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Magnetic Moment of a Lee Particle*

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A version of the Lee model is constructed, in which the particles are charged and the interaction resembles that of pseudoscalar meson theory. The allowed processes are $V \leftrightarrow N+\theta$, $\theta \leftrightarrow X+\bar{X}$, where V, N, and X are all fermions which are formally treated as distinct. Relativistic energy-momentum relations are used for all the particles. In this model, the eigenstate of the physical V particle is determined, and its magnetic moment is calculated. Some problems concerning the mass and charge renormalizations are discussed. A cutoff is employed in order to avoid the difficulties with the state normalizations which are characteristic of the Lee model. If the renormalized coupling constant G is small enough, all the components of the physical eigenstate have positive amplitudes. For somewhat larger values of G, the expressions for the moments are still well defined, even though parts of the eigenstate have negative amplitudes. Some modifications of the model are considered. The analog of the meson-nucleon static model gives moments which differ by a factor of about 2 from the exact results.

I. INTRODUCTION

SINCE the introduction of the model field theory of Lee,[1] several variations and extensions have appeared. A number of these studies have sought to incorporate some additional feature of more complicated field theories, while maintaining the soluble nature of the model.

In the original Lee model the only virtual process permitted was

$$V \leftrightarrow N+\theta, \tag{1}$$

in which V and N are fermions and θ is a boson. Machida[2] considered a pair model in which the allowed process is

$$\theta \leftrightarrow X+\bar{X}, \tag{2}$$

while Goldstein[3] combined the Lee and Machida models to allow both processes (1) and (2). In these treatments all the particles are considered uncharged and the spins of the fermions do not affect their description.

We have considered the problem of a charged Lee particle, which interacts with a charged boson in a manner suggestive of pseudoscalar meson theory. Such a particle possesses an anomalous magnetic moment, whose calculation is our primary object.

The virtual processes allowed in our model are just those of the Goldstein model, namely, (1) and (2) above. Likewise, we find it necessary to treat all three fermions as distinct, in order to keep the theory soluble in closed form. However, the particles are assumed to be charged. All the fermions are assigned isotopic spin $\frac{1}{2}$, and the θ is considered to be a pseudoscalar meson with isospin one. A γ_5 type of coupling is assumed, and relativistic forms for the energies of the fermions as well as the bosons are employed. In this model we solve for the eigenstates corresponding to the physical θ and V particles, and calculate the magnetic moments of the latter.

Just as in other versions of the Lee model, a major problem which must be faced is that of the normalization constants of the physical states. These constants, which give the fraction of bare θ and V in their respective physical states, and therefore ought to have values between zero and one, turn out to be negatively infinite in the theory. As a consequence, the spurious states ("ghosts"), first discussed by Källén and Pauli,[4] are present in our theory in the spectra of both the θ and V states. The difficulty is reflected in the integrals for the magnetic moments, which contain infinite factors and hence are not well defined.

In order to obtain meaningful results, we have therefore included a cutoff factor in the interaction, following Källén and Pauli.[4] With the cutoff present, there exists a range of values for the coupling constant within which all the normalization constants are between zero and one. In this range, no ghost states appear and the theory is subject to a standard interpretation. The maximum value of the rationalized, renormalized coupling constant which satisfies this condition is rather small, $G^2/4\pi \approx 1.5$. The integrals for the moments actually converge for somewhat larger values of G, as seen in Sec. III.

If the pair interaction is dropped, what remains is a pseudoscalar Lee model with recoil. The expressions for the moments in this case are convergent, and it is possible to compare the predictions of the theory with and without cutoff. However, the normalization constants without cutoff are still, of course, infinite for any value of the coupling constant.

With a cutoff present and no pair interaction, the normalization difficulties do not appear until the coupling constant is considerably larger, $G^2/4\pi \approx 35$. It is therefore possible to evaluate the theory when the

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coupling is of the order suggested by ordinary meson theory. We can then study the effect of a further approximation, namely, dropping the relativistic energy-momentum relation for the fermions. One arrives in this fashion at a theory very similar to the meson-nucleon static model, i.e., a $\boldsymbol{\sigma}\cdot\mathbf{k}$ type of Hamiltonian. The correspondence, however, is not exact. That is, putting $E=m$ is not entirely equivalent to neglecting recoil, which is the assumption of the static model. In the former approximation of the γ_5 theory there still remains a contribution to the magnetic moment from the orbital motion of the fermions, while such a term is, of course, absent in the fixed-source approximation. In either kind of static limit, the predictions for the moments differ by a factor of order 2 from the "exact" cutoff results.

II. DETAILS OF THE MODEL

We are concerned with three distinct fermion fields, ψ_V, ψ_N, and ψ_X. Each of these exists in two charge states, and may be represented by an eight-component spinor which is expanded in the usual way:

$$\psi_V(\mathbf{x})=\psi_V^{(+)}(\mathbf{x})+\psi_V^{(-)}(\mathbf{x})$$
$$=\frac{1}{\sqrt{\Omega}}\sum_{\mathbf{p};ij}\left(\frac{m}{E_\mathbf{p}}\right)^{1/2}[b_{V;ij}(\mathbf{p})u^i(\mathbf{p})U^je^{i\mathbf{p}\cdot\mathbf{x}}+d_{V;ij}^*(p)u^{i+2}(-\mathbf{p})U^je^{-i\mathbf{p}\cdot\mathbf{x}}, \quad (3)$$

with similar expressions for ψ_N and ψ_X. All three particles are assumed to have the same mass m. The indices i, j refer to spin and isospin, respectively, and run from one to two always. The $u^i(p)$ are Dirac spinors and the U^j are two-component Pauli isospinors. The b and b^* are destruction and creation operators for the particles of the type indicated by the subscripts, while d and d^* are the corresponding antiparticle operators. As in all versions of the Lee model, the antiparticle parts of V and N, namely, $\psi_V^{(-)}$, $\psi_N^{(-)}$, $\bar{\psi}_V^{(+)}$, and $\bar{\psi}_N^{(+)}$ play no part and must be suppressed.

The θ particle is to be a symmetric pseudoscalar meson; its field expansion is then

$$\theta_\alpha(\mathbf{x})=A_\alpha^{(+)}(\mathbf{x})+A_\alpha^{(-)}(\mathbf{x})$$
$$=\frac{1}{\sqrt{\Omega}}\sum_{\mathbf{k}}(2\omega_\mathbf{k})^{-1/2}[a_\alpha(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}+a_\alpha^*(\mathbf{k})e^{-i\mathbf{k}\cdot\mathbf{x}}]. \quad (4)$$

