Interpretation of ρ and ζ Mesons

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The ρ and ζ mesons are assumed to have the same quantum numbers, $I=J=1$. The P-wave pion-pion scattering is then studied in the hope that the two resonances, ρ and ζ, can be ascribed to the existence of one unstable vector meson B of isospin one. Besides the process $2\pi \rightleftarrows B$, a strong attractive interaction between pions in the $I=J=1$ state, which is caused by the exchange of B between pions and also by baryon loops, etc., is considered. The pion-pion scattering amplitude is calculated by the chain-diagram approximation and two resonances, which can be identified with the ρ and ζ mesons, are obtained. The two resonances in the kaon-pion system are discussed on the same lines.

I. INTRODUCTION

IN addition to the well-established resonance in the $I=J=1$ pion-pion scattering, called ρ meson, some evidence has been found for the existence of another $I=1$ resonance, called ζ meson.[1,2] The spin and parity of the ζ meson is unknown as yet. In this paper we assume that the ρ and ζ mesons have the same quantum numbers. From a theoretical point of view, one may say that such a resonance is either dynamical or kinematical in nature. By kinematical we mean that the resonance is due to the existence of a corresponding unstable elementary particle. There are, therefore, three possible interpretations of the two resonances, involving zero, one, or two elementary mesons in the $I=J=1$ state. Although such particles which underlie resonances could not directly be observed, they would be significant in an attempt to construct a more fundamental theory of elementary particles.

Among the three possibilities, we are interested in the second interpretation in which one $I=J=1$ meson, hereafter called B, is postulated. To visualize the mechanism of the resonances, we make explicit appeal to the conventional method. An equivalent treatment in terms of dispersion relations will also be discussed. In addition to the process $2\pi\rightleftarrows B$, we shall consider a strong attractive interaction between pions in the $I=J=1$ state, which is caused by the exchange of B between pions and also by baryon loops, etc. The P-wave pion-pion scattering amplitude is calculated by the chain-diagram approximation and two resonances, which can be identified with the ρ and ζ mesons, are obtained. This interpretation is simpler than the other two. In the first interpretation, in order to obtain the two dynamical resonances, we must be involved in considerable complication because the familiar effective range approach can give only one resonance. The third interpretation is less interesting simply because we wish to assume as few elementary particles as possible.

In the kaon-pion system, a similar situation seems to exist. In addition to the resonance K^* at about 880 MeV, evidence for a new resonance in the kaon-pion scattering at about 730 MeV has been reported.[3] These two resonances are discussed on the same lines.

II. PION-PION SCATTERING

We assume that the vector meson B interacts with pions through

$$H_{\mathrm{I}}=f\sum \epsilon_{\alpha\beta\gamma}\phi_{\mu,\alpha}(\partial_\mu\varphi_\beta)\varphi_\gamma, \tag{2.1}$$

ϕ_μ and φ are the B and pion field operators, respectively. The isospins are specified by indices α, β, and γ. We shall calculate the P-wave pion-pion scattering amplitude by using the chain-diagram approximation. Two links of a chain may be joined by diagrams of various kinds. We treat the process $2\pi\rightarrow B\rightarrow 2\pi$ (diagram I in Fig. 1) separately from all other possible processes, which are lumped as II in Fig. 1. Among the latter we notice the exchange of B between pions and baryon loops (Fig. 2). The exchange of B, which gives rise to the so-called bootstrap mechanism,[4] produces an attraction in the $I=J=1$ state. It has been shown using perturbation theory that baryon loops also give a strong attraction in this state.[5,6] We may assume, therefore,

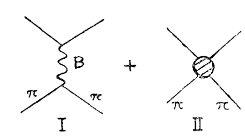

FIG. 1. Diagrams for the π-π interaction.

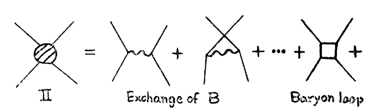

FIG. 2. Diagrams for the interaction II.

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[1] R. Barloutaud, J. Heughebaert, A. Leveque, J. Meyer, and R. Omnes, Phys. Rev. Letters **8**, 32 (1962); B. Sechi Zorn, *ibid.* **8**, 282, 386 (1962). Further references are found in these papers.

