Anomalous Thresholds in Unitary Theories*

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Consistency requirements for a Mandelstam representation are shown to imply identical anomalous threshold properties in perturbation theory and any unitary theory without the need to continue in external masses.

THE analytic structure described by the Landau equations has recently been interpreted[1,2] as resulting from the self-consistent iteration in unitary integrals of the normal threshold singularities and single-particle poles in crossed channels. However, in order to make use of results derived in perturbation theory it is necessary to establish an identity of physical sheet properties between perturbation theory and any unitary theory. Such an identity is easily made for the unitary integrals themselves[1] but these simply represent the difference of the scattering amplitude between two of its Riemann sheets, and there is a question of how these singularities are distributed between the two sheets. For anomalous thresholds, it has previously seemed necessary to make a continuation in external masses to study this point.[1] However, unitary singularities possess an intricate interlocking structure and, therefore, it is not unreasonable to expect that an "unnatural" choice of anomalous threshold properties will involve a penalty in the form of unwanted other singularities. It is the purpose of this note to show that this is indeed the case.

We consider the unitary singularities corresponding to the diagram Fig. 1[3] whose leading Landau curve is denoted by Σ. Let us consider the case in which the external masses are such that in perturbation theory there is one singular anomalous threshold in one channel only on the physical sheet. (Other cases follow similar lines of argument.) We wish to compare this with the unnatural case in which no anomalous threshold is singular on the physical sheet. In either circumstance, by continuity, the complex curve Σ is either wholly singular or wholly nonsingular on the physical sheet. We wish to choose the latter alternative. In this case there will be contribution corresponding to this diagram in the Bergman-Weil representation of the scattering amplitude which will be of the pure Mandelstam form, and the boundary of the region in which the spectral function is nonzero must be given by the real arc of Σ lying in the crossed cuts.[4] Moreover, the scattering amplitude must be singular along this curve in the inappropriate limit onto the boundary of the physical sheet. The form of this arc, Γ_1, of Σ is as shown in Fig. 2, where N_1 and N_2 are the normal thresholds and A is the conventionally singular anomalous threshold. However, if A is nonsingular, Γ_1 projects beyond the crossed cuts and is nonsingular in both appropriate and inappropriate limits. This is a contradiction. Thus, the existence of the anomalous threshold on the physical sheet would follow from a more than usually literal application of the principle of maximal analyticity—to rule out the possibility that Σ is wholly singular.

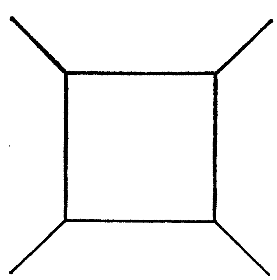

FIG. 1. The Landau diagram considered.

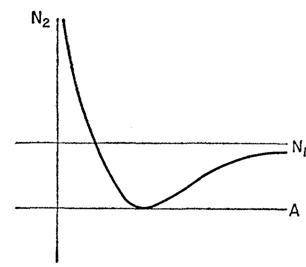

FIG. 2. The form of Γ_1.

In making this argument we have made use of specific knowledge about the form of the fourth-order Landau curve Σ. For more complicated anomalous thresholds, this information is not available; nevertheless it seems reasonable to conjecture that the corresponding necessary properties are, in fact, true. If this is so, one major obstacle to the use of the ideas of analyticity and unitarity in a pure S-matrix theory will have been removed.

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[4] This is because the discontinuity across a normal threshold in one channel does not contain the normal threshold or anomalous threshold singularities in the other channel. When the Mandelstam representation is obtained by two successive applications of Cauchy's theorem, the result follows.