The Hamiltonian which allows the processes (1), (2), and no others may be written

$$H=H_0+H_I+H_{\mathrm{m.r.}}, \quad (5)$$

where

$$H_0=\sum_{\mathbf{p};ij}E_\mathbf{p}[b_{V;ij}^*(\mathbf{p})b_{V;ij}(\mathbf{p})+b_{N;ij}^*(\mathbf{p})b_{N;ij}(\mathbf{p})+b_{X;ij}^*(\mathbf{p})b_{X;ij}(\mathbf{p})+d_{X;ij}^*(\mathbf{p})d_{X;ij}(\mathbf{p})]+\sum_{\mathbf{k},\alpha}\omega_\mathbf{k}a_\alpha^*(\mathbf{k})a_\alpha(\mathbf{k}), \quad (6)$$

$$H_I=g_1\int d^3x[\bar{\psi}_N^{(-)}(\mathbf{x})\gamma_5\tau_\alpha\psi_V^{(+)}(\mathbf{x})A_\alpha^{(-)}(\mathbf{x})+\bar{\psi}_V^{(-)}(\mathbf{x})\gamma_5\tau_\alpha\psi_N^{(+)}(\mathbf{x})A_\alpha^{(+)}(\mathbf{x})]$$
$$+g_2\int d^3x[\bar{\psi}_X^{(-)}(\mathbf{x})\gamma_5\tau_\alpha\psi_X^{(-)}(\mathbf{x})A_\alpha^{(+)}(\mathbf{x})+\bar{\psi}_X^{(+)}(\mathbf{x})\gamma_5\tau_\alpha\psi_X^{(+)}(\mathbf{x})A_\alpha^{(-)}(\mathbf{x})], \quad (7)$$

and

$$H_{\mathrm{m.r.}}=H_{\mathrm{m.r.}}(V)+H_{\mathrm{m.r.}}(\theta). \quad (8)$$

The last term represents the mass renormalization of the V and θ, the two particles which require renormalization. The form of these terms presents a special problem, which is discussed presently. The interaction term (7) represents symmetric pseudoscalar coupling, with the appropriate modification to limit the allowed processes to those desired. Such a limitation of course destroys the Lorentz invariance of the theory.

When H_I is expanded in momentum space using (3) and (4), it becomes

$$H_I=\frac{g_1}{\sqrt{\Omega}}\sum_{\mathbf{p},\mathbf{p}';ii'jj';\alpha}\left(\frac{m^2}{2\omega_{\mathbf{p}'-\mathbf{p}}E_\mathbf{p}E_{\mathbf{p}'}}\right)^{1/2}[\bar{u}^i(\mathbf{p})\gamma_5u^{i'}(\mathbf{p}')]$$
$$\times(\tilde{U}^j\tau_\alpha U^{j'})[b_{N;ij}^*(\mathbf{p})b_{V;i'j'}(\mathbf{p}')a_\alpha^*(\mathbf{p}'-\mathbf{p})+b_{V;ij}^*(\mathbf{p})b_{N;i'j'}(\mathbf{p}')a_\alpha(\mathbf{p}'-\mathbf{p})]$$
$$+\frac{g_2}{\sqrt{\Omega}}\sum_{\mathbf{p},\mathbf{p}';ii'jj';\alpha}\left(\frac{m^2}{2\omega_{\mathbf{p}'+\mathbf{p}}E_\mathbf{p}E_{\mathbf{p}'}}\right)^{1/2}(\tilde{U}^j\tau_\alpha U^{j'})$$
$$\times[\bar{u}^i(\mathbf{p})\gamma_5u^{i'+2}(-\mathbf{p}')b_{X;ij}^*(\mathbf{p})d_{X;i'j'}^*(\mathbf{p}')a_\alpha(\mathbf{p}+\mathbf{p}')+\bar{u}^{i+2}(-\mathbf{p})\gamma_5u^{i'}(\mathbf{p}')d_{X;ij}(\mathbf{p})b_{X;i'j'}(\mathbf{p}')a_\alpha^*(\mathbf{p}+\mathbf{p}')]. \quad (9)$$

We now consider the one-particle eigenstates of this Hamiltonian. It is clear that, just as in the Goldstein model, the vacuum and the bare N, X, and $\bar{X}$ eigenstates of H_0 are also eigenstates of H. We have to find the states corresponding to physical θ and V. The physical θ state with charge α and momentum k may be written

$$|\theta_\alpha(\mathbf{k})\rangle=A(\mathbf{k})[a_\alpha^*(\mathbf{k})|\rangle+g_2\sum_{\mathbf{q};ii'jj'}\phi_\alpha(ii'jj';\mathbf{kq})b_{X;ij}^*(\boldsymbol{\kappa}_+)d_{X;i'j'}^*(\boldsymbol{\kappa}_-)|\rangle], \quad (10)$$

where

$$\boldsymbol{\kappa}_\pm=\tfrac{1}{2}(\mathbf{k}\pm\mathbf{q}), \quad (11)$$

and $|\rangle$ denotes the vacuum. The wave function $\phi_\alpha(ii'jj';\mathbf{kq})$ is to be determined from the Schrödinger equation

$$H|\theta_\alpha(\mathbf{k})\rangle=\omega_\mathbf{k}|\theta_\alpha(\mathbf{k})\rangle, \quad (12)$$

and the normalization function $A(\mathbf{k})$ from

$$\langle\theta_\alpha(\mathbf{k})|\theta_\alpha(\mathbf{k})\rangle=1. \quad (13)$$

In order to obtain a solution of (12) for all values of $\mathbf{k}$, it is necessary to assign the mass renormalization counterterm (8) an unusual form. Normally, such a counterterm for a boson field would be written

$$H_{\mathrm{m.r.}}(\theta)=\tfrac{1}{2}\delta\mu^2\sum_\alpha\int d^3x\,\phi_\alpha(\mathbf{x})\phi_\alpha(\mathbf{x}), \tag{14}$$

and an identical term absorbed into the mass term of H_0. This procedure is what is meant by mass renormalization. In the momentum expansion (14) has the form

$$H_{\mathrm{m.r.}}(\theta)=\tfrac{1}{2}\delta\mu^2\sum_{\mathbf{k},\alpha}(1/\omega_{\mathbf{k}})[a_\alpha^*(\mathbf{k})a_\alpha(\mathbf{k})-\tfrac{1}{2}+\tfrac{1}{2}a_\alpha^*(\mathbf{k})a_\alpha^*(-\mathbf{k})+\tfrac{1}{2}a_\alpha(\mathbf{k})a_\alpha(-\mathbf{k})]. \tag{15}$$

It is within the spirit of the Lee model to discard the pair creation and annihilation terms in (15). This amounts to writing instead of (14)

$$H_{\mathrm{m.r.}}(\theta)=\tfrac{1}{2}\delta\mu^2\sum_\alpha\int d^3x[\phi_\alpha^{(+)}(\mathbf{x})\phi_\alpha^{(-)}(\mathbf{x})+\phi_\alpha^{(-)}(\mathbf{x})\phi_\alpha^{(+)}(\mathbf{x})]. \tag{16}$$

Such a procedure is implied in the work of Machida,[2] although it is not explicitly discussed by his paper, nor by Goldstein, in whose model the same difficulty is present. It must be recognized that with the "correct" counterterm (14) the Machida model is not soluble, since a three-meson state will be coupled by $H_{\mathrm{m.r.}}$ to the one-meson state. With the modified form (16) one can solve for the zero-momentum eigenstate. The result is only a slight generalization of Machida's, involving spin and isospin indices.

