will write down explicitly the components of the wave functions in isospace. Although this is generally not done, it greatly clarifies the manipulations and so reduces the possibility of errors.[21] The treatment of the isospin is put on exactly the same footing as the treatment of the ordinary spin.

The pion field and the Σ-hyperon field are vectors in isospace; their wave functions can, therefore, be written as

$$\boldsymbol{\pi}=\sum_m(-)^m\phi_{-m}\mathbf{e}_m^{(1)}, \tag{1a}$$

$$\boldsymbol{\Sigma}=\sum_m(-)^m\psi_{-m}(\Sigma)\mathbf{e}_m^{(1)}. \tag{1b}$$

The Λ-hyperon field is an isoscalar. The wave function can be written as

$$\Lambda=\psi(\Lambda)e_0^{(0)}. \tag{1c}$$

Here $e_{I_z}^{(I)}$ is the isospin wave function[22] for a total isospin I and z component I_z. The dependence on the space-time and ordinary spin is included in the functions ϕ_m, $\psi_m(\Sigma)$, and $\psi(\Lambda)$. Here we have used the spherical components a_1, a_0, and a_{-1} of a vector $\mathbf{a}$, which are related to the Cartesian components a_x, a_y, and a_z through the formulas

$$\begin{aligned} a_1&=-(a_x+ia_y)/\sqrt{2},\\ a_0&=a_z,\\ a_{-1}&=(a_x-ia_y)/\sqrt{2}. \end{aligned} \tag{2}$$

From the formulas (1a) and (2) one sees clearly that the field[23] associated with the positive pion ($I_z=1$) is $\phi_{-1}=(\phi_x-i\phi_y)/\sqrt{2}$.

It is advantageous to combine the Λ and Σ wave function into one column vector Y defined by

$$Y=\begin{bmatrix}\psi(\Lambda)\\ \psi_x(\Sigma)\\ \psi_y(\Sigma)\\ \psi_z(\Sigma)\end{bmatrix}. \tag{3}$$

The invariant $(\boldsymbol{\Sigma}^\dagger\times\boldsymbol{\Sigma})\cdot\boldsymbol{\pi}$ in isospace can then be written as

$$(Y^\dagger\mathbf{I}Y)\cdot\boldsymbol{\phi}=-i(\boldsymbol{\Sigma}^\dagger\times\boldsymbol{\Sigma})\cdot\boldsymbol{\pi}. \tag{4}$$

Here $\mathbf{I}$ is the isospin operator, which can be represented in matrix form as

$$I_x=\left\|\begin{matrix}0&0&0&0\\0&0&0&0\\0&0&0&-i\\0&0&i&0\end{matrix}\right\|,\quad I_y=\left\|\begin{matrix}0&0&0&0\\0&0&0&i\\0&0&0&0\\0&-i&0&0\end{matrix}\right\|,\quad I_z=\left\|\begin{matrix}0&0&0&0\\0&0&-i&0\\0&i&0&0\\0&0&0&0\end{matrix}\right\|, \tag{5}$$

and $\boldsymbol{\phi}$ are the three components ϕ_x, ϕ_y, and ϕ_z of the pion field.

[20] K. A. Brueckner and K. M. Watson, Phys. Rev. **92**, 1023 (1953).
[21] See footnotes 23 and 26.
[22] We use here the phase convention of E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, New York, 1935).
[23] In the formation of second quantitation the wave function gets associated with the destruction operator. Thus the destruction operator of the positive pion is ϕ_{-1}. In some treatises one finds the $\phi_1=\pm(\phi_x+i\phi_y)/\sqrt{2}$ erroneously associated with the positive pion.

The other relevant invariants are

$$(Y^\dagger \boldsymbol{\rho}_1 Y)\cdot\boldsymbol{\phi}=(\Lambda^\dagger\boldsymbol{\Sigma}+\boldsymbol{\Sigma}^\dagger\Lambda)\cdot\boldsymbol{\pi}, \tag{6a}$$

$$(Y^\dagger \boldsymbol{\rho}_2 Y)\cdot\boldsymbol{\phi}=i(\Lambda^\dagger\boldsymbol{\Sigma}-\boldsymbol{\Sigma}^\dagger\Lambda)\cdot\boldsymbol{\pi}. \tag{6b}$$

Here we have introduced the "vectors" $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ by

$$\rho_{1x}=\left\|\begin{matrix}0&1&0&0\\1&0&0&0\\0&0&0&0\\0&0&0&0\end{matrix}\right\|,\quad \rho_{1y}=\left\|\begin{matrix}0&0&1&0\\0&0&0&0\\1&0&0&0\\0&0&0&0\end{matrix}\right\|,\quad \rho_{1z}=\left\|\begin{matrix}0&0&0&1\\0&0&0&0\\0&0&0&0\\1&0&0&0\end{matrix}\right\|, \tag{7a}$$

$$\rho_{2x}=\left\|\begin{matrix}0&-i&0&0\\i&0&0&0\\0&0&0&0\\0&0&0&0\end{matrix}\right\|,\quad \rho_{2y}=\left\|\begin{matrix}0&0&-i&0\\0&0&0&0\\i&0&0&0\\0&0&0&0\end{matrix}\right\|,\quad \rho_{2z}=\left\|\begin{matrix}0&0&0&-i\\0&0&0&0\\0&0&0&0\\i&0&0&0\end{matrix}\right\|. \tag{7b}$$

It is worth noting here, that the invariants (6a) and (6b) can easily be transformed into each other by a change of the phase of the Λ isospin wave function (relative to the Σ isospin wave function). If we limit ourselves to the pion-hyperon interactions, then there is no difference between (6a) and (6b). In the following, we will base our choice between them on convenience.[24]

The nucleon wave function

$$\mathfrak{N}=\psi(p)e_{1/2}{}^{(1/2)}+\psi(n)e_{-1/2}{}^{(1/2)}$$

is commonly written in the vector notation

$$N_1=\begin{pmatrix}\psi(p)\\ \psi(n)\end{pmatrix},$$

and the invariant in isospace is then

$$(N_1{}^\dagger\boldsymbol{\tau}N_1)\cdot\boldsymbol{\phi}. \tag{8}$$

Here $\boldsymbol{\tau}$ are the three Pauli isospin operators,

$$\tau_x=\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \tau_y=\begin{pmatrix}0&-i\\i&0\end{pmatrix},\quad \tau_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}.$$

The interaction Lagrangian densities between the pion field and baryon fields, neglecting the explicit dependence on the space, time, and spin can be written as

$$[g_{NN}(N_1{}^\dagger\boldsymbol{\tau}N_1)+Y^\dagger(g_{\Sigma\Sigma}\mathbf{I}+g_{\Lambda\Sigma}\boldsymbol{\rho})Y]\cdot\boldsymbol{\phi}, \tag{9}$$

where $\boldsymbol{\rho}$ is $\boldsymbol{\rho}_1$ or $\boldsymbol{\rho}_2$ depending on our convenience.

For even Σ parity, a model[3,4,16] which is of some interest is the "doublet approximation." This model neglects the $\Lambda\Sigma$ mass difference and assumes the relation $g_{\Lambda\Sigma}=g_{\Sigma\Sigma}$ (or $f_{\Lambda\Sigma}=f_{\Sigma\Sigma}$) commonly known as "restricted symmetry." The isospin of the hyperon is considered to be composed of two isospins of one-half

$$\mathbf{I}=\mathbf{i}+\mathbf{k}.$$

The isospin wave functions of the hyperons can then be written as

$$e_1{}^{(1)}(\Sigma)=\alpha(i)\alpha(k),$$

$$e_0{}^{(1)}(\Sigma)=[\alpha(i)\beta(k)+\beta(i)\alpha(k)]/\sqrt{2},$$

$$e_{-1}{}^{(1)}(\Sigma)=\beta(i)\beta(k),$$

and

$$e_0{}^{(0)}(\Lambda)=[\alpha(i)\beta(k)-\beta(i)\alpha(k)]/\sqrt{2}.$$

Here, $\alpha(i)$ and $\beta(i)$ are the isospin wave functions for $i=1/2$, $i_z=1/2$ and $-1/2$.

If one assumes that the pion field is coupled only to the i part of the total isospin of the hyperon, then a convenient basis to use in isospin space is the set $\alpha(i)\alpha(k)$, $\beta(i)\alpha(k)$, $\alpha(i)\beta(k)$, and $\beta(i)\beta(k)$. Thus, the four 4-dimensional $\mathbf{I}$ space is the product space of two 2-dimensional spaces:

$$(I \text{ space}=i \text{ space}\otimes k \text{ space}).$$

With respect to this basis,[25] Y takes on the form

$$Y=\begin{pmatrix}N_2\\N_3\end{pmatrix},$$

where N_2 and N_3 are the spinors[26]

$$N_2=\begin{pmatrix}-\psi_{-1}(\Sigma)\\ [\psi_0(\Sigma)-\psi(\Lambda)]/\sqrt{2}\end{pmatrix},\quad N_3=\begin{pmatrix}[\psi_0(\Sigma)+\psi(\Lambda)]/\sqrt{2}\\ -\psi_1(\Sigma)\end{pmatrix}.$$

[24] In Secs. 3 and 4 we have chosen ρ in such a way as to make the potential [see Sec. 5, Eqs. (31) and (32)] symmetric. This will insure us a symmetric scattering matrix (see reference 18).

[25] We note that this basis is different from the one used before [Eq. (3)] and therefore Y is different.

[26] We prefer the notation used here to the more suggestive one, where one writes

$$N_2=-\begin{pmatrix}\Sigma^+\\(\Lambda^0-\Sigma^0)/\sqrt{2}\end{pmatrix},\quad N_3=\begin{pmatrix}(\Sigma^0+\Lambda^0)/\sqrt{2}\\ \Sigma^-\end{pmatrix},$$

(the $-$ sign in N_2 comes from our specific phase convention). This last notation can lead to confusion. We would like to save the symbol Σ^+ for the expression $\psi_{-1}(\Sigma)e_1{}^{(1)}(\Sigma)$, while it should not be used for *parts* of this expression. If one does not do this, the following confusion can arise: The Σ^+ as written down in N_2 should be decomposed into Cartesian components as $\Sigma^+=(\Sigma_x-i\Sigma_y)/\sqrt{2}$ because it is really $\psi_{-1}=(\psi_x-i\psi_y)/\sqrt{2}$. If one considers the $T=3/2$ isospin wave function, Σ^+p, then one has to decompose Σ^+ as $\Sigma^+=-(\Sigma_x+i\Sigma_y)/\sqrt{2}$ because it is really $e_1{}^{(1)}=-(e_x{}^{(1)}+ie_y{}^{(1)})/\sqrt{2}$.

In this notation we have

$$Y^\dagger(\boldsymbol{\varrho}_1+\mathbf{I})Y\cdot\boldsymbol{\phi}=(N_2{}^\dagger\boldsymbol{\tau}N_2+N_3{}^\dagger\boldsymbol{\tau}N_3)\cdot\boldsymbol{\phi},$$

and the interaction Lagrangian densities can be written in the highly symmetric form

$$g_{NN}(N_1{}^\dagger\boldsymbol{\tau}N_1)\cdot\boldsymbol{\phi}+g_Y(N_2{}^\dagger\boldsymbol{\tau}N_2+N_3{}^\dagger\boldsymbol{\tau}N_3)\cdot\boldsymbol{\phi},\quad(10)$$

where $g_Y=g_{\Sigma\Sigma}=g_{\Lambda\Sigma}$.