[2] After submitting this paper, we learned that the spin of the ζ meson is more likely to be even than odd [E. A. Everett, Phys. Rev. Letters **9**, 74 (1962)]. On the other hand, evidence has been reported for two new resonances in the π^+-π^- system [N. P. Samios, A. H. Bachman, R. M. Lea, T. E. Kalogeropoulos, and W. D. Shephard, Phys. Rev. Letters **9**, 139 (1962)]. Our theory will be applicable if either the ζ meson or one of the two new resonances occurs in the same channel as the ρ meson.

[3] G. Alexander, G. R. Kalbfleisch, D. H. Miller, and G. A. Smith, Phys. Rev. Letters **8**, 447 (1962).

[4] G. F. Chew, in *Dispersion Relations and Elementary Particles*, edited by C. DeWitt and R. Omnes (Herman & Cie, Paris, 1960), p. 455.

[5] K. Igi and K. Kawarabayashi, Progr. Theoret. Phys. (Kyoto) **20**, 576 (1958).

[6] S. Ishida and K. Nakazawa, Progr. Theoret. Phys. (Kyoto) **27**, 33 (1962).

that the interaction II is attractive. Moreover, we replace it, for simplicity, by a point interaction of separable form:

$$H_{\mathrm{II}}=-(g^2/6\pi m^2)(\varphi_\alpha\partial_\mu\varphi_\beta-\varphi_\beta\partial_\mu\varphi_\alpha)^2, \qquad (2.2)$$

where m is the mass of B. This approximation will not be very unrealistic because the interaction II does not depend sensitively on the energy of the scattering system, although it may be rather sensitive to the momentum transfer.

The calculation of the chain diagram has been done by several authors,[6,7] so we do not give any details. The following formula is obtained for the scattering amplitude.

$$f(p)=\frac{\exp(i\delta)\sin\delta}{p}=\frac{p^2}{6\pi W}\left(\frac{f^2}{m^2-W^2}+\frac{g^2}{m^2}\right)\times\left[1-\frac{I(p)}{6\pi^2}\left(\frac{f^2}{m^2-W^2}+\frac{g^2}{m^2}\right)\right]^{-1}, \qquad (2.3)$$

with

$$I(p)=\int_0^\Lambda\frac{q^4dq}{\omega_q(q^2-p^2-i\epsilon)}, \qquad (2.4)$$

where p is the pion momentum in the center-of-mass system. W is the total energy of the two pions; $W=2\omega_p=2(p^2+\mu^2)^{1/2}$, μ being the pion mass.

The integral $I(p)$, which comes from the two intermediate pion propagators, diverges quadratically. The degree of divergence can be reduced to the logarithmic one by a subtraction procedure, i.e., renormalization of the coupling constants[6,7]; by substituting

$$f_r^2=Zf^2,\quad g_r^2=Zg^2,$$
$$Z^{-1}=1+\frac{I(0)}{6\pi^2}\left(\frac{f^2}{m^2-\mu^2}+\frac{g^2}{m^2}\right), \qquad (2.5)$$

into Eq. (2.3), f^2 and g^2 can be replaced by f_r^2 and g_r^2, respectively, and $I(p)$ by

$$J(p)=I(p)-I(0)=p^2\int_0^\Lambda\frac{q^2dq}{\omega_q(q^2-p^2-i\epsilon)}$$
$$\approx p^2[\log2\Lambda-(p/\omega_p)\log(p+\omega_p)+i\pi p/2\omega_p]. \qquad (2.6)$$

If the cutoff, Λ, is large, we may meet trouble in renormalizing the coupling constants, just as in the Lee model. Here we take the view, however, that such a difficulty is due to improper assumptions on the form of the interaction, and hope that, by a proper treatment of the interactions, $I(0)$ becomes so small that Z is not much smaller than unity.

Resonance energies are obtained as roots of the equation

$$\mathrm{Re}[p^2f(p)^{-1}]=p^3\cot\delta$$
$$=6\pi W\left[\left(\frac{f_r^2}{m^2-W^2}+\frac{g_r^2}{m^2}\right)^{-1}-\frac{1}{6\pi^2}\mathrm{Re}J(p)\right]=0. \qquad (2.7)$$

By examining the behavior of the terms in the square brackets as functions of W^2, it is easily seen that the Eq. (2.7) has two roots.