However, in order to obtain a consistent solution for nonzero k, even this procedure is not sufficient. It is necessary to write the renormalization term as

$$H_{\mathrm{m.r.}}(\theta)=\sum_{\mathbf{k},\alpha}\delta\omega_{\mathbf{k}}a_\alpha^*(\mathbf{k})a_\alpha(\mathbf{k}), \tag{17}$$

that is, the counterterm is momentum dependent. We then obtain the following solution:

$$\phi_\alpha(ii'jj';\mathbf{kq})=-m\frac{\bar{u}^i(\boldsymbol{\kappa}_+)\gamma_5u^{i'+2}(-\boldsymbol{\kappa}_-)\tilde{U}^i\tau_\alpha U^{i'}}{(2\Omega\omega_{\mathbf{k}}E_+E_-)^{1/2}(E_++E_--\omega_{\mathbf{k}})}, \tag{18}$$

$$[A(\mathbf{k})]^{-2}=1+\frac{g_2^2}{\Omega}\sum_q\frac{m^2+E_+E_--\boldsymbol{\kappa}_+\cdot\boldsymbol{\kappa}_-}{\omega_{\mathbf{k}}E_+E_-(E_++E_--\omega_{\mathbf{k}})^2}, \tag{19}$$

$$\delta\omega_{\mathbf{k}}=\frac{g_2^2}{\Omega}\sum_q\frac{E_+E_-+m^2-\boldsymbol{\kappa}_+\cdot\boldsymbol{\kappa}_-}{\omega_{\mathbf{k}}E_+E_-(E_++E_--\omega_{\mathbf{k}})}, \tag{20}$$

where

$$E_\pm\equiv E(\boldsymbol{\kappa}_\pm)=(m^2+\boldsymbol{\kappa}_\pm)^{1/2}. \tag{21}$$

It must be pointed out that this method is not, strictly speaking, a valid mass renormalization, since the counterterm (17) can no longer be associated with the mass term in H_0. Thus if the bare θ particle is a Klein-Gordon particle, then the dressed particle is not, and conversely.

The same problem is present in the work of Goldstein.[3] His use of μ instead of $\omega_{\mathbf{k}}$ in the pair interaction term does not avoid a momentum-dependent renormalization, unless one also puts $E_{\mathbf{p}}=m$ for all the fermions, as in Lee's original paper. We have adopted the relativistic forms for both $\omega_{\mathbf{k}}$ and $E_{\mathbf{p}}$. We then view the momentum dependence of the renormalization as being a consequence of the lack of complete Lorentz invariance, which is the price one pays for having a soluble theory. It is in this spirit that we accept the unpleasant form of Eq. (17).[5]

A similar difficulty arises, in principle, in the construction of the physical V state. However, for our magnetic moment calculation it will be sufficient to consider a V particle at rest, so that the momentum dependence of the V renormalization will not affect our results. In order to obtain the V eigenstate for arbitrary momentum it is necessary to write the renormalization term as

$$H_{\mathrm{m.r.}}(V)=\sum_{\mathbf{p};ij}\delta E_{\mathbf{p}}b_{V;ij}^*(\mathbf{p})b_{V;ij}(\mathbf{p}), \tag{22}$$

after which the eigenstate

$$\begin{aligned}|V_{i_0j_0}(\mathbf{p})\rangle=\mathcal{A}(\mathbf{p})[b_{V;ij}^*(\mathbf{p})|\rangle&+g_1\sum_{k;ij\alpha}\phi_{i_0j_0}(ij\alpha;\mathbf{pk})b_{N;ij}^*(\mathbf{p}-\mathbf{k})a_\alpha^*(\mathbf{k})|\rangle\\&+g_1g_2\sum_{\mathbf{kq};ii'i''jj'j''}\psi_{i_0j_0}(ii'i''jj'j'';\mathbf{pkq})\\&\times b_{N;ij}^*(\mathbf{p}-\mathbf{k})b_{X;i'j'}^*(\boldsymbol{\kappa}_+)d_{X;i''j''}^*(\boldsymbol{\kappa}_-)|\rangle]\end{aligned} \tag{23}$$

can be determined. The θ renormalization cancels the quadratically divergent part of the wave function, and we obtain for the functions ϕ and ψ

$$\phi_{i_0j_0}(ij\alpha;\mathbf{pk})=\frac{-m\bar{u}^i(\mathbf{p}-\mathbf{k})\gamma_5u^{i_0}(\mathbf{p})\tilde{U}^i\tau_\alpha U^{i_0}}{(2\Omega\omega_{\mathbf{k}}E_{\mathbf{p}}E_{\mathbf{p}-\mathbf{k}})^{1/2}(E_{\mathbf{p}-\mathbf{k}}-E_{\mathbf{p}}+\omega_{\mathbf{k}})[1+g_2^2f_1(\mathbf{p},\mathbf{k})]}, \tag{24}$$

$$\psi_{i_0j_0}(ii'i''jj'j'';\mathbf{pkq})=-m\sum_\alpha\frac{\phi_{i_0j_0}(ij\alpha;\mathbf{p},\mathbf{k})\bar{u}^{i'}(\boldsymbol{\kappa}_+)\gamma_5u^{i''+2}(-\boldsymbol{\kappa}_-)\tilde{U}^{i'}\tau_\alpha U^{i''}}{(2\Omega\omega_{\mathbf{k}}E_+E_-)^{1/2}(E_{\mathbf{p}-\mathbf{k}}-E_{\mathbf{p}}+E_++E_-)}, \tag{25}$$

where

$$f_1(\mathbf{p},\mathbf{k})=\frac{1}{\Omega}\sum_q\frac{m^2+E_+E_--\boldsymbol{\kappa}_+\cdot\boldsymbol{\kappa}_-}{\omega_{\mathbf{k}}E_+E_-(E_++E_--\omega_{\mathbf{k}})(E_++E_-+E_{\mathbf{p}-\mathbf{k}}-E_{\mathbf{p}})}. \tag{26}$$

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[5] In a formal sense, one could extract the momentum-independent part of (17) and call it the mass renormalization, interpreting the remainder as a part of the interaction which would vanish if the theory were Lorentz invariant. None of the subsequent results would be altered by such an interpretation.

The wave function normalization factor is

$$[\mathfrak{A}(\mathbf{p})]^{-2}=1+\frac{3}{8}\frac{g_1^2}{\Omega}\times\sum_{\mathbf{k}}\frac{[2E_{\mathbf{p}}E_{\mathbf{p-k}}-2m^2+\mathbf{p}\cdot(\mathbf{p}-\mathbf{k})][1+g_2^2f_2(\mathbf{p},\mathbf{k})]}{(\omega_{\mathbf{k}}E_{\mathbf{p}}E_{\mathbf{p-k}})(E_{\mathbf{p-k}}-E_{\mathbf{p}}+\omega_{\mathbf{k}})^2[1+g_2^2f_1(\mathbf{p},\mathbf{k})]^2},\tag{27}$$

where

$$f_2(\mathbf{p},\mathbf{k})=\frac{1}{\Omega}\sum_{\mathbf{q}}\frac{m^2+E_+E_--\boldsymbol{\kappa}_+\cdot\boldsymbol{\kappa}_-}{\omega_{\mathbf{k}}E_+E_-(E_{\mathbf{p-k}}-E_{\mathbf{p}}+E_++E_-)^2}.\tag{28}$$