A special case is global symmetry, where $g_{NN}=g_{\Sigma\Sigma}=g_{\Lambda\Sigma}=g$. The interaction Lagrangian densities are then

$$g(N_1{}^\dagger\boldsymbol{\tau}N_1+N_2{}^\dagger\boldsymbol{\tau}N_2+N_3{}^\dagger\boldsymbol{\tau}N_3)\cdot\boldsymbol{\phi}.\quad(11)$$

3. CONNECTION BETWEEN DERIVATIVE AND NONDERIVATIVE COUPLING[27]

The Lagrangian density for the hyperon and pion fields can be written as

$$\mathfrak{L}=\mathfrak{L}_Y+\mathfrak{L}_\pi+\mathfrak{L}_{\Sigma\Sigma\pi}+\mathfrak{L}_{\Lambda\Sigma\pi}.\quad(12)$$

Here $\mathfrak{L}_Y$ is the Lagrangian density of the free-hyperon fields. Then

$$\mathfrak{L}_Y=\tfrac{1}{2}(\partial_\mu\bar{Y}\gamma^\mu Y-\bar{Y}\gamma^\mu\partial_\mu Y)-\bar{Y}MY,\quad(13)$$

where M is the matrix

$$M=\begin{Vmatrix} M_\Lambda & 0 & 0 & 0\\ 0 & M_\Sigma & 0 & 0\\ 0 & 0 & M_\Sigma & 0\\ 0 & 0 & 0 & M_\Sigma\end{Vmatrix},\quad(14)$$

with M_Λ the Λ mass and M_Σ the Σ mass.

The Lagrangian density of the free pion-field is

$$\mathfrak{L}_\pi=\tfrac{1}{2}\textstyle\sum_i\phi_i(\Box-\mu^2)\phi_i,\quad(15)$$

where

$$\Box\equiv\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial t^2}$$

and μ is the pion mass.

In case of nonderivative coupling we can write for the interaction Lagrangian densities

$$\mathfrak{L}_{\Lambda\Sigma\pi}=-g_{\Lambda\Sigma}(4\pi)^{1/2}(\bar{Y}O\boldsymbol{\varrho}_1Y)\cdot\boldsymbol{\phi},\quad(16a)$$

$$\mathfrak{L}_{\Sigma\Sigma\pi}=g_{\Sigma\Sigma}(4\pi)^{1/2}(\bar{Y}i\gamma_5\mathbf{I}Y)\cdot\boldsymbol{\phi},\quad(16b)$$

where

$O=1$ for odd Σ parity (*S* coupling)

$O=-i\gamma_5$ for even Σ parity (*PS* coupling).

In case of derivative coupling the interaction Lagrangian densities are

$$\mathfrak{L}_{\Lambda\Sigma\pi}=-\frac{f_{\Lambda\Sigma}}{\mu}(4\pi)^{1/2}(\bar{Y}O^\mu\boldsymbol{\varrho}Y)\cdot\partial_\mu\boldsymbol{\phi},\quad(17a)$$

$$\mathfrak{L}_{\Sigma\Sigma\pi}=-\frac{f_{\Sigma\Sigma}}{\mu}(4\pi)^{1/2}(\bar{Y}i\gamma^\mu\gamma_5\mathbf{I}Y)\cdot\partial_\mu\boldsymbol{\phi},\quad(17b)$$

[27] E. J. Kelly, Phys. Rev. **79**, 399 (1950).

where

$O^\mu\boldsymbol{\varrho}=i\gamma^\mu\boldsymbol{\varrho}_2$ for odd Σ parity (*V* coupling),

$O^\mu\boldsymbol{\varrho}=i\gamma^\mu\gamma_5\boldsymbol{\varrho}_1$ for even Σ parity (*PV* coupling).

The field equations are left invariant if one adds to the Lagrangian densities a four-divergence.[28] We will add to $\mathfrak{L}_{\Lambda\Sigma\pi}$ as given by (17a) the four-divergence

$$\frac{f_{\Lambda\Sigma}}{\mu}(4\pi)^{1/2}\partial_\mu\{(\bar{Y}O^\mu\boldsymbol{\varrho}Y)\cdot\boldsymbol{\phi}\},$$

and to $\mathfrak{L}_{\Sigma\Sigma\pi}$ as given by (17b) the four-divergence

$$\frac{f_{\Sigma\Sigma}}{\mu}(4\pi)^{1/2}\partial_\mu\{(\bar{Y}i\gamma^\mu\gamma_5\mathbf{I}Y)\cdot\boldsymbol{\phi}\}.$$

The interaction Lagrangian densities become then

$$\mathfrak{L}'_{\Lambda\Sigma\pi}=\frac{f_{\Lambda\Sigma}}{\mu}(4\pi)^{1/2}\boldsymbol{\phi}\cdot\partial_\mu(\bar{Y}O^\mu\boldsymbol{\varrho}Y),\quad(18a)$$

and

$$\mathfrak{L}'_{\Sigma\Sigma\pi}=\frac{f_{\Sigma\Sigma}}{\mu}(4\pi)^{1/2}\boldsymbol{\phi}\cdot\partial_\mu(\bar{Y}i\gamma^\mu\gamma_5\mathbf{I}Y).\quad(18b)$$

The field equations in the case of derivative couplings are

$(\gamma^\mu\partial_\mu+M)Y=$ terms proportional to $f_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$,

$\bar{Y}(\gamma^\mu\overleftarrow{\partial}_\mu-M)=$ terms proportional to $f_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$.

From these field equations one can obtain directly:

$\partial_\mu(\bar{Y}i\gamma^\mu\boldsymbol{\varrho}_2Y)=-(M_\Sigma-M_\Lambda)(\bar{Y}\boldsymbol{\varrho}_1Y)$ + terms proportional to $f_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$,

$\partial_\mu(\bar{Y}i\gamma^\mu\gamma_5\boldsymbol{\varrho}_1Y)=(M_\Sigma+M_\Lambda)(\bar{Y}i\gamma_5\boldsymbol{\varrho}_1Y)$ + terms proportional to $f_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$,

$\partial_\mu(\bar{Y}i\gamma^\mu\gamma_5\mathbf{I}Y)=2M_\Sigma(\bar{Y}i\gamma_5\mathbf{I}Y)$ + terms proportional to $f_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$.

Substituting this in the expressions (18a) and (18b) for the interaction Lagrangian densities and keeping only the terms linear in the coupling constants gives

for odd Σ parity

$$\mathfrak{L}'_{\Lambda\Sigma\pi}=-\left(\frac{M_\Sigma-M_\Lambda}{\mu}\right)f_{\Lambda\Sigma}(4\pi)^{1/2}(\bar{Y}\boldsymbol{\varrho}_1Y)\cdot\boldsymbol{\phi},\quad(19a)$$

for even Σ parity

$$\mathfrak{L}'_{\Lambda\Sigma\pi}=\frac{M_\Sigma+M_\Lambda}{\mu}f_{\Lambda\Sigma}(4\pi)^{1/2}(\bar{Y}i\gamma_5\boldsymbol{\varrho}_1Y)\cdot\boldsymbol{\phi},\quad(19b)$$

and for either Σ parity

$$\mathfrak{L}'_{\Sigma\Sigma\pi}=\frac{2M_\Sigma}{\mu}f_{\Sigma\Sigma}(4\pi)^{1/2}(\bar{Y}i\gamma_5\mathbf{I}Y)\cdot\boldsymbol{\phi}.\quad(20)$$

[28] G. Wentzel, *Einführung in die Quantentheorie der Wellenfelder* (Franz Deuticke, Leipzig, Germany, 1943), p. 2.

Comparing (19a) and (19b) with (16a) and (20) with (16b) shows that, if we neglect terms of higher order in the coupling constants, V and S coupling and PV and PS coupling are equivalent. The relations between the coupling constants for S and V coupling are

$$g_{\Lambda\Sigma}=\frac{M_\Sigma-M_\Lambda}{\mu}f_{\Lambda\Sigma}=0.558f_{\Lambda\Sigma}, \tag{21a}$$

and for PS and PV coupling are

$$g_{\Lambda\Sigma}=\frac{M_\Sigma+M_\Lambda}{\mu}f_{\Lambda\Sigma}=16.70f_{\Lambda\Sigma}, \tag{21b}$$

$$g_{\Sigma\Sigma}=\frac{2M_\Sigma}{\mu}f_{\Sigma\Sigma}=17.26f_{\Sigma\Sigma}. \tag{21c}$$

The corresponding relation for the pion-nucleon coupling is well known:

$$g_{NN}=\frac{2M_N}{\mu}f_{NN}=13.60f_{NN}. \tag{21d}$$

These relations have the consequence that there is a distinction between global PS and global PV coupling. Due to the baryon masses appearing in Eq. (21) global PV coupling is not global in the PS sense and vice versa.

4. HYPERON-NUCLEON POTENTIALS

The hyperon-nucleon scattering will be studied by using a potential. Various authors[29] have discussed the problem of constructing the potential corresponding to a field-theoretic Hamiltonian. We shall employ the straightforward prescription of Brueckner and Watson[20] which gives good agreement with the low-energy nucleon-nucleon scattering data.

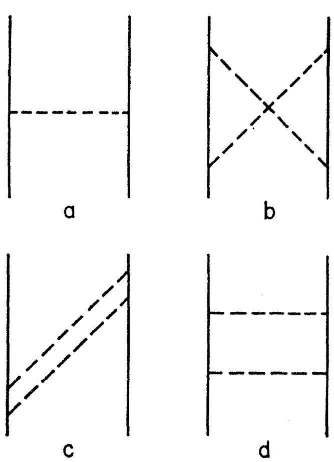

FIG. 1. Graphs included in the Brueckner-Watson potential are shown in (a), (b), and (c). Graph (d), two-meson exchange with an intermediate state of two nucleons only, is omitted.

[29] Reviews can be found in R. J. N. Phillips, *Reports on Progress in Physics* (The Physical Society, London, 1959), Vol. 22, p. 562 and M. J. Moravcsik and H. P. Noyes, Ann. Rev. Nucl. Sci. **11**, 95 (1961).

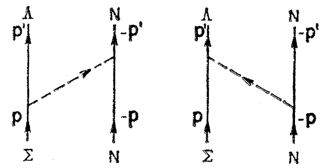

FIG. 2. Second-order contributions to $V_{\Lambda\Sigma}$. The effect of the $\Lambda\Sigma$ mass difference in this graph is discussed in Sec. 4.