The scattering amplitude can be rewritten in a more convenient form to see its resonant structure. If we set

$$\mathrm{Re}J(p)\equiv\pi C(W^2-4\mu^2), \qquad (2.8)$$

then C is a slowly varying function of W. Assuming that C is a constant, we have

$$f(p)=\frac{p^2}{W}\left[\frac{Cg_r^2(W^2-m_\zeta^2)(W^2-m_\rho^2)}{f_r^2m^2+g_r^2(m^2-W^2)}-i\frac{p^3}{W}\right]^{-1}, \qquad (2.9)$$

where m_p and m_ζ are resonance energies which correspond to the ρ and ζ mesons, respectively. The half-widths of the resonances are given by

$$\frac{\Gamma_\rho}{2}=-\frac{p_\rho^3}{m_\rho}\frac{f_r^2m^2+g_r^2(m^2-m_\rho^2)}{Cg_r^2(m_\rho^2-m_\zeta^2)2m_\rho}, \qquad (2.10)$$

and a similar formula for $\Gamma_\zeta/2$. Here p_ρ is the pion momentum at the resonance ρ.

In order to obtain the two resonances at the observed positions, the bare mass m must lie between m_ρ and m_ζ, and $(f_r/g_r)^2\ll1$. For larger values of $(f_r/g_r)^2$, there is a tendency that the two resonances become broader and more distant from each other. Provided that C can be regarded as a constant, the resonance energies do not depend on g_r^2 and C separately but only on their product g_r^2C. For the value of $C=(1/4\pi)[\log2\Lambda-(p/\omega_p)\log(p+\omega_p)]$, let us replace p and ω_p by their average values taken over the two observed resonances. Then, taking a cutoff $\Lambda=M$ (or $2M$), where M is the nucleon mass, we have $4\pi C=1.92$ (2.62). Examples of the solution are given in Table I. The experimental values are $m_\rho\approx750$ MeV, $\Gamma_\rho/2=50\sim100$ MeV, $m_\zeta\approx570$ MeV, $\Gamma_\zeta/2\lesssim35$ MeV.[1]

As is seen from the examples of the solution, f_r^2 is much smaller than g_r^2, so the exchange of B is not the main part of the interaction II. This does not necessarily mean, however, that the bootstrap mechanism is unimportant. The interaction II may be better described as the exchange of the ρ and ζ mesons, etc., rather than as the exchange of bare B, etc. Such a treatment will be possible by means of double dispersion relations.[4] It may well be that the effective interaction of $2\pi\rightleftarrows\rho$ or ζ is much stronger than that of $2\pi\rightleftarrows B$ and gives rise to a strong bootstrap effect. We feel, however, besides the exchange of the ρ and ζ mesons, inelastic intermediate processes, particularly baryon loops, are important candidates for the strong attraction between

TABLE I. Energies are in units of MeV. Numbers in parentheses are for the case of $\Lambda=2M$.

m	$(f_r/g_r)^2$	$(\mu/m)^2g_r^2C$	m_ζ	$\Gamma_\zeta/2$	m_ρ	$\Gamma_\rho/2$
640	0.1	1	571	78 (57)	757	121 (89)
600	0.07	1.1	567	47 (35)	741	190 (140)

[7] S. Okubo, Phys. Rev. **118**, 357 (1960); B. W. Lee and M. T. Vaughn, Phys. Rev. Letters **4**, 578 (1960); Y. Miyamoto, Progr. Theoret. Phys. (Kyoto) **24**, 840 (1960).

pions. If only the nucleon single loop is considered in the interaction II, $(\mu/m)^2(g_r^2/4\pi)$ will be replaced, apart from the renormalization factor Z, by $(2\mu^2/3\pi M^2)\times(g_N^2/4\pi)^2\approx 1$. Here $g_N^2/4\pi\approx 15$ is the pion-nucleon coupling constant. Inclusion of hyperon loops will reinforce this attraction, if the Σ-Λ relative parity is even.[6] So, the effects of baryon loops seem to give rise to a sufficiently strong attractive force between pions in the $I=J=1$ state. Blankenbecler[8] and Bransden *et al.*[9] pointed out that the four-pion intermediate state also produces attraction between pions in the $I=J=1$ state.