The sums in the functions f_1 and f_2 are still linearly divergent. A further reduction of the divergence is achieved by charge renormalization. For this purpose we rewrite (26) as

$$f_1(\mathbf{p},\mathbf{k})=\frac{1}{\Omega}\sum_{\mathbf{q}}\frac{m^2+E_+E_--\boldsymbol{\kappa}_+\cdot\boldsymbol{\kappa}_-}{\omega_{\mathbf{k}}E_+E_-(E_++E_--\omega_{\mathbf{k}})^2}\times\left(1-\frac{E_{\mathbf{p-k}}-E_{\mathbf{p}}+\omega_{\mathbf{k}}}{E_++E_-+E_{\mathbf{p-k}}-E_{\mathbf{p}}}\right).\tag{29}$$

In (29) the first sum is just that which appears in the expression for $[A(\mathbf{k})]^{-2}$, Eq. (19), while the remaining term is only logarithmically divergent. Thus, we can rewrite

$$1+g_2^2f_1(\mathbf{p},\mathbf{k})=[A(\mathbf{k})]^{-2}[1-G_2^2F_1(\mathbf{p},\mathbf{k})],\tag{30}$$

where

$$F_1(\mathbf{p},\mathbf{k})=\frac{1}{\Omega}\sum_{\mathbf{q}}\frac{(m^2+E_+E_--\boldsymbol{\kappa}_+\cdot\boldsymbol{\kappa}_-)(E_{\mathbf{p-k}}-E_{\mathbf{p}}+\omega_{\mathbf{k}})}{\omega_{\mathbf{k}}E_+E_-(E_++E_--\omega_{\mathbf{k}})^2(E_++E_-+E_{\mathbf{p-k}}-E_{\mathbf{p}})},\tag{31}$$

and

$$G_2=g_2A(\mathbf{k})\tag{32}$$

is defined as the renormalized coupling constant. Accompanying this charge renormalization there must be a wave function renormalization. The renormalized field operator, defined by

$$a_\alpha^{(r)}(\mathbf{k})=a_\alpha(\mathbf{k})[A(\mathbf{k})]^{-1},\tag{33}$$

will satisfy

$$\langle|a_\alpha^{(r)}(\mathbf{k})|\theta_\alpha(\mathbf{k})\rangle=1.\tag{34}$$

Thus our renormalization procedure is consistent with that of Källén.[4]

It will be observed that the charge renormalization "constant" is in fact also momentum dependent. If G_2 is interpreted as the meaningful coupling constant, then the unrenormalized g_2 must be momentum dependent. Such a velocity-dependent charge renormalization has already been discussed by Fried.[6]

To achieve a V renormalization which satisfies a relation analogous to (34), we must define

$$G_1=g_1\mathfrak{A}(\mathbf{p})A(\mathbf{k}),\tag{35}$$

$$b_V^{(r)}(\mathbf{p})=b_V(\mathbf{p})[\mathfrak{A}(\mathbf{p})]^{-1}.\tag{36}$$

The interaction (7) is unchanged if g_1 and g_2 are replaced by G_1 and G_2, respectively, and at the same time the θ and V operators are replaced by the corresponding renormalized ones. The renormalization procedure represented by Eqs. (32) and (35) is equivalent to that of Scarfone.[7]

We can now rewrite the physical V state (23), for zero momentum, in terms of the renormalized coupling constants:

$$\begin{aligned}|V_{i_0j_0}\rangle=&\mathfrak{A}b_{V;i_0j_0}^*(0)|\rangle\\&+G_1\sum_{\mathbf{k};ij\alpha}A(\mathbf{k})\Phi_{i_0j_0}(ij\alpha;\mathbf{k})b_{N;ij}^*(-\mathbf{k})a_\alpha^*(\mathbf{k})|\rangle\\&+G_1G_2\sum_{\mathbf{kq};ii'i''jj'j''}\Psi_{i_0j_0}(ii'i''jj'j'';\mathbf{kq})\\&\times b_{N;ij}^*(-\mathbf{k})b_{X;i'j'}^*(\boldsymbol{\kappa}_+)d_{X;i''j''}^*(\boldsymbol{\kappa}_-)|\rangle,\end{aligned}\tag{37}$$

where

$$\Phi_{i_0j_0}(ij\alpha;\mathbf{k})=-m\frac{\bar{u}^i(-\mathbf{k})\gamma_5u^{i_0}(0)\tilde{U}^j\tau_\alpha U^{j_0}}{(2\Omega\omega_{\mathbf{k}}E_{\mathbf{k}}m)^{1/2}(E_{\mathbf{k}}-m+\omega_{\mathbf{k}})[1-G_2^2F_1(\mathbf{k})]},\tag{38}$$

and Ψ is obtained from Φ by the same relation as (25). The normalization constants A and $\mathfrak{A}$ are expressed in terms of the renormalized G's by

$$[A(\mathbf{k})]^2=1-\frac{G_2^2}{\Omega}\sum_{\mathbf{q}}\frac{m^2+E_+E_--\boldsymbol{\kappa}_+\cdot\boldsymbol{\kappa}_-}{(\omega_{\mathbf{k}}E_+E_-)(E_++E_--\omega_{\mathbf{k}})^2},\tag{39}$$

$$\mathfrak{A}^2=1-\frac{3}{4}\frac{G_1^2}{\Omega}\times\sum_{\mathbf{k}}\frac{(E_{\mathbf{k}}-m)[1-G_2^2F_2(\mathbf{k})]}{\omega_{\mathbf{k}}E_{\mathbf{k}}(E_{\mathbf{k}}+\omega_{\mathbf{k}}-m)^2[1-G_2^2F_1(\mathbf{k})]^2},\tag{40}$$

where

$$F_2(\mathbf{k})=\frac{1}{\Omega}\sum_{\mathbf{q}}\frac{(m^2+E_+E_--\boldsymbol{\kappa}_+\cdot\boldsymbol{\kappa}_-)(E_{\mathbf{k}}+\omega_{\mathbf{k}}-m)}{(\omega_{\mathbf{k}}E_+E_-)(E_++E_--\omega_{\mathbf{k}})^2(E_++E_-+E_{\mathbf{k}}-m)}\times\left(2-\frac{E_{\mathbf{k}}-m+\omega_{\mathbf{k}}}{E_++E_-+E_{\mathbf{k}}-m}\right).\tag{41}$$

In these equations we have written $\mathfrak{A}$, $F_1(\mathbf{k})$, and $F_2(\mathbf{k})$ rather than $\mathfrak{A}(0)$, $F_1(0,\mathbf{k})$, and $F_2(0,\mathbf{k})$ since we shall only consider the V state with momentum zero. Finally, we write the V mass renormalization:

$$\delta E_0=\delta m=\frac{3}{4}\frac{G_1^2}{\Omega}\sum_{\mathbf{k}}\frac{E_{\mathbf{k}}-m}{\omega_{\mathbf{k}}E_{\mathbf{k}}(E_{\mathbf{k}}+\omega_{\mathbf{k}}-m)[1-G_2^2F_1(\mathbf{k})]}.\tag{42}$$

[6] H. M. Fried, Phys. Rev. 118, 1427 (1960).

[7] L. M. Scarfone, Nuovo cimento 19, 377 (1961).

The difficulties with the normalization constants are apparent from Eqs. (39) and (40). Since the integrals in these expressions diverge, both A and $\mathcal{A}$ are minus infinity for finite G_1, G_2, unless the integrals are cut off.