We have shown in Sec. 3 that the PS and PV coupling as well as the S and V coupling are equivalent if we neglect terms of higher order in the coupling constants. Thus, we are led to unique nonrelativistic interaction Hamiltonian densities. For even Σ parity:

$$\begin{aligned}\mathcal{H}_{\text{int}}=\frac{f_{NN}}{\mu}(4\pi)^{1/2}(N_1^\dagger\sigma_i\boldsymbol{\tau}N_1)\cdot\nabla_i\boldsymbol{\phi}\\+\frac{f_{\Sigma\Sigma}}{\mu}(4\pi)^{1/2}(Y^\dagger\sigma_i\mathbf{I}Y)\cdot\nabla_i\boldsymbol{\phi}\\+\frac{f_{\Lambda\Sigma}}{\mu}(4\pi)^{1/2}(Y^\dagger\sigma_i\boldsymbol{\varrho}_1Y)\cdot\nabla_i\boldsymbol{\phi},\end{aligned} \tag{22a}$$

for odd Σ parity:

$$\begin{aligned}\mathcal{H}_{\text{int}}=\frac{f_{NN}}{\mu}(4\pi)^{1/2}(N_1^\dagger\sigma_i\boldsymbol{\tau}N_1)\cdot\nabla_i\boldsymbol{\phi}\\+\frac{f_{\Sigma\Sigma}}{\mu}(4\pi)^{1/2}(Y^\dagger\sigma_i\mathbf{I}Y)\cdot\nabla_i\boldsymbol{\phi}\\+g_{\Lambda\Sigma}(4\pi)^{1/2}(Y^\dagger\boldsymbol{\varrho}_1Y)\cdot\boldsymbol{\phi}.\end{aligned} \tag{22b}$$

In these nonrelativistic interaction Hamiltonian densities we have kept only the leading terms corresponding to the three different pion-baryon vertices and neglected all the higher order corrections.

Using these interaction Hamiltonian densities, we compute the scattering matrix in second and fourth order omitting those terms with intermediate states of two baryons only. Thus all graphs of the form shown in Figs. 1(a), 1(b), and 1(c) are included, but those like Fig. 1(d) are omitted. For the nucleon-nucleon case the matrix element of the potential is defined as the static limit of this scattering matrix. By "static limit" is meant the neglect of all kinetic energies of the baryons. The situation is slightly more complicated in the hyperon-nucleon case. Consider the exchange of one pion in the transition $\Sigma+N\leftrightarrow\Lambda+N$. For the two possible time orderings of Fig. 2 we find the contributions to the Fourier transform of the potential

$$V_{\Lambda\Sigma}=-\frac{R}{2\omega}\left(\frac{1}{\omega+\gamma}+\frac{1}{\omega-\gamma}\right)=\frac{-R}{\omega^2-\gamma^2},$$

where

$$\gamma=\frac{p^2}{2M_\Sigma}-\frac{p'^2}{2M_\Lambda}+M_\Sigma-M_\Lambda=\frac{p'^2}{2M_N}-\frac{p^2}{2M_N},$$

$$\omega^2=(\mathbf{p}'-\mathbf{p})^2+\mu^2.$$

Here, **p** and **p**′ are the momenta of each baryon in the center-of-mass frame before and after the collision. R contains all the operators in spin and isospin space, which are the same for both graphs. In the limit that the kinetic energy in the Σ channel is zero we obtain

$$\gamma=\gamma_0=M_\Lambda(M_\Sigma-M_\Lambda)/(M_\Lambda+M_N)=0.30\,\mu.$$

Comparison with the nucleon-nucleon one-pion-exchange potential shows, that the two effects of the mass difference between the initial and final state are:

A $4\frac{1}{2}\%$ increase of the range of the potential from μ^{-1} to μ'^{-1}, where

$$\mu'=\mu[1-(M_\Lambda(M_\Sigma-M_\Lambda)/\mu(M_\Lambda+M_N))^2]^{1/2}=131.6\text{ MeV}.$$

A 14% decrease of the strength of the potential. This is due to a multiplicative factor $(\mu'/\mu)^3=0.86$.

Instead of defining the "static limit" as the scattering at zero ΣN kinetic energy, we could just as well have taken zero ΛN kinetic energy. This gives

$$\gamma_0=M_\Sigma(M_\Sigma-M_\Lambda)/(M_\Sigma+M_N)=0.31\,\mu.$$

Thus, the value μ' for the one pion part of the $V_{\Lambda\Sigma}$ potential is quite well determined, being rather insensitive to our zero-energy reference point. Similar corrections, arising from the $\Lambda\Sigma$ mass difference are present in the fourth-order potentials. We have ignored these, because our recipe for the construction of the potential consistently ignores the recoil kinetic energies of the baryons in all fourth-order processes. Since these recoil energies are presumably of the same order as the mass differences, there is no reason to include one effect and ignore the other.

A straightforward calculation gives then the following results.[30] If the Σ parity is even, then

$$\begin{aligned}V=f_{NN}[&f_{\Sigma\Sigma}\mathbf{I}\cdot\boldsymbol{\tau}V^{(2)}+f_{\Lambda\Sigma}\boldsymbol{\varrho}_1\cdot\boldsymbol{\tau}\,{}^{M}V^{(2)}]\\&+f_{NN}^2[f_{\Sigma\Sigma}^2(\mathbf{I}^2+\mathbf{I}\cdot\boldsymbol{\tau})+f_{\Lambda\Sigma}^2(3-\mathbf{I}^2+\mathbf{I}\cdot\boldsymbol{\tau})\\&+2f_{\Lambda\Sigma}f_{\Sigma\Sigma}\boldsymbol{\varrho}_1\cdot\boldsymbol{\tau}]\,{}^{X}V^{(4)}+f_{NN}^2[f_{\Sigma\Sigma}^2(\mathbf{I}^2-\mathbf{I}\cdot\boldsymbol{\tau})\\&+f_{\Lambda\Sigma}^2(3-\mathbf{I}^2-\mathbf{I}\cdot\boldsymbol{\tau})-2f_{\Lambda\Sigma}f_{\Sigma\Sigma}\boldsymbol{\varrho}_1\cdot\boldsymbol{\tau}]\,{}^{II}V^{(4)}.\end{aligned}\quad(23)$$

Here the contributions of the second-order graphs 1(a) are

$$V^{(2)}=[V_\sigma^{(2)}(x)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+V_T^{(2)}(x)S_{12}]\mu,\quad(24\text{a})$$

$${}^{M}V^{(2)}=[V_\sigma^{(2)}(x')\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+V_T^{(2)}(x')S_{12}](\mu'/\mu)^3\mu.\quad(24\text{b})$$

Here $x=\mu r$ and $x'=\mu' r$. The tensor operator S_{12} is defined as

$$S_{12}=\frac{3(\boldsymbol{\sigma}_1\cdot\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{r})}{r^2}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2.$$

The contributions of the fourth-order crossed graphs 1(b) are

$${}^{X}V^{(4)}=[{}^{X}V_1^{(4)}(x)+{}^{X}V_\sigma^{(4)}(x)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+{}^{X}V_T^{(4)}(x)S_{12}]\mu,\quad(25\text{a})$$

and of the fourth-order uncrossed graphs 1(c) are

$${}^{II}V^{(4)}=[{}^{II}V_1^{(4)}(x)+{}^{II}V_\sigma^{(4)}(x)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+{}^{II}V_T^{(4)}(x)S_{12}]\mu.\quad(25\text{b})$$

The explicit forms of the radial functions will be given in Appendix B.

If the Σ parity is odd, then

$$\begin{aligned}V=f_{NN}[&f_{\Sigma\Sigma}\mathbf{I}\cdot\boldsymbol{\tau}V^{(2)}+g_{\Lambda\Sigma}\boldsymbol{\varrho}_1\cdot\boldsymbol{\tau}U^{(2)}]\\&+f_{NN}^2f_{\Sigma\Sigma}^2[(\mathbf{I}^2+\mathbf{I}\cdot\boldsymbol{\tau})\,{}^{X}V^{(4)}+(\mathbf{I}^2-\mathbf{I}\cdot\boldsymbol{\tau})\,{}^{II}V^{(4)}]\\&+f_{NN}^2g_{\Lambda\Sigma}^2[(3-\mathbf{I}^2+\mathbf{I}\cdot\boldsymbol{\tau})\,{}^{X}U^{(4)}+(3-\mathbf{I}^2-\mathbf{I}\cdot\boldsymbol{\tau})\,{}^{II}U^{(4)}]\\&+f_{NN}^2f_{\Sigma\Sigma}g_{\Lambda\Sigma}\boldsymbol{\varrho}\cdot\boldsymbol{\tau}\,{}^{T}U^{(4)}.\end{aligned}\quad(26)$$

Here

$$U^{(2)}=\left[U_S^{(2)}(x')\frac{\mathbf{S}\cdot\mathbf{r}}{r}+U_D^{(2)}(x')\frac{\mathbf{D}\cdot\mathbf{r}}{r}\right]\left(\frac{\mu'}{\mu}\right)^2\mu,\quad(27\text{a})$$

$$\boldsymbol{\varrho}\cdot\boldsymbol{\tau}\,{}^{T}U^{(4)}=\boldsymbol{\varrho}_1\cdot\boldsymbol{\tau}\left[U_S^{(4)}(x)\frac{\mathbf{S}\cdot\mathbf{r}}{r}+U_D^{(4)}(x)\frac{\mathbf{D}\cdot\mathbf{r}}{r}\right]\mu+\boldsymbol{\varrho}_2\cdot\boldsymbol{\tau}U_Q^{(4)}(x)\frac{\mathbf{Q}\cdot\mathbf{r}}{r}\mu,\quad(27\text{b})$$

$${}^{X}U^{(4)}={}^{X}U_1^{(4)}(x)\mu,\quad(27\text{c})$$

$${}^{II}U^{(4)}={}^{II}U_1^{(4)}(x)\mu.\quad(27\text{d})$$

The explicit forms of the radial functions will be found in Appendix B.

We have introduced here the operators

$$\mathbf{S}=\tfrac{1}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2),\quad(28\text{a})$$

$$\mathbf{D}=\tfrac{1}{2}(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2),\quad(28\text{b})$$

$$\mathbf{Q}=\tfrac{1}{2}(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2).\quad(28\text{c})$$

Because the operators **D** and **Q** are not symmetric in the particles it is necessary to say that we take the hyperon as particle 1 and the nucleon as particle 2.

The complete potential is now given by the above equations for all x outside a repulsive core. The hard core is a phenomenological way of taking our ignorance into account. For either Σ parity we shall assume that the core in the ΣN channel is the same as that in the ΛN channel.

5. EVEN Σ-PARITY POTENTIALS

The most general charge-independent YN potential can be written as

$$V_{YN}=A_1+A_2\mathbf{I}^2+A_3\mathbf{I}\cdot\boldsymbol{\tau}+A_4\boldsymbol{\varrho}_1\cdot\boldsymbol{\tau},\quad(29)$$

where A_1, A_2, A_3, and A_4 are functions of p^2, **r**, **L**, $\boldsymbol{\sigma}_1$, and $\boldsymbol{\sigma}_2$. The $T=3/2$ potential is

$$V_{3/2}=A_1+2A_2+A_3.\quad(30)$$

[30] For completeness we give the momentum space integrals leading to these forms in Appendix A.