Let us now discuss our model in terms of dispersion relations. According to Chew and Mandelstam[10] and to Miyamoto,[7] the invariant amplitude $T_{I=J=1}\equiv(W/2)f(p)$ can be written as

$$T(\nu)=N(\nu)/D(\nu), \tag{2.11}$$

where $\nu=p^2$. On the complex ν plane, N is analytic except along the branch cut $-\infty<\nu<-\mu^2$. D is analytic except along the branch cut $0<\nu<\infty$. Considering analyticity properties of N and D, we can set

$$N(\nu)/\nu=a+\frac{\nu}{\pi}\int_{-\infty}^{-\mu^2}d\nu'\frac{\mathrm{Im}T(\nu')D(\nu')}{\nu'^2(\nu'-\nu)}, \tag{2.12}$$

and

$$D(\nu)=b-\frac{c}{\nu-\nu_0}-\frac{\nu}{\pi}\int_0^{\infty}d\nu'\left(\frac{\nu'}{\nu'+\mu^2}\right)^{1/2}\frac{N(\nu')}{\nu'(\nu'-\nu-i\epsilon)}, \tag{2.13}$$

where a, b, c, and ν_0 are real constants. Here we have introduced in D a Castillejo, Dalitz, and Dyson (CDD) pole[11]: $c/(\nu-\nu_0)$. Neglecting the contribution from the negative cut, we put $N/\nu=a$ and

$$D(\nu)=b-\frac{c}{\nu-\nu_0}-\frac{a\nu}{\pi}\int_0^{\infty}\left(\frac{\nu'}{\nu'+\mu^2}\right)^{1/2}\frac{d\nu'}{\nu'-\nu-i\epsilon}, \tag{2.14}$$

which gives

$$\left(\frac{\nu^3}{\nu+\mu^2}\right)^{1/2}\cot\delta=a^{-1}\left[b-\frac{c}{\nu-\nu_0}-\frac{a\nu}{\pi}\times\mathrm{Re}\int_0^{\infty}\left(\frac{\nu'}{\nu'+\mu^2}\right)^{1/2}\frac{d\nu'}{\nu'-\nu-i\epsilon}\right]. \tag{2.15}$$

The Eq. (2.7), etc., can be derived by putting

$$\begin{gathered}b/a=12\pi m^2/g_r^2,\quad c/b=\tfrac{1}{4}m^2(f_r/g_r)^2,\\ \nu_0+\mu^2=\tfrac{1}{4}m^2[1-(f_r/g_r)^2],\end{gathered} \tag{2.16}$$

and cutting the integral off at Λ. As is to be expected,[10] our model is, thus, equivalent to introducing a CDD singularity.

Bransden *et al.*[9] have solved the coupled equations of S- and P-wave pion-pion scatterings, using a computer, and found a very interesting solution which shows two P-wave resonances. They do not introduce any CDD singularity, so their two resonances are considered to be dynamical. However, the mechanism of the resonances in their theory cannot easily be visualized.

III. DISCUSSION

In the kaon-pion scattering, it seems very probable that the so-called K^* resonance at about 880 MeV is in the $I=\frac{1}{2}$, $J=1$ state. The same I-J assignment also has been suggested for the new resonance at about 730 MeV.[3] Assuming that both the resonances are in the same I-J state, we may expect that a mechanism similar to that of the ρ and ζ mesons is operating here, namely, the two resonances may be ascribed to the existence of one fundamental meson. The recent experiment[3] gives the width of the K^* resonance as $\Gamma=60\pm5$ MeV, which is much larger than that given before. This broader width makes it easier to reproduce the resonances by our mechanism. The kaon-pion interaction due to a single closed baryon loop has been investigated by Nakayama,[12] who has found some combinations of signs and magnitude of coupling constants which give a definitely attractive interaction between kaon and pion in the $I=\frac{1}{2}$, $J=1$ state. These combinations are especially interesting in connection with the pion-hyperon and kaon-nucleon scatterings. We do not give any detailed numerical analysis here, because we are interested only in qualitative consequences of our model.

As was mentioned in the introduction, our meson B would be more significant than the observed ρ and ζ mesons, in the attempt at more fundamental theory of elementary particles, such as Sakurai's[13] or Gell-Mann's.[14] In their theories one, and only one, $I=1$, $J=1^-$, $S=0$ meson is predicted. Gell-Mann's theory predicts one $I=\frac{1}{2}$, $J=1$, $S=\pm1$ meson, called M, also. We may take the view that their particles should be identified with our "hidden particles" rather than with the observed resonances.

Note added in proof. Gatland and Moffat have given an analytic approximation of the solution of the inverse amplitude dispersion relations,[9] showing that their inverse amplitude has a CDD pole. I wish to thank Dr. I. R. Gatland and Dr. J. W. Moffat for sending preprints of their work.

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