III. V-PARTICLE MAGNETIC MOMENT

The four-current associated with each fermion field is

$$j_\mu = ie\bar{\psi}(\mathbf{x})\gamma_\mu \tfrac{1}{2}(1+\tau_3)\psi(\mathbf{x}), \tag{43}$$

while the boson current may be written

$$J_\mu = ie[\phi^*(\mathbf{x})\partial_\mu\phi(\mathbf{x}) - (\partial_\mu\phi^*(\mathbf{x}))\phi(\mathbf{x})], \tag{44}$$

where ϕ is the charged part of the field,

$$\phi = (1/\sqrt{2})(\phi_1 + i\phi_2). \tag{45}$$

The interaction with an external electromagnetic field A_μ is then

$$H_{\mathrm{e.m.}} = \int d^3x (j_{\mu;V} + j_{\mu;N} + j_{\mu,X} + J_\mu) A_\mu. \tag{46}$$

For the computation of the magnetic moment we can put

$$A_\mu = (y,0,0,0), \tag{47}$$

which describes a uniform magnetic field of unit magnitude in the z direction. We then have

$$\mu_{\mathrm{op}} = \mu_{\mathrm{op};V} + \mu_{\mathrm{op};N} + \mu_{\mathrm{op};X} + \mu_{\mathrm{op};\theta}, \tag{48}$$

where

$$\mu_{\mathrm{op};V} = ie\int \bar{\psi}_V(\mathbf{x})\gamma_1\, y[(1+\tau_3)/2]\psi_V(\mathbf{x}) d^3x, \tag{49}$$

$$\mu_{\mathrm{op};\theta} = ie\int \left(\phi^*(\mathbf{x})\frac{\partial\phi(\mathbf{x})}{\partial x} - \frac{\partial\phi^*(\mathbf{x})}{\partial x}\phi(\mathbf{x})\right) y d^3x. \tag{50}$$

The expressions for $\mu_{\mathrm{op};N}$ and $\mu_{\mathrm{op};X}$ are identical to (49), with the appropriate fields. The operators ψ_V and ϕ in these expressions are to be interpreted as renormalized operators according to Sec. II.

To expand these operators in momentum space, we can make use of the formal identity

$$\int d^3x\, e^{i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{x}} y dx = i\Omega \frac{\partial}{\partial p_y'}(\delta_{\mathbf{p}'\mathbf{p}}), \tag{51}$$

and obtain

$$\mu_{\mathrm{op};V} = e\mathcal{A}^{-2} \sum_{\mathbf{p},\mathbf{p}';ii'jj'} \left(\frac{m^2}{E_\mathbf{p}E_{\mathbf{p}'}}\right)^{1/2} \tilde{U}^j[(1+\tau_3)/2]U^{j'}\bar{u}^i(\mathbf{p}) \times\gamma_1 u^{i'}(\mathbf{p}')\frac{\partial}{\partial \mathbf{p}_y'}(\delta_{\mathbf{p}'\mathbf{p}}) b_{V;ij}^*(\mathbf{p}) b_{V;i'j'}(\mathbf{p}'), \tag{52}$$

$$\mu_{\mathrm{op};N} = e \sum_{\mathbf{p}\mathbf{p}';ii'jj'} \left(\frac{m^2}{E_\mathbf{p}E_{\mathbf{p}'}}\right)^{1/2} \tilde{U}^j[(1+\tau_3)/2]U^{j'}\bar{u}^i(\mathbf{p}) \times\gamma_1 u^{i'}(\mathbf{p}')\frac{\partial}{\partial \mathbf{p}_y'}(\delta_{\mathbf{p}'\mathbf{p}}) b_{N;ij}^*(\mathbf{p}) b_{N;i'j'}(\mathbf{p}'), \tag{53}$$

$$\begin{aligned}\mu_{\mathrm{op};X} = e\sum \left(\frac{m^2}{E_\mathbf{p}E_{\mathbf{p}'}}\right)^{1/2} \tilde{U}^j\left(\frac{1+\tau_3}{2}\right)U^{j'}\Big\{\frac{\partial}{\partial \mathbf{p}_y'}(\delta_{\mathbf{p}'\mathbf{p}}) \\ \times[\bar{u}^i(\mathbf{p})\gamma_1 u^{i'}(\mathbf{p}') b_{X;ij}^*(\mathbf{p}) b_{X;i'j'}(\mathbf{p}') \\ - \bar{u}^{i+2}(-\mathbf{p})\gamma_1 u^{i'+2}(-\mathbf{p}') d_{X;ij}^*(\mathbf{p}) d_{X;i'j'}(\mathbf{p}')] \\ + \frac{\partial}{\partial \mathbf{p}_y'}(\delta_{\mathbf{p}',-\mathbf{p}})[\bar{u}^{i+2}(-\mathbf{p})\gamma_1 u^{i'}(\mathbf{p}') b_{X;ij}(\mathbf{p}) \\ \times d_{X;i'j'}(\mathbf{p}') - \bar{u}^i(\mathbf{p})\gamma_1 u^{i'+2}(-\mathbf{p}') \\ \times b_{X;ij}^*(\mathbf{p}) d_{X;i'j'}^*(\mathbf{p}')]\Big\},\end{aligned} \tag{54}$$

$$\begin{aligned}\mu_{\mathrm{op};\theta} = ie\sum_{\mathbf{k},\mathbf{k}'} \frac{[A(\mathbf{k})A(\mathbf{k}')]^{-1}}{(\omega_\mathbf{k}\omega_{\mathbf{k}'})^{1/2}} \mathbf{k}_x \frac{\partial}{\partial \mathbf{k}_y'}(\delta_{\mathbf{k}'\mathbf{k}}) \\ \times[a_1^*(\mathbf{k})a_2(\mathbf{k}') - a_2^*(\mathbf{k}')a_1(\mathbf{k}) \\ + a_1^*(\mathbf{k})a_2^*(-\mathbf{k}') - a_1(-\mathbf{k})a_2(\mathbf{k}')].\end{aligned} \tag{55}$$

The renormalization constants have been explicitly written in (52) and (55), so the operators in these equations satisfy the ordinary commutation relations. The pair terms in (54) and (55) have been included for completeness, even though they cannot contribute to the moments in the present theory.

The magnetic moments are now obtained as the matrix elements of (48), using (52)–(55), in the physical V states with $i_0=1$ (spin up) and $j_0=1, 2$. The evaluation is straightforward, though laborious. The most complicated part of the moment is that coming from the N particles, which appear in both the second and third terms of the state vector (37). The contributions of each of these terms is badly divergent, but when they are combined, with $[A(\mathbf{k})]^2$ expressed by (39), most of the divergence cancels. After all the spin sums are carried out and the remaining sum over momentum is transformed to an integral, we obtain for the N-particle contribution to the moment, in units of the magneton $e/2m$,

$$\mu_N = \frac{1}{2\pi}\left(\frac{G_1^2}{4\pi}\right)\left\langle\frac{3-\tau_3}{2}\right\rangle \times \int k^2 dk \frac{m(E_\mathbf{k}-m)[1-G_2^2F_2(\mathbf{k})]}{\omega_\mathbf{k}E_\mathbf{k}^2(E_\mathbf{k}-m+\omega_\mathbf{k})^2[1-G_2^2F_1(\mathbf{k})]^2}, \tag{56}$$

where F_1 and F_2 are given by Eqs. (31) and (41). The remaining parts of the moment are

$$\mu_V = \tfrac{1}{2}\langle 1+\tau_3\rangle, \tag{57}$$

$$\mu_\theta = \frac{2}{3\pi}\left(\frac{G_1^2}{4\pi}\right)\langle\tau_3\rangle \times \int k^2 dk \frac{m(E_\mathbf{k}-m)}{\omega_\mathbf{k}^2 E_\mathbf{k}(E_\mathbf{k}-m+\omega_\mathbf{k})^2[1-G_2^2F_1(\mathbf{k})]^2}, \tag{58}$$

$$\mu_X = 0. \tag{59}$$

$\langle\tau_3\rangle$ is 1 for the charged V ("proton") state, -1 for the neutral state ("neutron").