In the $T=1/2$ states the potential can convert the $T=1/2$ ΛN state to a $T=1/2$ ΣN state. In terms of the $T=1/2$ isospin wave functions η_Λ and η_Σ, the effect of the potential is

$$\begin{aligned} V\eta_\Lambda &= V_{\Lambda\Lambda}\eta_\Lambda + V_{\Sigma\Lambda}\eta_\Sigma, \\ V\eta_\Sigma &= V_{\Lambda\Sigma}\eta_\Lambda + V_{\Sigma\Sigma}\eta_\Sigma, \end{aligned} \tag{31}$$

where

$$\begin{aligned} V_{\Lambda\Lambda} &= A_1, \\ V_{\Sigma\Lambda} &= V_{\Lambda\Sigma} = -\sqrt{3}A_4, \\ V_{\Sigma\Sigma} &= A_1 + 2A_2 - 2A_3. \end{aligned} \tag{32}$$

If we restrict ourselves only to the second- and fourth-order pion potentials as calculated in Sec. 4, we get

$$\begin{aligned} A_1 &= 3f_{NN}^2 f_{\Lambda N}^2[{}^XV^{(4)} + {}^{II}V^{(4)}], \\ A_2 &= f_{NN}^2(f_{\Sigma\Sigma}^2 - f_{\Lambda\Sigma}^2)[{}^XV^{(4)} + {}^{II}V^{(4)}], \\ A_3 &= f_{NN}f_{\Sigma\Sigma}V^{(2)} + f_{NN}^2(f_{\Sigma\Sigma}^2 + f_{\Lambda\Sigma}^2)[{}^XV^{(4)} - {}^{II}V^{(4)}], \\ A_4 &= f_{NN}f_{\Lambda\Sigma}\,{}^MV^{(2)} + 2f_{NN}^2 f_{\Sigma\Sigma}f_{\Lambda\Sigma}[{}^XV^{(4)} - {}^{II}V^{(4)}]. \end{aligned} \tag{33}$$

For the case of global symmetry it is instructive to compare these YN potentials with the nucleon-nucleon potentials.

The most general charge-independent NN potential can be written as

$$V_{NN} = B_1 + B_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \tag{34}$$

where

$$\begin{aligned} B_1 &= \tfrac{1}{4}(V_0 + 3V_1), \\ B_2 &= \tfrac{1}{4}(V_1 - V_0). \end{aligned} \tag{35}$$

Here V_0 and V_1 are the potentials in the $T=0$ and $T=1$ nucleon-nucleon states.

If we restrict ourselves here to second- and fourth-order pion potentials, as calculated by Brueckner and Watson,[20] then

$$\begin{aligned} B_1 &= 3f_{NN}^4[{}^XV^{(4)} + {}^{II}V^{(4)}], \\ B_2 &= f_{NN}^2V^{(2)} + 2f_{NN}^4[{}^XV^{(4)} - {}^{II}V^{(4)}]. \end{aligned} \tag{36}$$

From a comparison of (33) and (36) we notice that if we assume

(1) global PV coupling ($f_{NN} = f_{\Lambda\Sigma} = f_{\Sigma\Sigma}$),

and

(2) that we may neglect the effect of the $\Lambda\Sigma$ mass difference on the off-diagonal potential (${}^MV^{(2)} = V^{(2)}$), then

$$\begin{aligned} A_1 &= B_1, \\ A_2 &= 0, \\ A_3 &= A_4 = B_2, \end{aligned} \tag{37}$$

and we can relate the pion component of the hyperon-nucleon potential directly to the pion component of the nucleon-nucleon potential. Of course this result (37) is only strictly true for these specific potentials. If recoil effects were taken into account, then the potentials would have a more complicated dependence on the baryon masses. But if instead of assumption 2 we assume that

(2′) We may neglect the mass differences between all the baryons,

then the expressions (37) hold for the general pion contribution to the YN and NN potentials. This is easily shown with the help of the specific form (11) for the interaction lagrangian density.

If in addition to (1) and (2′) we assume that

(3) The potentials are due solely to the pions (we can neglect the influence of the K mesons on both the YN and NN potential, for example),

then we get:

$$\begin{aligned} V_{3/2} &= V_1, \\ V_{\Lambda\Lambda} &= \tfrac{1}{4}(V_0 + 3V_1), \\ V_{\Lambda\Sigma} &= \tfrac{1}{4}\sqrt{3}(V_0 - V_1), \\ V_{\Sigma\Sigma} &= \tfrac{1}{4}(3V_0 + V_1). \end{aligned} \tag{38}$$

These relations were first pointed out by Lichtenberg and Ross.[16] They are quite useful, because one can now use (semi)phenomenological nucleon-nucleon potentials instead of pion theoretic ones to obtain the YN potentials. Quite extensive use of the relations (38) has been made by de Swart and Dullemond[10,18,19] to investigate the YN scattering in the case of global PV coupling.

6. ODD Σ-PARITY POTENTIALS

For odd Σ parity one has the operator $\boldsymbol{\varrho}_2 \cdot \boldsymbol{\tau}\, \mathbf{Q}\cdot\mathbf{r}/r$. Using the relation $(\boldsymbol{\sigma}\cdot\mathbf{a})(\boldsymbol{\sigma}\cdot\mathbf{b}) = \mathbf{a}\cdot\mathbf{b} + i\boldsymbol{\sigma}\cdot(\mathbf{a}\times\mathbf{b})$, we note that we can write

$$-i\frac{\mathbf{Q}\cdot\mathbf{r}}{r} = \frac{\mathbf{D}\cdot\mathbf{r}}{r}\left(\frac{1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{2}\right) = -\left(\frac{1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{2}\right)\frac{\mathbf{D}\cdot\mathbf{r}}{r}.$$

Therefore

$$\varrho_2\cdot\boldsymbol{\tau}\frac{\mathbf{Q}\cdot\mathbf{r}}{r} = \varrho_+\cdot\boldsymbol{\tau}\frac{\mathbf{D}\cdot\mathbf{r}}{r}\left(\frac{1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{2}\right) + \varrho_-\cdot\boldsymbol{\tau}\left(\frac{1+\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{2}\right)\frac{\mathbf{D}\cdot\mathbf{r}}{r}, \tag{39}$$

where

$$\varrho_+ = (\varrho_1 + i\varrho_2)/2, \quad \varrho_- = (\varrho_1 - i\varrho_2)/2.$$

We can rewrite (39) as

$$\varrho_2\cdot\boldsymbol{\tau}\frac{\mathbf{Q}\cdot\mathbf{r}}{r} = \varrho_1\cdot\boldsymbol{\tau}\frac{\mathbf{D}\cdot\mathbf{r}}{r}P, \tag{40}$$

where the operator P has the values $P=1$ if it operates on a triplet ΣN state or a singlet ΛN state and $P=-1$ if it operates on a singlet ΣN state or a triplet ΛN state.

The most general charge-independent YN potential can therefore be written again in the form (29) provided

we write

$$A_4=A_4'+A_4''P. \quad (41)$$

This means that the formulas (30), (31), and (32) are also valid for the odd Σ-parity case. Reference to Sec. 4, Eq. (26), establishes the definition of A_i for the odd Σ-parity case.

The operator $\mathbf{D}$ does not commute with the total spin $\mathbf{S}$, therefore S is not a good quantum number. There is a coupling between the singlet and triplet states. This implies that for a fixed J the two triplet states $l=J\pm1$ in a given hyperon-nucleon channel are coupled to the singlet and triplet $l=J$ states in the other hyperon-nucleon channel. For example, the $^3S_1$ Σ-nucleon state is coupled via the tensor force with the $^3D_1$ Σ-nucleon state and via the operators $\mathbf{S}\cdot\mathbf{r}/r$ and $\mathbf{D}\cdot\mathbf{r}/r$ with the $^3P_1$ and $^1P_1$ Λ-nucleon states.

In general, the Schrödinger equations consist of four coupled second-order differential equations. The sole exception is the $^1S_0$ state which is coupled only to the $^3P_0$ state in the other hyperon-nucleon channel. It is interesting to note, that when we restrict ourselves to static potentials, then $V_{\Lambda\Sigma}$ cannot contain other operators besides the operators $\mathbf{S}\cdot\mathbf{r}/r$, $\mathbf{D}\cdot\mathbf{r}/r$, and $\mathbf{Q}\cdot\mathbf{r}/r$ now considered.

If we take as the order of the channels the singlet state with $l=J$, the triplet state with $l=J$, the triplet state with $l=J-1$, and the triplet state with $l=J+1$, then we can write the operators

$$\frac{\mathbf{S}\cdot\mathbf{r}}{r}=\frac{-1}{(2J+1)^{1/2}}\left\|\begin{matrix}0 & 0 & 0 & 0\\ 0 & 0 & (J+1)^{1/2} & J^{1/2}\\ 0 & (J+1)^{1/2} & 0 & 0\\ 0 & J^{1/2} & 0 & 0\end{matrix}\right\|, \quad (42)$$

and

$$\frac{\mathbf{D}\cdot\mathbf{r}}{r}=\frac{-1}{(2J+1)^{1/2}}\left\|\begin{matrix}0 & 0 & -J^{1/2} & (J+1)^{1/2}\\ 0 & 0 & 0 & 0\\ -J^{1/2} & 0 & 0 & 0\\ (J+1)^{1/2} & 0 & 0 & 0\end{matrix}\right\|. \quad (43)$$

A very special case occurs, when $f_{\Sigma\Sigma}=0$. In this case $V_{\Lambda\Lambda}$ and $V_{\Sigma\Sigma}$ are spin independent, and $V_{\Lambda\Sigma}=(\mathbf{D}\cdot\mathbf{r}-\mathbf{S}\cdot\mathbf{r})V(r)/r$. As is shown in the Appendix C this special combination of operators allows us to simplify the equations; after which it is quite evident that the Λ-nucleon force in the S states is spin independent. To obtain the required spin dependence of the Λ-nucleon potential we have to resort to $f_{\Sigma\Sigma}\neq0$. Of course we could also introduce a spin dependence by requiring that the cores be spin dependent.

7. NUMERICAL RESULTS

From different analyses[8,9] of the light hyperfragments de Swart and Dullemond have estimated the Λ-nucleon scattering lengths. If the Λ-nucleon interaction is due primarily to pion exchange, then the singlet and triplet scattering lengths are[10]

$$a_0=-(3.6_{-1.8}^{+3.6})\ \mathrm{F},$$
$$a_1=(-0.53\pm0.12)\ \mathrm{F}.$$

The pion-nucleon coupling constant is quite well determined from pion-nucleon scattering experiments[13] and

$$f_{NN}^2=0.080\pm0.005.$$

Thus, knowing the scattering lengths, it should be possible to fix the values of the two pion-hyperon coupling constants $f_{\Lambda\Sigma}$ and $f_{\Sigma\Sigma}$. However, in addition to the coupling constants, the potential has another unknown parameter, the hard core radius. For fixed scattering length, variations in the coupling constants can be compensated by a change of the core radius. Of course, this ambiguity is also present in the nucleon-nucleon potentials, but in this case the knowledge of f_{NN}^2 fixes the core radii. We have done the calculations for the nucleon-nucleon problem using these potentials. We find that to fit the experimental scattering lengths,[31] $a_0=-23.74$ F in the $^1S_0$ state and $a_1=5.39$ F in the $^3S_1$ state with $f_{NN}^2=0.080$, we need a core $x_0=0.495$ F ($0.347\ \mu^{-1}$) in the $^1S_0$ state and a core $x_1=0.455$ F ($0.318\ \mu^{-1}$) in the $^3S_1$ state.[32] The corresponding effective ranges are: $r_0=2.35$ F (experimental $r_0=2.670\pm0.023$ F) and $r_1=1.73$ F (experimental $r_1=1.704\pm0.028$ F).[31]

It is very encouraging to discover that the cores required in the $^1S_0$ and $^3S_1$ states are rather similar and of the same order of magnitude as the cores generally used in phenomenological or semiphenomenological analyses of the nucleon-nucleon problem. In these analyses[33] the cores are generally taken in the range $0.3\ \mu^{-1}\lesssim x_c\lesssim 0.4\ \mu^{-1}$.