It is evident that, in the absence of a cutoff, the theory is in difficulty, unless $G_2=0$. For the integrals which define the functions $F_1(k)$, $F_2(k)$ diverge logarithmically. If the theory were completely Lorentz invariant, this would be a distressing result, since one would expect observable quantities to be convergent after renormalization. However, it has already been noted that such is not the case, and a cutoff is necessary in order to avoid the difficulties with the state normalizations. Even when $G_2=0$, in which case all the integrals converge, the cutoff is desirable for the latter reason. The logarithmic divergence remaining after renormalization also appears in the scattering problem.[3,7]

Even with a cutoff, the moments are not defined for arbitrarily large G_2, since the integrands in (56) and (58) will certainly contain poles unless

$$G_2^2F_1(\mathbf{k})<1 \qquad (60)$$

for all k. However, it is easily shown that (60) is automatically satisfied if the coupling is sufficiently weak to keep $[A(\mathbf{k})]^2$ positive throughout the range of integration. For, on comparison of expressions (31) and (39), it is observed that the integrand in the former is the same as that in the latter, multiplied by the factor $(E_{\mathbf{p-k}}-E_{\mathbf{p}}+\omega_{\mathbf{k}})/(E_{\mathbf{p-k}}-E_{\mathbf{p}}+E_++E_-)$, which is certainly smaller than one. This factor is small enough, in fact, that the inequality (60) is satisfied (and therefore the moments well defined) for coupling constants considerably larger than those allowed by the condition on $A(\mathbf{k})$.

A few qualitative features of the expressions for the moments may be noted. According to Eq. (57), the V particles contribute just the magnetic moments of the bare particles, i.e., the nonanomalous parts of the moments. This is a consequence of the wave function renormalization. The θ particles, it is seen, contribute only to the vector part of the anomalous moment,

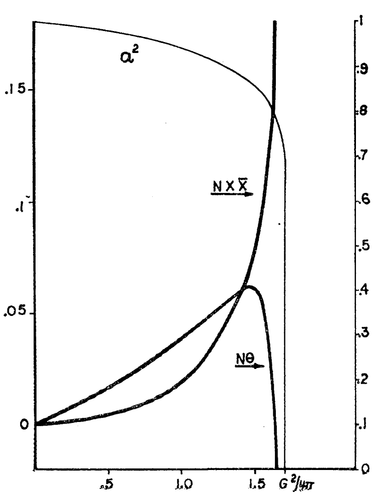

FIG. 1. Amplitudes of the bare V, $N\theta$, and $NX\bar{X}$ parts of the physical V eigenstate. The scale for α^2, the amplitude of the bare V part of the state, is at the right of the figure.

TABLE I. θ-state normalization factors: $A(k)=1-(G_2^2/4\pi)I(\mathbf{k})$.

k	$I(\mathbf{k})$	$F_1(\mathbf{k})$	$F_2(\mathbf{k})$
0	0.652	0.00370	0.00670
1	0.490	0.00400	0.00727
2	0.346	0.00450	0.00826
3	0.279	0.00525	0.00925
4	0.243	0.00606	0.01020
5	0.224	0.00688	0.01110
6	0.214	0.00770	0.01197
7	0.210	0.00859	0.01275

where the vector and scalar parts (coefficients of τ_3 and of unity, respectively) are related to the "neutron" and "proton" moments in the usual way:

$$\mu_v=\tfrac{1}{2}(\mu_p-\mu_n-1), \qquad (61)$$

$$\mu_s=\tfrac{1}{2}(\mu_p+\mu_n-1). \qquad (62)$$

The scalar moment comes entirely from the N particles. The pair contribution, according to Eq. (59), vanishes. This may be demonstrated by verifying that the pair moment is independent of i_0, the spin state of the original V particle. Thus, the pairs contribute to the V-particle moment only indirectly, by altering the wave functions of the other particles. This feature is, of course, a peculiarity of the present model, since in a more realistic theory one would certainly expect to find contributions from nucleon pairs.

IV. RESULTS

The integrals of Sec. III have been evaluated numerically as a function of G_1, for each of the following two cases:

$$\text{(I)}\ G_2=G_1,$$

$$\text{(II)}\ G_2=0.$$

In all of the computations the meson mass has been taken as unity, and the "nucleon" mass as seven. For the cutoff calculations, the source function was taken as square, with the cutoff at the nucleon mass.

Our first task is to calculate the renormalization constants, which determine the extent of the ghost-free region, and the amplitudes of the various components of the physical states. For the physical θ these are determined from Eq. (39), which we rewrite as

$$[A(\mathbf{k})]^2=1-(G_2^2/4\pi)I(\mathbf{k}). \qquad (63)$$

The values of the integral $I(k)$ are given in Table I. It is seen that $A(\mathbf{k})$ is monotonic increasing with $\mathbf{k}$, and $[A(0)]^2$, the worst case, becomes negative at $G_2^2/4\pi=1.54$. In order to avoid all difficulties with the state amplitudes, as long as the pair interaction is present, the coupling constant must be smaller than this value (which we define as $G_c^2/4\pi$). Also listed in Table I are the functions $F_1(\mathbf{k})$ and $F_2(\mathbf{k})$, which are needed in the computation of the moments. $F_1(\mathbf{k})$ has its maximum at $k=7$, where its value is 0.0085. Hence the factor $1-G^2F_1(\mathbf{k})$, which appears in the denomi-

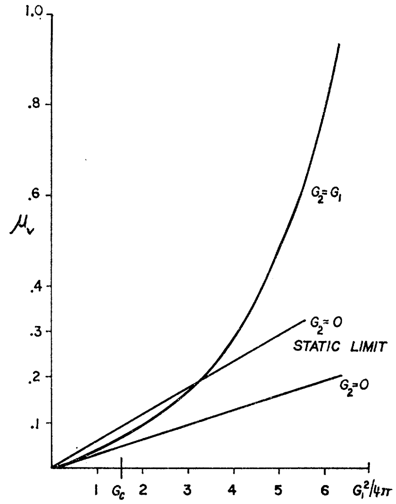

FIG. 2. The vector part of the anomalous magnetic moment.

nator of (56) and (58), is positive definite only as long as $G^2<120$ (i.e., $G^2/4\pi<\sim 10$). The moments are not defined if the coupling constant exceeds this value, but this condition is considerably less stringent than the one on $A(k)$.

The V-particle normalization $\mathcal{A}^2$, given by Eq. (40), is shown in Fig. 1 as a function of the coupling constant for the case $G_2=G_1$. Also shown are the amplitudes of the $|N\theta)$ and $|NX\bar{X})$ parts of the state vector, integrated and summed over all momenta and spins. The amplitudes are slowly varying until G^2 approaches G_c^2, when both the $|N\theta)$ and the pair amplitudes begin to increase sharply in magnitude. This behavior is due to the rapid decrease of $A(\mathbf{k})$, which appears in the expression for the individual amplitudes even though it has dropped out in Eq. (40). In particular, the $|N\theta)$ amplitude is

$$(N\theta|V\rangle=-\frac{3}{4}\frac{G_1^2}{\Omega}\times\sum_{\mathbf{k}}\frac{[A(\mathbf{k})]^2(E_{\mathbf{k}}-m)}{\omega_{\mathbf{k}}E_{\mathbf{k}}(E_{\mathbf{k}}+\omega_{\mathbf{k}}-m)^2[1-G_2^2F_1(\mathbf{k})]^2}. \quad (64)$$

As soon as $[A(\mathbf{k})]^2$ becomes negative for some $\mathbf{k}$ (i.e., as soon as $G^2>G_c^2$), the corresponding portion of the $|N\theta)$ state acquires a negative amplitude. When $[A(\mathbf{k})]^2$ has become negative over a sufficient portion of the range of integration, the total $|N\theta)$ amplitude changes sign. Simultaneously, the pair amplitude, which contains the factor $1-G^2F_2(\mathbf{k})-[A(\mathbf{k})]^2$, increases rapidly with G. The normalization constant $\mathcal{A}^2$ decreases and becomes negative at $G^2/4\pi=1.72$. For values of G larger than those shown in the figure, the amplitudes oscillate, but $\mathcal{A}^2$ remains negative. For $G^2/4\pi>10$, the amplitudes are no longer defined, and neither are the expressions for the moments. However, there exists a region, for $G^2/4\pi$ between 1.5 and 10, in which the moments are well defined even though parts of the state vector have negative amplitude. We shall present the results in this region, although they are certainly to be regarded with suspicion. It is unfortunate that the critical coupling constant G_c turns out to be as small as it does. Within the region in which all the amplitudes are positive, the maximum admixture of pair state is only 10%, and this makes it difficult to assess the effect of the pair interaction on the theory.

In the case $G_2=0$, Eq. (40) has, after integration, the form

$$\mathcal{A}^2=1-0.0284G_1^2/4\pi. \quad (65)$$

For this theory there are no difficulties with the θ states, since $A(\mathbf{k})\equiv 1$. From (65) we conclude that the V normalization does not become negative until $G^2/4\pi\approx 35$. In the absence of the pair interaction, the ghost difficulties are evidently far less serious.

The calculated values for the vector part of the anomalous moments are shown in Fig. 2 for the two cases $G_2=G_1$ and $G_2=0$. Figure 3 shows the scalar moments for the same two cases. When the pair interaction is absent the anomalous moments must be proportional to G_1^2, and we have

$$\mu_v=0.0323G_1^2/4\pi, \quad (66a)$$

$$\mu_s=0.0204G_1^2/4\pi. \quad (66b)$$

It is clear from the figures that, in the region $G<G_c$, the pair interaction has little effect on the vector moment and almost none on the scalar moment. This is not surprising in view of the small amplitude of the pair state in this region.[8] The most interesting behavior takes place when $G>G_c$. Here, it is seen, the vector moment is enhanced as a result of the pair interaction, whereas the scalar moment reaches a maximum and

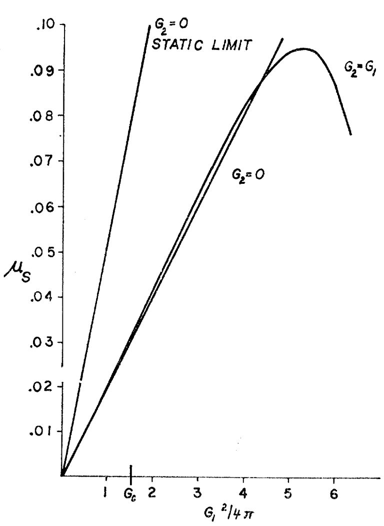

FIG. 3. The scalar part of the anomalous magnetic moment.

[8] However, it should not be concluded that this is still a weak-coupling region, since the pair amplitude, which is of fourth order, is larger than the second-order $|N\theta)$ amplitude.

then decreases. This behavior is caused by the factor $1-G^2F_2(\mathbf{k})$ in Eq. (56), which contributes only to the scalar moment. It is therefore not directly attributable to the states with negative amplitude. But, as already emphasized, the theory is in a dubious state for such large values of the coupling.

It was mentioned in the previous section that, for $G_2=0$, the integrals for the moments converge without a cutoff. The calculated values in this case are

$$\mu_v=0.0592G_1^2/4\pi, \tag{67a}$$

$$\mu_s=0.0635G_1^2/4\pi. \tag{67b}$$

It is worth noting that values of k between the cutoff and infinity contribute appreciably to the integrals. Again, however, it must be remembered that $\mathfrak{A}^2$ is minus infinity for all values of G_1.

We next examine the effect of neglecting recoil. The integrals of course now diverge, even when $G_2=0$. This is the only case we consider since it would be inconsistent to include pairs in such an approximation. In the no-recoil limit, $E=m$, the N and θ contributions to the moment, Eqs. (58), (60), take the forms

$$\mu_N=\left(\frac{G_1^2}{4\pi}\right)\frac{1}{4\pi m^2}\left\langle\frac{3-\tau_3}{2}\right\rangle\int\frac{k^4}{\omega^3}dk, \tag{68}$$

$$\mu_\theta=\left(\frac{G_1^2}{4\pi}\right)\frac{1}{3\pi m}\langle\tau_3\rangle\int\frac{k^4}{\omega^4}dk. \tag{69}$$

The V particle contribution, as before, is the nonanomalous part of the moments. On evaluating the integrals in (68) and (69), one obtains for the vector and scalar moments,

$$\mu_v=0.0570G_1^2/4\pi, \tag{70a}$$

$$\mu_s=0.0530G_1^2/4\pi. \tag{70b}$$

These values, which are also shown in Figs. 2 and 3, are seen to be larger by about a factor of 2 than the "exact" cutoff results (66). This is the kinematic effect of the smaller energies in the denominators of the integrals.[9]