In all numerical work, the mass of an isomultiplet was taken as the average mass of the components. Our values are

$$m_\pi=138.1\ \mathrm{MeV}/c^2,$$
$$M_N=938.9\ \mathrm{MeV}/c^2,$$
$$M_\Lambda=1115\ \mathrm{MeV}/c^2,$$
$$M_\Sigma=1192\ \mathrm{MeV}/c^2,$$

and therefore $1\ \mu^{-1}=1.4289$ F. For the pion-nucleon coupling constant we happened to choose $f_{NN}=0.28504$ ($f_{NN}^2=0.08125$).

Using these values, we have calculated the potentials up to $5\ \mu^{-1}$. The Schrödinger equation was integrated

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[31] M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Ann. Rev. Nuclear Sci. **10**, 291 (1960).

[32] A measure for the dependence of f_{NN}^2 on the core is given by df^2/dx_c. We have evaluated this at $f_{NN}^2=0.080$ and find

$$df_{NN}^2/dx_0=0.53\ \mathrm{F}^{-1} \text{ for the } {}^1S_0 \text{ state},$$

and

$$df_{NN}^2/dx_1=0.42\ \mathrm{F}^{-1} \text{ for the } {}^3S_1 \text{ state}.$$

[33] The latest proposed semiphenomenological nucleon-nucleon potential uses $x_c=0.35\ \mu^{-1}$ in all states [K. E. Lassila, M. H. Hull, Jr., H. M. Ruppel, F. A. McDonald, and G. Breit, Phys. Rev. **126**, 881 (1962)].

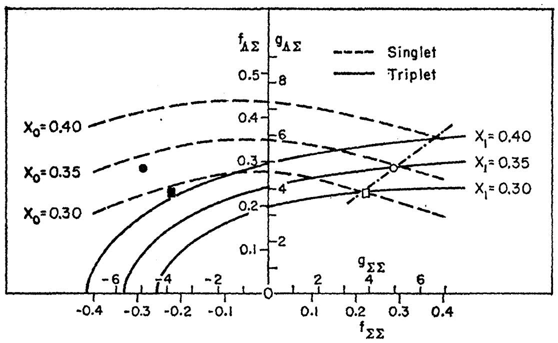

FIG. 3. Results for even Σ parity. Solid lines show $f_{\Lambda\Sigma}$ vs $f_{\Sigma\Sigma}$ required to fit the triplet scattering length for three choices of the hard core (x_1). Dashed lines show the results of fitting the singlet scattering length with three different cores (x_0). (Core radii are given in μ^{-1}.) Dashed-dotted line is $f_{\Lambda\Sigma}$ vs $f_{\Sigma\Sigma}$ for equal cores in singlet and triplet. For a singlet and triplet core of 0.35 μ^{-1} the coupling constants are very nearly those of global PV symmetry (open circle). Global PS symmetry would be possible with a somewhat smaller core (open square). Changing the sign of $f_{\Sigma\Sigma}$ gives the "antiglobal symmetry" values (solid square and circle) which are clearly unfavored.

numerically using the methods outlined by de Swart and Dullemond.[18] We have determined $f_{\Lambda\Sigma}$ as a function of $f_{\Sigma\Sigma}$ subject to the requirement that the scattering length and core radius are fixed. The results are shown in Fig. 3 for even Σ parity and in Fig. 4 for odd Σ parity. The complete numerical results are given in Tables I to IV. The effect of the different cores is illustrated in these figures. To illustrate the effects of the uncertainty in the scattering lengths on our analysis, we have given in Fig. 5 the results for even Σ parity and in Fig. 6 the results for odd Σ parity for the most interesting case $x_0 = x_1 = 0.35\ \mu^{-1}$.

8. DISCUSSION

The main question is: How accurately does this analysis pin down the different coupling constants? The answer depends almost totally on the philosophy which one has about the cores. The introduction of an infinite repulsive, hard core is a phenomenological way of taking into account our ignorance about higher order corrections to the potential (like recoil), to include contributions of heavier mesons (like pion resonances), etc. The presence of these hard cores is now pretty well established in the nucleon-nucleon potentials. We have assumed that they will also be present in the hyperon-nucleon potentials.

TABLE I. Even Σ parity. $f_{\Lambda\Sigma}$ which fit the $^1S_0$ Λ-nucleon scattering length ($a_0 = -3.6$ F) as function of $f_{\Sigma\Sigma}$ and x_0. x_0 in pion Compton wavelengths.

$f_{\Sigma\Sigma}$	$x_0=0.30$	0.35	0.40
0.4	0.177	0.260	0.351
0.3	0.205	0.288	0.379
0.2	0.234	0.316	0.403
0.1	0.262	0.339	0.424
0	0.276	0.352	0.435
−0.1	0.272	0.351	0.439
−0.2	0.249	0.335	0.429
−0.3	0.219	0.309	0.409
−0.4	0.184	0.277	0.384

TABLE II. Even Σ parity. $f_{\Lambda\Sigma}$ which fit the $^3S_1$ Λ-nucleon scattering length ($a_1 = -0.53$ F) as function of $f_{\Sigma\Sigma}$ and x_1. x_1 in pion Compton wavelengths.

$f_{\Sigma\Sigma}$	$x_1=0.25$	0.30	0.35	0.40
0.4		0.240	0.298	0.356
0.285	0.193	0.239	0.290	0.344
0.15		0.224	0.273	0.328
0	0.153	0.197	0.244	0.298
−0.15		0.134	0.191	0.251
−0.285			0.096	0.178
−0.4				0.06
−0.20		0.098		
−0.25		0.034		
−0.317			0.050	
−0.327			0.025	

We have noticed in Sec. 7 that in this kind of approach one needs slightly different cores in the $^1S_0$ and $^3S_1$ nucleon-nucleon states to fit the observed scattering lengths. From a phenomenological analysis of the $T=0$ $^3S_1$ and $T=1$ $^1S_0$ NN states, it is not possible to say whether this difference in the cores arises because of spin-dependent or isospin-dependent effects (if we could calculate these effects we could of course find their explicit dependence on spin and isospin).

Spin dependence or isospin dependence generalizes differently when we consider the hyperon-nucleon system. A spin-dependent core would give different cores in the different spin states, but would still require the same core in the ΛN and ΣN channels. An isospin-dependent core would introduce different cores in the Λ-nucleon and Σ-nucleon channels, but would require the same core in the different spin states.

What is the difference between the cores due to? We can think of two basic reasons. First of all, one can

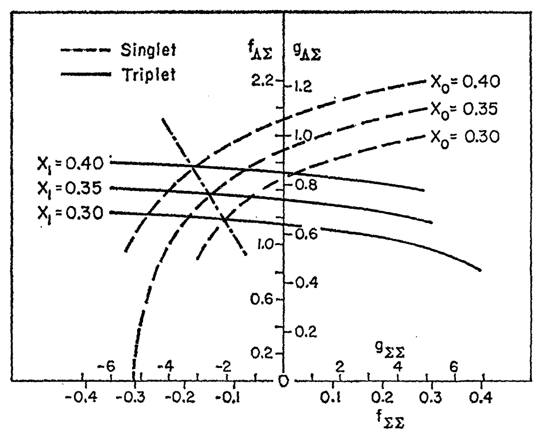

FIG. 4. Results for odd Σ parity. The meaning of the curves is the same as in Fig. 3. Favored values of $f_{\Sigma\Sigma}$ are smaller than f_{NN} and have the opposite sign. The scalar coupling constant $g_{\Lambda\Sigma}$ is smaller than other estimates (see Sec. 8). For the relation between the scalar coupling constant $g_{\Lambda\Sigma}$ and the vector coupling constant $f_{\Lambda\Sigma}$, see Sec. 3, Eq. (21).

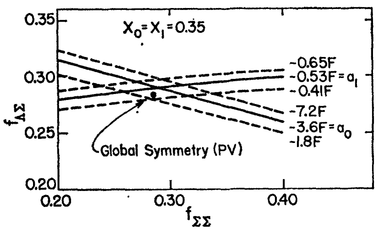

FIG. 5. Sensitivity of our analysis to uncertainties in the scattering lengths for the case of even Σ parity. For a singlet and triplet core of 0.35 μ^{-1} we show a portion of Fig. 3 ($a_1 = -0.53$ F, $a_0 = -3.6$ F). Also shown are the neighboring curves resulting when the scattering lengths are taken at their extreme values. ($-0.41\text{ F} \leqslant a_1 \leqslant -0.65$ F, $-1.8\text{ F} \leqslant a_0 \leqslant -7.2$ F, see the discussion of Sec. 8.) Much greater variation in $f_{\Sigma\Sigma}$ is possible than in $f_{\Lambda\Sigma}$.

assume that these cores are basically the same (for example, due to the same heavier mesons giving spin- and isospin-independent potentials). But, because our calculation of the inner part of the potential (just outside the hard core) is not quite correct (due to recoil corrections, 3π exchange, etc.) we cannot exclude the possibility of small differences between the cores.

The second way of looking at this problem would be to allow the basic mechanisms responsible for the core to be different in the different states (for example, due to heavier mesons giving rise to spin or isospin-dependent potentials). In this case larger differences between the cores would be permissible, because the reasons of the first viewpoint still hold here and therefore could introduce some additional differences between the cores.

From our results and the present knowledge of the N-N problem it is impossible to decide which one of these viewpoints is to be held. However, as we have seen in Sec. 7, the cores required in the $^3S_1$ and $^1S_0$ nucleon-nucleon states are only slightly different.

The simplest assumption about the cores would therefore be that they are the same in the singlet and triplet spin states as well as in the Λ-nucleon and Σ-nucleon channels. In this case we obtain the dashed-dotted line in Fig. 3 and Fig. 4. We notice that in the case of even Σ parity this leads straightforwardly to restricted symmetry. If we make the additional assumption that the core is the same as the commonly accepted mean in the nucleon-nucleon case ($x_0 = x_1 = 0.35\ \mu^{-1}$), then we obtain the solutions, for even Σ parity

$$x_0 = x_1 = 0.35\ \mu^{-1},$$
$$f_{NN} = 0.285 \quad (f_{NN}^2 = 0.08125),$$
$$f_{\Sigma\Sigma} = 0.290 \quad (f_{\Sigma\Sigma}^2 = 0.0841),$$
$$f_{\Lambda\Sigma} = 0.290 \quad (f_{\Lambda\Sigma}^2 = 0.0841),$$

and for odd Σ parity

$$x_0 = x_1 = 0.35\ \mu^{-1},$$
$$f_{NN} = 0.285 \quad (f_{NN}^2 = 0.08125),$$
$$f_{\Sigma\Sigma} = -0.150 \quad (f_{\Sigma\Sigma}^2 = 0.0225),$$
$$g_{\Lambda\Sigma} = 0.763 \quad (g_{\Lambda\Sigma}^2 = 0.578).$$

In the even Σ-parity case this last assumption fixes the coupling constants at practically the values required by global PV symmetry.

If one relaxes the requirement of equal cores, then many more possibilities become available. We did not investigate all these possibilities. In our work we always assumed the same cores in the Λ-nucleon and Σ-nucleon channels. However, we did look into the possibility of different cores in the different spin states. In this case one can get away with the very interesting possibility that $f_{\Sigma\Sigma} = 0$ for either even or odd Σ parity. A drawback

TABLE III. Odd Σ parity. $g_{\Lambda\Sigma}$ which fit the $^1S_0$ Λ-nucleon scattering length ($a_0 = -3.6$ F) as function of $f_{\Sigma\Sigma}$ and x_0. x_0 in pion Compton wavelengths.

$f_{\Sigma\Sigma}$	$x_0 = 0.25$	0.30	0.35	0.40	0.45	0.50
0.4	0.933		1.159		1.403	
0.285	0.896	1.000	1.110	1.225	1.356	1.493
0.2	0.857		1.067		1.316	
0.1	0.800		1.015		1.263	
0	0.718	0.826	0.945	1.069	1.205	1.345
−0.1	0.553		0.839		1.122	
−0.2			0.657		1.015	
−0.285			0.251		0.885	1.083
−0.4					0.564	
−0.12		0.655				
−0.15	0.357					
−0.17				0.890		

TABLE IV. Odd Σ parity. $g_{\Lambda\Sigma}$ which fit the $^3S_1$ Λ-nucleon scattering length ($a_1 = -0.53$ F) as function of $f_{\Sigma\Sigma}$ and x_1. x_1 in pion Compton wavelengths.

$f_{\Sigma\Sigma}$	$x_1 = 0.25$	0.30	0.35	0.40	0.45	0.50
0.4		0.451				1.004
0.285	0.442	0.544	0.662	0.779	0.900	1.028
0.2		0.585				1.045
0.1		0.615				1.064
0	0.550	0.640	0.738	0.845	0.955	1.079
−0.1		0.660				1.094
−0.2		0.676				1.109
−0.285	0.595	0.685	0.783	0.889	1.000	1.118
−0.4		0.696				1.134

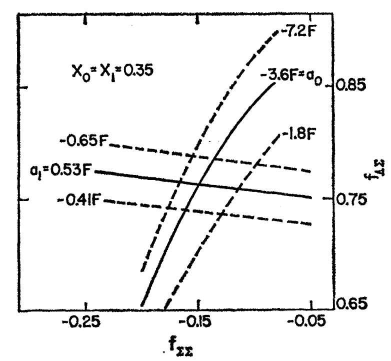

FIG. 6. Sensitivity of our analysis to uncertainties in the scattering lengths for the case of odd Σ parity. The meaning of the curves is exactly the same as in Fig. 5. Singlet and triplet cores are 0.35 μ^{-1}.

to this choice of $f_{\Sigma\Sigma}$ is, that the difference $\Delta x = x_1 - x_0$ between the cores in the $^3S_1$ and $^1S_0$ states has to be about $0.09\ \mu^{-1}$. This difference seems rather large (in the NN case the difference was only $\Delta x = -0.029\ \mu^{-1}$). If $f_{\Sigma\Sigma}=0$ then possible choices for the cores and coupling constants are: for even Σ parity

$$x_0 = 0.31\ \mu^{-1}, \qquad x_1 = 0.39\ \mu^{-1},$$
$$f_{NN} = 0.285 \qquad (f_{NN}^2 = 0.08125),$$
$$f_{\Sigma\Sigma} = 0,$$
$$f_{\Lambda\Sigma} = 0.285 \qquad (f_{\Lambda\Sigma}^2 = 0.08125),$$

and for odd Σ parity

$$x_0 = 0.30\ \mu^{-1}, \qquad x_1 = 0.39\ \mu^{-1},$$
$$f_{NN} = 0.285 \qquad (f_{NN}^2 = 0.08125),$$
$$f_{\Sigma\Sigma} = 0,$$
$$g_{\Lambda\Sigma} = 0.826 \qquad (g_{\Lambda\Sigma}^2 = 0.682).$$

If we require the cores to be of the same order of magnitude as the cores in the NN problem ($0.3\ \mu^{-1} \lesssim x_c \lesssim 0.4\ \mu^{-1}$), then the $\Lambda\Sigma\pi$ coupling constant is rather well established. For even Σ parity we get $f_{\Lambda\Sigma} \sim 0.25$ to 0.35 and for odd Σ parity $g_{\Lambda\Sigma} \sim 0.65$ to 0.85. This means that for even Σ parity the $\Lambda\Sigma\pi$ coupling is about of the same strength as the $NN\pi$ coupling. For odd Σ parity this estimate does not agree with the estimates of this coupling constant made from the $\Lambda\Sigma$ mass difference,[34,35] where the value $g_{\Lambda\Sigma} \sim 1.25$ was obtained.[36]

The coupling constant $f_{\Sigma\Sigma}$ is much less firmly established. We have shown, however, that for even Σ parity $f_{\Sigma\Sigma}$ is surely not large and negative and for odd Σ parity definitely not large and positive. For example with even parity, the case of antiglobal symmetry ($f_{NN} = -f_{\Sigma\Sigma} = \pm f_{\Lambda\Sigma}$) can definitely be ruled out on the basis of these calculations,[37] because this case would require an unreasonably large core ($x_1 \approx 0.5\ \mu^{-1} = 0.7$ F) in the $^3S_1$ state.

9. EFFECTS DUE TO STRONG FORCES IN THE CLOSED CHANNELS

For the moment we will consider only even Σ parity and cores of $0.35\ \mu^{-1}$. We note from Fig. 3 that we cannot find a triplet solution, having no Λ-nucleon bound state, if $f_{\Sigma\Sigma} \leqslant -0.33$. The explanation is that at the values $f_{\Sigma\Sigma} = -0.33$ and $f_{\Lambda\Sigma} = 0$, there exists a $^3S_1 + {}^3D_1$ bound Σ-nucleon state equal in rest mass to $M_\Lambda + M_N$. Under the condition $f_{\Sigma\Sigma} \leqslant -0.33$ any attempt to couple the Λ-nucleon system to the Σ-nucleon system results in a bound state, no matter how weak the $\Lambda\Sigma\pi$ coupling is. That this is the correct explanation why we cannot find a solution can be seen in two ways.

First, because even with a very small $\Lambda\Sigma\pi$ coupling constant we have obtained a bound Λ-nucleon state if $f_{\Sigma\Sigma} \leqslant -0.33$.

Secondly, we can compare the calculations for odd and even Σ parity, because for very small $f_{\Lambda\Sigma}$ the Σ-nucleon forces ($V_{\Sigma\Sigma}$) are the same for either Σ parity. If, therefore, a strongly bound $^3S_1 + {}^3D_1$ Σ-nucleon bound state exists, with rest mass $M_\Lambda + M_N$ when $f_{\Lambda\Sigma} = 0$, then for even Σ parity the $^3S_1 + {}^3D_1$ Λ-nucleon states and for odd parity the $^3P_1 + {}^1P_1$ Λ-nucleon states should show binding at the *same* value of $f_{\Sigma\Sigma}$ and very small values of $f_{\Lambda\Sigma}$. We have checked that this is indeed the case.

For the odd Σ-parity case no singlet solution can be found if $f_{\Sigma\Sigma} \leqslant -0.30$. Here we have a $^3P_0$ Σ-nucleon bound state with rest mass $M_\Lambda + M_N$ if $f_{\Sigma\Sigma} = -0.30$ and $g_{\Lambda\Sigma} = 0$; therefore for even Σ-parity the $^3P_0$ Λ-nucleon state should be bound for very small $f_{\Lambda\Sigma}$ and the same value of $f_{\Sigma\Sigma}$ ($f_{\Sigma\Sigma} = -0.30$). We have also verified this.

We notice from the foregoing discussion that if $f_{\Lambda\Sigma} = 0$, then there exists a $^3P_0$ bound state with a larger binding energy than the $^3S_1$ bound state. This can be inferred from the smaller absolute value of $f_{\Sigma\Sigma}$ required for a $^3P_0$ bound state of rest mass $M_\Lambda + M_N$, than required for a $^3S_1 + {}^3D_1$ bound state of the same rest mass. This situation is *not* in contradiction with a theorem about the ordering of levels in a central potential,[38] which would require the S state to be the lowest bound state. This because our potential is not central at all, but the tensor force in $V_{\Sigma\Sigma}$ is the most important one.

A numerical evaluation of Eq. (23) shows, that for $f_{\Lambda\Sigma} = 0$ the most important potential in the Σ-nucleon system is the second-order tensor force. The sign of the second-order tensor potential is opposite to the sign of the $f_{\Sigma\Sigma}$ coupling constant. Since the fourth-order tensor potential[39] is always positive, it cancels part of the second-order tensor potential for positive $f_{\Sigma\Sigma}$ and adds to the second-order term for negative $f_{\Sigma\Sigma}$. Because $\langle S_{12} \rangle = -4$ for a $^3P_0$ state, the tensor force will give a large attraction in this state if the tensor potential is positive, therefore if $f_{\Sigma\Sigma} < 0$. For the $^3S_1$ state $\langle S_{12} \rangle = 0$ and the tensor force supplies the attraction only via the coupling with the $^3D_1$ state, which is, of course, much less effective than the direct term in the $^3P_0$ state.

10. TOTAL CROSS SECTIONS

The earlier attempts to fit the high-energy nucleon-nucleon scattering data ($E_{\text{lab}} = 100$ to 300 MeV) with static potentials were unsuccessful.[40] Static pion theoretic potentials (like the Gartenhaus potential[41]) which did fit the low-energy data failed to reproduce the high-

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[34] Y. Nambu and J. J. Sakurai, Phys. Rev. Letters **6**, 377 (1961).

[35] J. Bernstein and R. Oehme, Phys. Rev. Letters **6**, 639 (1961).

[36] To fit the triplet scattering length with $g_{\Lambda\Sigma} \simeq 1.25$ would require a core $x_1 \simeq 0.55\ \mu^{-1} \simeq 0.8$ F, which is unreasonably large.

[37] Other arguments are given in references (10) and (19).

[38] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1953), Appendix 1.

[39] In the $T=1/2$ potential, $V_{\Sigma\Sigma}$, only uncrossed diagrams [Fig. 1(c)] contribute if $g_{\Lambda\Sigma} = 0$.

[40] J. Gammel, R. Christian, and R. Thaler, Phys. Rev. **105**, 311 (1957).

[41] S. Gartenhaus, Phys. Rev. **100**, 900 (1955).

energy data. Signell and Marshak[42] overcame this difficulty by adding a phenomenological spin-orbit potential[43] ($\mathbf{L}\cdot\mathbf{S}$) to the Gartenhaus potential. Field-theoretic $\mathbf{L}\cdot\mathbf{S}$ potentials have been proposed by several authors.[44–50,7] It is now generally accepted that spin-orbit terms are present in the nucleon-nucleon potentials. It can therefore be expected that spin-orbit potentials will also be important for higher energies in the hyperon-nucleon system. For this reason our predictions of the total cross section are probably not reliable for higher energies, although they should be essentially correct at low energies.

We have calculated the total cross section as a function of energy for the case of equal cores $x_0=x_1=0.35\ \mu^{-1}$. The most important phase shifts for the even Σ-parity case with $f_{NN}=0.285$ and $f_{\Sigma\Sigma}=f_{\Lambda\Sigma}=0.290$ are given in Table V, for the odd Σ-parity case with

TABLE V. Even Σ parity. Phase shifts and coupling parameters for the Λ-nucleon scattering. $E_{\rm lab}$ in MeV. Phase shifts and coupling parameters in degrees.