A theory very similar, though not identical, to the preceding is obtained by modifying the Chew-Low-Wick static model Hamiltonian so as to limit the allowed processes to those of Eq. (1). This may be accomplished formally by defining a third kind of spin space (in addition to ordinary and isotopic spin), in which the V particle is represented by the state $\begin{pmatrix}1\\0\end{pmatrix}$, and the N state by $\begin{pmatrix}0\\1\end{pmatrix}$. If we let $\boldsymbol{\eta}$ denote the set of Pauli matrices in this space, then the interaction Hamiltonian can be written

$$H_I=\sum_{\mathbf{k},\alpha}a_\alpha(\mathbf{k})V_{\mathbf{k}\alpha}+a_\alpha^*(\mathbf{k})V_{\mathbf{k}\alpha}^*, \tag{71}$$

where

$$V_{\mathbf{k}\alpha}=ifv(k)(2\Omega\omega)^{-1/2}\boldsymbol{\sigma}\cdot\mathbf{k}\tau_\alpha\eta_+, \tag{72}$$

$$\eta_\pm=(1/\sqrt{2})(\eta_x\pm i\eta_y), \tag{73}$$

and $v(k)$ is the Fourier transform of the source function. It is known that such a Hamiltonian is closely related to the no-recoil, no-pairs limit of (9), with a pseudovector coupling constant related to g by

$$f=g/2m. \tag{74}$$

With Hamiltonian (71) it is straightforward to calculate the V eigenstate, and from it the magnetic moments. It might be expected that the results would be identical to those of the zero-order approximation (67), just discussed. Indeed, the meson contributions to the moments in the two theories do turn out to be identical. However, the contributions from the N particles differ. In the static model the N-particle moment comes only from the spin. In the zero-order approximation to the γ_5 theory, on the other hand, the N particles retain an orbital moment even in the limit $E=m$; that is, putting $E=m$ is not equivalent to neglecting recoil. It turns out in fact that the N-particle moment in the static model (and hence also the scalar moment) is just the negative of (68). The numerical results are then as follows:

$$\mu_v=0.0923G_1^2/4\pi, \tag{75a}$$

$$\mu_s=-0.0530G_1^2/4\pi. \tag{75b}$$

These differ from (70) as well as from the more exact results (66).

In obtaining the static model predictions (75), the step analogous to wave function renormalization has been included. That is, the source contribution to the moment has been multiplied by the ratio of renormalized to unrenormalized coupling constants. This makes it again equal to the nonanomalous moment. Such a procedure seems called for; it was not carried out in previous static model computations of the moments,[10] in which charge renormalization was included. The numerical effect is not insignificant.

It is clear that in our model theory, the analog of the static model gives predictions which are considerably different from the "exact" cutoff results. If the results for the larger coupling constants may be ascribed qualitative significance, the neglect of pairs also causes a considerable change in the behavior of the moments.

One may speculate about the possible implication of these results to the situation in actual meson theory, where the calculation of the anomalous moments in the γ_5 theory remains an unsolved problem. Because of the large value of the coupling constant, no perturbation calculation can be ascribed more than qualitative

[9] A second-order expansion of the energies, in which terms of order $(p/m)^2$ are retained, leads to no noticeable improvement in the calculated values.

[10] F. R. Halpern, L. Sartori, K. Nishimura, and R. Spitzer, Ann. Phys. (New York) 7, 154 (1959).

significance. In the static model, a reasonably accurate physical eigenstate has been constructed,[10] and used to compute the moments. The result, for a reasonable value of the coupling constant, was a vector moment in good agreement with experiment, but a scalar moment which was much too large. Other static model computations have given similar results.

It seems fair to ask whether such results reflect a real failure of pseudoscalar meson theory, or whether the rather drastic assumptions of the static model might be responsible for the lack of agreement. Our results with the Lee model argue mildly in favor of the latter interpretation. Of course, we would not be so rash as to claim that our numerical results have any relevance to the problem of the nucleon moments. However, if the neglect of recoil and of pairs alter the magnetic moment prediction in this model theory, it seems at least plausible that they may likewise do so in a more realistic one.

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Perturbation Theory of Many-Boson Systems*

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A noncanonical transformation of the boson creation and annihilation operators is performed in order to obtain a Hamiltonian which can be treated by the standard methods of field-theoretic perturbation theory. The standard results of Belyaev (with a slight modification) are rederived by this technique.

I. INTRODUCTION

IT has been shown by Belyaev[1] that the many-boson system can be treated by the methods of field-theoretic perturbation theory. His proof of this fact, however, is outside the realm of field-theoretic perturbation theory. The purpose of this paper is an alternative derivation of Belyaev's result, using only standard perturbative techniques. This new derivation is somewhat more exact in its treatment of the zero-momentum state, and a slight correction to Belyaev's result is found. Further light is thrown on the nature of the approximation of large numbers.

II. FORMULATION OF THE PROBLEM

We consider a system consisting of a large number of identical bosons interacting via two-body forces. The units are chosen so that $\hbar = 2m = 1$, where m is the mass of a single boson. Then the Hamiltonian for the system is

$$\mathcal{E} = \sum_{\mathbf{k}} k^2 a_{\mathbf{k}}^\dagger a_{\mathbf{k}} + (2V)^{-1} \sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3\mathbf{k}_4} \times v(\mathbf{k}_1 - \mathbf{k}_3)\delta_{\mathbf{k}_1+\mathbf{k}_2,\mathbf{k}_3+\mathbf{k}_4} a_{\mathbf{k}_1}^\dagger a_{\mathbf{k}_2}^\dagger a_{\mathbf{k}_3} a_{\mathbf{k}_4}. \quad (1)$$

We use script type for operators, except the $a_{\mathbf{k}}$, $a_{\mathbf{k}}^\dagger$, where the $a_{\mathbf{k}}$, $a_{\mathbf{k}}^\dagger$, are the annihilation and creation operators, respectively, for a boson in the single-particle state with wave number $\mathbf{k}$ and wave function $V^{-1/2}e^{i\mathbf{k}\cdot\mathbf{r}}$. V is the volume of the system and only those $\mathbf{k}$'s necessary for completeness of the set of single-particle functions are included, i.e., the $\mathbf{k}$'s satisfying the usual periodic boundary conditions. $v(\mathbf{k})$ is the Fourier transform of the two-body interaction $u(\mathbf{r})$:

$$v(\mathbf{k}) = \int u(\mathbf{r}) e^{i\mathbf{k}\cdot\mathbf{r}} d^3r. \quad (2)$$

In order that $\mathcal{E}$ be Hermitian, we must have

$$v(-\mathbf{k}) = v^*(\mathbf{k}). \quad (3)$$

Since the $a_{\mathbf{k}}$, $a_{\mathbf{k}}^\dagger$ are Bose operators, they obey the commutation relations

$$[a_{\mathbf{k}}^\dagger, a_{\mathbf{k}'}^\dagger] = [a_{\mathbf{k}}, a_{\mathbf{k}'}] = 0, \quad [a_{\mathbf{k}}, a_{\mathbf{k}'}^\dagger] = \delta_{\mathbf{k},\mathbf{k}'}. \quad (4)$$

From these, it follows that $\mathfrak{N}_{\mathbf{k}} = a_{\mathbf{k}}^\dagger a_{\mathbf{k}}$ is the operator for the number of particles in the single-particle state $\mathbf{k}$ and has the eigenvalues $0, 1, 2, \cdots$. The total number operator is

$$\mathfrak{N} = \sum_{\mathbf{k}} \mathfrak{N}_{\mathbf{k}}. \quad (5)$$

$\mathfrak{N}$ and $\mathcal{E}$ commute, so that we can now formulate the problem as follows: find the simultaneous eigenvectors, $|E_i, N\rangle$ of $\mathcal{E}$ and $\mathfrak{N}$ and, in particular, the set of eigen-

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[1] S. T. Belyaev, J. Exptl. Theoret. Phys. (U.S.S.R.) 34, 417 (1958) [translation: Soviet Phys.—JETP 7, 289 (1958)].