$E_{\rm lab}$	$^1S_0$	$^1P_1$	$^1D_2$	$^3P_0$	$^3P_1$	$^3D_2$
30	41.4	3.2	...	3.8	4.3	...
60	37.0	8.2	...	8.8	11.2	...
100	31.1	15.2	1.1	14.4	22.1	1.7
130	27.5	20.1	1.9	17.4	31.3	2.9
160	25.0	24.5	2.8	19.6	43.7	4.4
167	25.0	25.5	3.0	20.1	48.4	4.8
$E_{\rm lab}$	$^3S_1$	ϵ_1	$^3D_1$	$^3P_2$	ϵ_2	$^3F_2$
30	9.0	2.9	...	1.9	...	...
60	6.8	14.0	1.0	4.2	...	...
100	7.9	50.8	...	6.6	...	...
130	16.2	58.0	−2.7	7.8	...	...
160	52.9	55.4	−5.0	8.5	3.1	2.0
167	86.9	53.0	−5.4	8.7	4.5	2.5
168	96.7	52.3	−5.4			

$f_{NN}=0.285$, $f_{\Sigma\Sigma}=-0.150$, and $g_{\Lambda\Sigma}=0.763$ are given in Table VI.

The total cross section σ_T can be expressed in terms of the partial cross sections $\sigma(l,S,J)$ as

$$\sigma_T=\sum_{l,S,J}\sigma(l,S,J), \tag{44}$$

TABLE VI. Odd Σ parity. Phase shifts and coupling parameters for the Λ-nucleon scattering. $E_{\rm lab}$ in MeV. Phase shifts and coupling parameters in degrees.

$E_{\rm lab}$	$^1S_0$	$^1P_1$	ϵ_P	$^3P_1$	$^1D_2$	ϵ_D	$^3D_2$
30	38.3	7.3	34.1	3.4	...	...	...
60	33.7	18.7	33.5	7.1	1.4	43.2	...
100	28.8	34.9	33.1	10.5	3.9	42.0	2.0
130	26.8	47.2	33.0	12.1	6.9	41.7	2.9
160	26.7	62.3	33.4	13.2	11.7	41.6	3.9
167	27.4	68.6	33.9	13.4	13.5	41.6	4.1
$E_{\rm lab}$	$^3P_0$	$^3S_1$	ϵ_1	$^3D_1$	$^3P_2$	$^3F_2$	$^3D_3$
30	4.4	6.0	...	...	2.9	...	...
60	9.8	1.5	...	...	5.7	...	...
100	16.3	−5.0	−2.5	3.2	8.1	...	1.8
130	21.1	−9.3	−1.8	5.3	9.0	...	2.7
160	28.6	−13.0	−1.3	8.4	9.3	1.5	3.5
167	33.3	−13.8	−1.1	9.5	9.4	1.7	3.7

where[51]

$$\sigma(l,S,J)=(\pi/k^2)(2J+1)\,\mathrm{Im}T_{ll}(S,J), \tag{45}$$

with $T=(S-1)/2i$.

The total cross section and the most important partial cross sections are given in Table VII for the even Σ-parity case and in Table VIII for the odd Σ-parity case. The cross section is determined experimentally in the energy region from 67 MeV to the Σ threshold by Alexander *et al.*[15] They find $\sigma_T=25\pm10$ mb; for the same energy interval, we find $\sigma_T=39$ mb for even Σ parity and $\sigma_T=45$ mb for odd Σ parity. The calculated cross sections are too large for both parities, however, due to the large experimental error no definite conclusion can be drawn.

TABLE VII. Even Σ parity. Partial and total cross sections for the Λ-nucleon scattering. $E_{\rm lab}$ is in MeV. Cross section in mb. σ_0 is singlet cross section. σ_1 is triplet cross section. σ_T is total cross section.

$E_{\rm lab}$	$^1S_0$	$^1P_1$	$^3S_1$	$^3D_1$	$^3P_1$	σ_T	σ_0	σ_1
30	37.4	...	6.3	...	1.4	47	153	11
60	15.6	2.6	1.7	...	4.9	27	73	12
100	6.9	5.3	...	...	11.0	28	49	21
130	4.2	7.0	1.4	3.3	16.1	36	45	33
160	2.9	8.3	10.1	20.9	23.1	70	45	78
167	2.7	8.5	16.9	29.5	25.8	88	46	102

TABLE VIII. Odd Σ parity. Partial and total cross section for the Λ-nucleon scattering. $E_{\rm lab}$ in MeV. Cross section in mb.

$E_{\rm lab}$	$^1S_0$	$^1P_1$	$^3P_1$	$^3P_0$	$^3S_1$	σ_T
30	32.9	3.1	1.9	...	2.8	42
60	13.2	9.7	5.4	1.2	...	32
100	6.0	18.5	9.3	2.0	...	40
130	4.0	23.3	11.3	2.6	1.5	48
160	3.2	27.1	13.2	3.7	2.5	57
167	3.3	28.4	14.2	4.6	2.6	62

[42] P. S. Signell and R. E. Marshak, Phys. Rev. **106**, 832 (1957); **109**, 1229 (1958).

[43] Simultaneously with Signell and Marshak, J. L. Gammel and R. M. Thaler, Phys. Rev. **107**, 291 and 1337 (1957); **108**, 163 (1957), also introduced a phenomenological spin-orbit potential. In contrast to Signell and Marshak, they used phenomenological central and tensor potentials.

[44] S. Sato, K. Itabaski, and I. Sato, Progr. Theoret. Phys. (Kyoto) **14**, 303 (1955).

[45] R. E. Marshak and S. Okubo, Ann. Phys. (New York) **4**, 166 (1958).

[46] A. Klein, R. Raphael, and N. Tzoar, Phys. Rev. Letters **2**, 433 (1959).

[47] M. Sugawara and S. Okubo, Phys. Rev. **117**, 605, 611 (1960).

[48] C. K. Iddings and P. M. Platzmann, Phys. Rev. **120**, 644 (1960).

[49] S. N. Gupta, Phys. Rev. Letters **2**, 124 (1959).

[50] G. Breit, Proc. Natl. Acad. Sci. U. S. **46**, 746 (1960).

[51] We note that this definition (45) of the partial cross sections is unique, even in the case of coupled channels.

In the even Σ-parity case a comparison with earlier calculations[18,19] of de Swart and Dullemond is interesting. They also used global PV symmetry, but made use of the relations (38) to construct the YN potentials from semiphenomenological NN potentials. The most important point is that their YN potentials differ from our pion-theoretic ones mainly through the presence of spin-orbit potentials. They find a total cross section (averaged over the same energy interval) $\sigma_T=25.5$ mb for the Bryan-Gartenhaus-Signell-Marshak potential and $\sigma_T=20$ mb for the Hamada potential. This is considerably lower than the values we find. Their better agreement with the experimental values could perhaps be considered an indication of the presence of spin-orbit potentials as we will show below.

Let us consider our results for even Σ parity in more detail. From Table VII we notice that 30 to 40% of the total cross section is contained in the $^3P_1$ partial wave. Only near Σ threshold, do the $^3S_1$ and $^3D_1$ partial waves contribute significantly. Agreement with experiment would be considerably improved if these three partial cross sections (especially the $^3P_1$) could somehow be reduced. The analysis of Sec. 9 shows that the tensor force gives a strong attraction in the $^3P_1$ Σ-nucleon channel for positive $f_{\Sigma\Sigma}$. The coupling of the $^3D_1$ Λ-nucleon state with the $^3S_1$ Σ-nucleon state via the tensor part of $V_{\Lambda\Sigma}$ should enhance the scattering in the $^3S_1+^3D_1$ Λ-nucleon partial waves near the Σ-nucleon threshold [in fact, we find a resonance in these states between $E_{\rm lab}=167$ and 168 MeV (cf. Table V), just below the Σ threshold].

An attractive spin-orbit potential[52] in $V_{\Sigma\Sigma}$ and/or $V_{\Lambda\Lambda}$ would result in an effective repulsion in the $^3P_1$ and $^3D_1$ states.

De Swart and Dullemond included such potentials in their calculations[18,19] and obtained much smaller values for the $^3P_1$, $^3S_1$, and $^3D_1$ partial cross sections. One can also see from their results that such spin-orbit potentials drastically reduce the $^3P_0$ phase shift (even changing its sign) and slightly enhance the $^3P_2$ phase shift.

For the singlet states, our potential is slightly more attractive than those used by de Swart and Dullemond (we have fitted a slightly larger scattering length than they obtain[10]) and our phase shifts are slightly larger.

As we noted above, there is a resonance in the $^3S_1+^3D_1$ states, just below the Σ threshold. Because the unitarity limit $3\pi/k^2$ for $J=1$ partial waves is not large at this resonance, this does not have much effect on the total cross section. It appears as a large cusp at the Σ threshold. Since the resonance is so close to the Σ threshold, its position is sensitive to small changes in the potential. For example, de Swart and Dullemond, using a slightly larger core and an attractive spin-orbit potential (in both $V_{\Lambda\Lambda}$ and $V_{\Sigma\Sigma}$) find no resonance below the Σ threshold. Traces of this resonance still survive, however; the diagonal element in the ΣN channel of the scattering length matrix is large and negative.

Inspection of Table VIII shows that in the odd Σ-parity case about 60–70% of the total cross section is due to the $^1P_1+^3P_1$ partial cross sections. Again the addition of an attractive spin-orbit potential to $V_{\Lambda\Lambda}$ or $V_{\Sigma\Sigma}$ would reduce these phase shifts. Of course velocity dependent terms (like $\mathbf{S}\cdot\mathbf{p}$, $\mathbf{D}\cdot\mathbf{p}$, etc.) may be expected in the $\Lambda\Sigma$ potential too.

As shown in Sec. 9 large negative values of $f_{\Sigma\Sigma}$ can give rise to bound states in the ΛN system. This is due to the strong attractive forces in the ΣN channel. Although $f_{\Sigma\Sigma}$ used here ($f_{\Sigma\Sigma}=-0.150$) is not as strongly negative as the values considered in Sec. 9, there is the possibility that some trace of the bound states remain; i.e., that there are resonances in the ΛN scattering below Σ threshold. That this is not the case may be seen from the phase shifts in Table VI.

ACKNOWLEDGMENTS

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APPENDIX A

For completeness, we will give here the momentum space integrals corresponding to the potentials of Sec. 4.

The second-order potentials corresponding to Fig. 1(a) are

$$V^{(2)}=-\frac{\mu}{2\pi^2}\int d^3k\, e^{i\mathbf{k}\cdot\mathbf{x}}\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})}{\omega^2},$$

$$^{M}V^{(2)}=-\frac{\mu}{2\pi^2}\left(\frac{\mu'}{\mu}\right)^3\int d^3k\, e^{i\mathbf{k}\cdot\mathbf{x}}\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})}{\omega^2-\Delta^2},$$

$$U^{(2)}=\frac{\mu}{2\pi^2}\left(\frac{\mu'}{\mu}\right)^2\int d^3k\, e^{i\mathbf{k}\cdot\mathbf{x}}\frac{i\boldsymbol{\sigma}_2\cdot\mathbf{k}}{\omega^2-\Delta^2},$$

where $\omega^2=k^2+1$ and $\Delta=\gamma_0/\mu=0.30$.

The fourth-order potentials corresponding to the crossed diagrams Fig. 1(b) are

$$^{X}V^{(4)}=\frac{\mu}{8\pi^4}\int d^3kd^3k'\, e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}}\frac{\omega^2+\omega'^2+\omega\omega'}{\omega^3\omega'^3(\omega+\omega')}A(\mathbf{k},\mathbf{k}'),$$

$$^{X}U^{(4)}=\frac{\mu}{8\pi^4}\int d^3kd^3k'\, e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}}\frac{\omega^2+\omega'^2+\omega\omega'}{\omega^3\omega'^3(\omega+\omega')}B(\mathbf{k},\mathbf{k}'),$$

$$^{X}W^{(4)}=\frac{\mu}{8\pi^4}\int d^3kd^3k'\, e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}}\frac{\omega^2+\omega'^2+\omega\omega'}{\omega^3\omega'^3(\omega+\omega')}C(\mathbf{k},\mathbf{k}'),$$

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[52] By an attractive spin-orbit potential we mean an $\mathbf{L}\cdot\mathbf{S}$ potential which is attractive in the states $J=(l+1)$.

where

$$A(\mathbf{k},\mathbf{k}')=-(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_1\cdot\mathbf{k}')(\boldsymbol{\sigma}_2\cdot\mathbf{k}')(\boldsymbol{\sigma}_2\cdot\mathbf{k}),$$
$$B(\mathbf{k},\mathbf{k}')=(\boldsymbol{\sigma}_2\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k}'),$$
$$C(\mathbf{k},\mathbf{k}')=2\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_+i(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k}')+2\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_-i(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k}')(\boldsymbol{\sigma}_2\cdot\mathbf{k}).$$

The fourth-order potentials corresponding to the uncrossed diagrams of Fig. 1(c) are

$${}^{\mathrm{II}}V^{(4)}=\frac{\mu}{8\pi^4}\int d^3kd^3k'\,e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}}\frac{D(\mathbf{k},\mathbf{k}')}{\omega^2\omega'^2(\omega+\omega')},$$
$${}^{\mathrm{II}}U^{(4)}=\frac{\mu}{8\pi^4}\int d^3kd^3k'\,e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}}\frac{B(\mathbf{k},\mathbf{k}')}{\omega^2\omega'^2(\omega+\omega')},$$
$${}^{\mathrm{II}}W^{(4)}=\frac{\mu}{8\pi^4}\int d^3kd^3k'\,e^{i(\mathbf{k}+\mathbf{k}')\cdot\mathbf{x}}\frac{E(\mathbf{k},\mathbf{k}')}{\omega^2\omega'^2(\omega+\omega')},$$

where

$$D(\mathbf{k},\mathbf{k}')=-(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_1\cdot\mathbf{k}')(\boldsymbol{\sigma}_2\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k}'),$$
$$E(\mathbf{k},\mathbf{k}')=-2\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_+i(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k}')(\boldsymbol{\sigma}_2\cdot\mathbf{k})-2\boldsymbol{\tau}\cdot\boldsymbol{\varrho}_-i(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k}').$$

The off-diagonal potential $\boldsymbol{\varrho}\cdot\boldsymbol{\tau}\;{}^TU^{(4)}$ is given by

$$\boldsymbol{\varrho}\cdot\boldsymbol{\tau}\;{}^TU^{(4)}={}^XW^{(4)}+{}^{\mathrm{II}}W^{(4)}.$$

APPENDIX B

The radial functions describing the potentials are

$$V_\sigma^{(2)}(x)=\frac{1}{3x}e^{-x},$$
$$V_T^{(2)}(x)=\frac{1}{3}\left(\frac{1}{x}+\frac{3}{x^2}+\frac{3}{x^3}\right)e^{-x},$$
$$U_D^{(2)}(x)=\left(\frac{1}{x}+\frac{1}{x^2}\right)e^{-x},$$
$$U_S^{(2)}(x)=-U_D^{(2)}(x),$$
$${}^XV_1^{(4)}(x)=\frac{-1}{2\pi}\left[\left(\frac{12}{x^2}+\frac{23}{x^4}\right)K_1(2x)+\left(\frac{4}{x}+\frac{23}{x^3}\right)K_0(2x)\right],$$
$${}^XV_\sigma^{(4)}(x)=\frac{4}{3\pi}\left[\left(\frac{2}{x^2}+\frac{3}{x^4}\right)K_1(2x)+\frac{3}{x^3}K_0(2x)\right],$$
$${}^XV_T^{(4)}(x)=\frac{-1}{3\pi}\left[\left(\frac{4}{x^2}+\frac{15}{x^4}\right)K_1(2x)+\frac{12}{x^3}K_0(2x)\right],$$
$${}^XU_1^{(4)}(x)=\frac{-1}{\pi}\left[\frac{3}{x^2}K_1(2x)+\frac{2}{x}K_0(2x)\right],$$
$${}^{\mathrm{II}}V_1^{(4)}(x)=-{}^XV_1^{(4)}(x)-\frac{2}{\pi}\left[\left(\frac{1}{x^2}+\frac{4}{x^3}+\frac{4}{x^4}\right)e^{-x}K_1(x)+\left(\frac{1}{x}+\frac{2}{x^2}+\frac{2}{x^3}\right)e^{-x}K_0(x)\right],$$
$${}^{\mathrm{II}}V_\sigma^{(4)}(x)={}^XV_\sigma^{(4)}(x)-\frac{4}{3\pi}\left[\left(\frac{1}{x^2}+\frac{2}{x^3}+\frac{2}{x^4}\right)e^{-x}K_1(x)+\left(\frac{1}{x^2}+\frac{1}{x^3}\right)e^{-x}K_0(x)\right],$$
$${}^{\mathrm{II}}V_T^{(4)}(x)={}^XV_T^{(4)}(x)+\frac{2}{3\pi}\left[\left(\frac{1}{x^2}+\frac{5}{x^3}+\frac{5}{x^4}\right)e^{-x}K_1(x)+\left(\frac{1}{x^2}+\frac{1}{x^3}\right)e^{-x}K_0(x)\right],$$
$${}^{\mathrm{II}}U_1^{(4)}(x)=-{}^XU_1^{(4)}(x)=\frac{2}{\pi}\left(\frac{1}{x}+\frac{1}{x^2}\right)e^{-x}K_1(x),$$
$$U_D^{(4)}(x)=-\frac{2}{\pi}\left[\left(\frac{1}{x}+\frac{3}{x^2}+\frac{3}{x^3}\right)\times e^{-x}K_1(x)+\left(\frac{1}{x}+\frac{1}{x^2}\right)e^{-x}K_0(x)-\left(\frac{4}{x}+\frac{9}{x^3}\right)K_1(2x)-\frac{8}{x^2}K_0(2x)\right],$$
$$U_S^{(4)}(x)=U_D^{(4)}(x),$$
$$U_Q^{(4)}(x)=-\frac{8}{\pi}\left(\frac{1}{x^2}+\frac{1}{x^3}\right)e^{-x}K_1(x),$$

where we have used the functions $K_n(x)$ defined by

$$K_n(x)=\frac{\Gamma(n+\frac{1}{2})}{\sqrt{\pi}}\left(\frac{2}{x}\right)^n\int_0^\infty\frac{\cos kx}{(k^2+1)^{n+\frac{1}{2}}}dk.$$

APPENDIX C. THE CASE $f_{\Sigma\Sigma}=0$ FOR ODD Σ PARITY

In the special case that $f_{\Sigma\Sigma}=0$ we notice from Eqs. (26) and (27) that $V_{\Lambda\Lambda}$ and $V_{\Sigma\Sigma}$ are spin independent if the Σ parity is odd. The off-diagonal potential $V_{\Lambda\Sigma}$ has the specific form

$$V_{\Lambda\Sigma}=\left(\frac{\mathbf{D}\cdot\mathbf{r}}{r}-\frac{\mathbf{S}\cdot\mathbf{r}}{r}\right)b,\qquad\text{(C1)}$$

where b is a function of r only.

If we take the order of the states as ${}^1J_J$, ${}^3J_J$, ${}^3(J-1)_J$, and ${}^3(J+1)_J$, then the Schrödinger equation takes on the special form

$$D\psi\equiv\begin{pmatrix}A & B\\ B^T & C\end{pmatrix}\psi=0.\qquad\text{(C2)}$$

Notice that this result does not depend upon which channels refer to the Λ-nucleon and which to the Σ-nucleon system. In (C2) A is a diagonal operator, with equal diagonal elements;

$$A=\begin{pmatrix} a_J & 0 \\ 0 & a_J \end{pmatrix}.$$

C is also a diagonal operator; however, due to the centrifugal barrier the diagonal elements are unequal:

$$C=\begin{pmatrix} C_{J-1} & 0 \\ 0 & C_{J+1} \end{pmatrix}.$$

The off-diagonal element B has the form

$$B=\frac{b}{(2J+1)^{1/2}}\begin{pmatrix} J^{1/2} & -(J+1)^{1/2} \\ -(J+1)^{1/2} & -J^{1/2} \end{pmatrix}.$$

We now perform a unitary transformation

$$u=\begin{pmatrix} U & 0 \\ 0 & 1 \end{pmatrix}$$

on ψ, and the transformed D' is

$$D'=uDu^{-1}=\begin{pmatrix} A & UB \\ B^T U^{-1} & C \end{pmatrix}.$$

It is possible to choose U such that UB becomes diagonal. Choose

$$U=\frac{1}{(2J+1)^{1/2}}\begin{pmatrix} -J^{1/2} & (J+1)^{1/2} \\ (J+1)^{1/2} & J^{1/2} \end{pmatrix},$$

TABLE IX. Λ-nucleon states (for the case of odd Σ parity, $f_{\Sigma\Sigma}=0$, and equal cores) which have the same eigen-phase shifts are listed in the same row. A double entry, e.g., ${}^1P_1$, ${}^3P_1$, refers to the mixture of these states required to diagonalize B (Appendix C).

ΛN channel	{Associated ΣN channel}	Same phase as ΛN channel	{Associated ΣN channel}
${}^1S_0$	$\{{}^3P_0\}$	${}^3S_1$	$\{{}^1P_1, {}^3P_1\}$
${}^1P_1$, ${}^3P_1$	$\{{}^3S_1\}$	${}^3P_0$	$\{{}^1S_0\}$
${}^1P_1$, ${}^3P_1$	$\{{}^3D_1\}$	${}^3P_2$	$\{{}^1D_2, {}^3D_2\}$
${}^1J_J$, ${}^3J_J$	$\{{}^3(J-1)_J\}$	${}^3J_{J-1}$	$\{{}^1(J-1)_{J-1}, {}^3(J-1)_{J-1}\}$
${}^1J_J$, ${}^3J_J$	$\{{}^3(J+1)_J\}$	${}^3J_{J+1}$	$\{{}^1(J+1)_{J+1}, {}^3(J+1)_{J+1}\}$

then

$$UB=\begin{pmatrix} -b & 0 \\ 0 & -b \end{pmatrix},$$

and the four coupled differential equations are split into two sets of two coupled differential equations.

The state ${}^3(J-1)_J$ is now coupled to a specific mixture of ${}^1J_J$ and ${}^3J_J$; the state ${}^3(J+1)_J$ is coupled to a mixture of ${}^1J_J$ and ${}^3J_J$ orthogonal to the first mixture. If we use the same cores in all states, then the result is, that there is a degeneracy; certain pairs of phase shifts are equal. This is shown in Table IX. We note from this table that the ${}^1S_0$ and ${}^3S_1$ Λ-nucleon phase shifts are equal; i.e., the potential in the S states is spin independent. In the states with $l\neq 0$ there are two independent phase shifts; i.e., in these states the potential is spin dependent. For example, in the P states of the Λ-nucleon system there are two phase shifts which are nonequal. These are the ${}^3P_0$ and the ${}^3P_2$ phases. There are two eigenphases for the coupled ${}^3P_1$ and ${}^1P_1$ states, these are equal to the ${}^3P_0$ and the ${}^3P_2$ phase shifts.